Advanced Algorithms 12/07, 2018

Exercise 11

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1 Optimal Offline Algorithm for Paging

Devise a polynomial-time algorithm for computing the optimal offline solution for the paging problem. Prove its correctness and analyze its time complexity.

2 Online Edge Coloring

Given a fixed set of vertices V, a set of edges $E \subseteq V \times V$ arrive over time and upon arrival of each edge, we should color it with one of colors $\{1, 2, \ldots, q\}$. This is the permanent color of that edge and cannot be changed later. The coloring should be a proper coloring at all times, i.e., no two edges that share an endpoint should receive the same color. Suppose that we are given the guarantee that at all times, the maximum degree of any node is at most Δ. Notice that by Vizing's theorem, the offline algorithm can color the edges using just $\Delta + 1$ colors.

(A) Devise an online algorithm that computes a $(2\Delta - 1)$-edge-coloring.

(B) More interestingly, prove that any deterministic online edge-coloring algorithm requires at least $2\Delta - 1$ colors, i.e., no deterministic online algorithm can get a competitive ratio better than 2.

3 Hungry Cow

Consider the following hungry cow problem—a cow stands on the x-axis at the origin, and is looking for a nice patch of yummy green grass, which it knows exists somewhere on the x-axis at some integer distance $d \geq 1$ either to the left or to the right of the origin. Neither the distance nor the side are known to the cow. Devise an algorithm for the cow that helps it to minimize the distance it needs to travel to get to the food, in the worst case.

(A) Prove that an exponential zigzag strategy provides a 9-competitive algorithm.

(B) Prove that this ratio is best possible for deterministic algorithms.