| **Advanced Algorithms** | 10/05, 2018 |
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| Exercise 02 | |
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## 1 2-Approximation for Knapsack (Vazirani 8.2)

In the knapsack problem (discussed in the class), discard all elements that are larger than the budget $B$, and then sort the remaining elements by decreasing ratio of profit to size, let this order be $a_1, a_2, \ldots, a_n$. Let $k$ be the smallest number such that the total size of the first $k$ elements $a_1, a_2, \ldots, a_k$ exceeds the budget $B$. Pick the more profitable of the following two options: $\{a_1, a_2, \ldots, a_{k-1}\}$ and $\{a_k\}$. Prove that this gives a 2 approximation for the most profitable set that fits in the knapsack.

## 2 Next-Fit Algorithm for Bin Packing (Vazirani 9.2)

In the bin packing problem (discussed in the class), consider the algorithm that tries to pack the next item only in the most recently started bin, and if it does not fit, starts a new bin. Prove that this is a 2-approximation, i.e., show that this algorithm uses at most a 2 factor more than the number of bins used by OPT.

## 3 Bin Covering (Vazirani 9.7)

Given $n$ items with sizes $a_1, a_2, \ldots, a_n \in [c, 1]$ for some fixed constant $c \in (0, 1)$, give an Asymptotic Polynomial-Time Approximation Scheme (APTAS) for the problem of maximizing the number of bins, subject to the constraint that each bin has items with total size at least 1.

## 4 Scheduling with Constant Machines (Vazirani 10.2)

Give a Fully Polynomial-Time Approximation Scheme (FPTAS) for the variant of the minimum makespan scheduling problem in which the number $m$ of machines is a fixed constant.