On the Necessity of Using Delaunay Triangulation Substrate in Greedy Routing Based Networks

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***Abstract*—Large scale decentralized communication systems have motivated a new trend towards online routing where routing decisions are performed based on a limited and localized knowledge of the network. Geometrical greedy routing has been among the simplest and most common online routing schemes. While a geometrical online routing scheme is expected to deliver each packet to the point in the network that is closest to the destination, geometrical greedy routing, when applied over generalized substrate graphs, does not guarantee such delivery as its forwarding decision might deliver packets to a localized minimum instead. This letter investigates the necessary and sufficient conditions of greedy supporting graphs that would guarantee such delivery when used as a greedy routing substrate.**

***Index Terms*—Delaunay Triangulation, Greedy Routing, Geometric Routing, P2P Networks, Wireless Networks**

I. Introduction

DISTRIBUTED routing has become a trend in a variety of communication architectures ranging from ad-hoc and wireless sensor networks to large-scale P2P networks such as Massively Multiuser Virtual Environments (MMVE) and online games. In many cases, complete knowledge about the environment in which routing takes place is not available beforehand, and the path must be determined through a localized routing decision process without having a global knowledge. This family of memory-less routing schemes are generally referred to as online routing algorithms [1].

Among online routing algorithms, greedy routing is one of the simplest and most commonly used. In greedy routing, when a node wants to forward a packet to one of its neighbors as the next routing hop, it tries to choose the neighbor that has the shortest distance (is the closest) to the destination. However, the algorithm only considers the neighbors that are closer to the destination compared to the current node and will retain the packet in this node if it fails to find such a neighbor. Distance may be defined in terms of different criteria. The simplest and most general implementations select the nearest neighbor to the destination in terms of Euclidean distance. While there are other metrics such as Most Forward within Radius (MFR), Nearest Forward Progress (NFP), Geographic Distance Routing (GEDIR), and others, there is a quite similar fundamental operation among these schemes: choosing the next hop from among the neighbors that leads to getting closer to the destination. A comprehensive survey of greedy routing algorithms and the variety of exploited metrics in the context of Ad Hoc Networks is provided in [2] where they have also been compared with other types of position-based routings.

Perhaps the main reason behind the popularity of greedy routing in decentralized systems is that its decision algorithm only requires information about the location of the current node, its immediate neighbors, and the destination node. This significantly reduces implementation complexity, especially in large scale systems where complete knowledge of the network is not easily accessible. However, the local routing decision suffers from a serious shortcoming: the possibility of getting stuck in a local optimum rather than a global one, hence failing to deliver a packet to its intended destination.

To avoid such cases, the routing graph must be chosen carefully. A graph $G = (V, E)$ is considered to support greedy routing *iff* greedy routing on G delivers each and every packet to the member of V that is closest to the destination. Obviously, the packet must be delivered to the exact destination if the destination itself exists as a member in V. According to this definition, greedy routing can also be used for multicasting to the closest neighborhood of a desired point, which is the main goal in geocasting applications, as well as a common operation in many Internet applications such as server selection, node clustering, and peer-to-peer overlay networks [3].

Delaunay Triangulation (DT) has been proven to provide a promising substrate graph for greedy routing: Dobkin et al. [4] showed that the shortest path in Delaunay Triangulation is within a constant time of the shortest path in the complete graph. Moreover, Bose et al. [1] and Lee et al. [3] proved that Delaunay Triangulation supports greedy routing. However, the reliability of greedy routing over other types of substrates has also been a subject of research since it was thought that other graphs could possibly support greedy routing as well. For example, the work in [5] studies a number of possible graphs that fail to guarantee greedy routing convergence over non-DT architectures, and hence it uses a modified version of greedy routing named GPSR (Greedy Perimeter Stateless Routing) that has been enhanced by a "recovery phase" to get out of local minima. Similarly, the work in [6] uses another modified version of greedy routing (GOFAR+: Greedy Other Adaptive Face Routing plus Adaptive Boundary Circle) to avoid this

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Manuscript received August 19, 2009; revised October 17, 2009.

problem, though it does state that doing so will lead to a far less efficient performance. While these and other similar protocols manage to avoid delivery failure, their modification of greedy routing results in increased complexity and reduced efficiency that have been the main motivations behind using greedy routing in the first place. Clearly, to know what graph is needed to support greedy routing (without modifying greedy routing and losing its simplicity and efficiency) would be of great interest to practitioners in this field.

In this article, we discuss this problem in a general case and we prove for the first time that in fact, containing Delaunay Triangulation without degenerate edges as a subset of substrate graph is a necessary and sufficient condition for supporting greedy routing with guaranteed closest-to-destination delivery of packets. In other words, greedy routing will not deliver packets to an incorrect localized minimum *iff* a Delaunay Triangulation substrate without degenerate edges is used. While the sufficiency case has been shown in [1] and [3], the necessity is a new finding that will have significant implications for applications where the routing graph can be selected by the application itself, such as P2P networks, Wireless networks, Ad-hoc networks, MMVEs, etc. For these and other similar applications, we show that it's not just a matter of choice to use or not use DT, but DT must be contained in all greedy routing substrates as a matter of necessity. We start the proof by first presenting some notations and definitions that will be used in the course of this work.

II. Definitions

1) Voronoi Cell: Consider a set of points V, in the Euclidean plane. For each point $v_i \in V$, the Voronoi Cell of v_i, $VC(v_i)$, is defined as the set of all points in the plane that are closer to v_i than any other point in V. It should be noted that according to this definition, Voronoi Cells of the members of V partition the Euclidean plane.

2) Delaunay Triangulation: Presuming a set of points V, Delaunay Triangulation of V, $DT(V)$, is a graph $G = (V, E)$ where $e = (v_i, v_j) \in E$, *iff* $VC(v_i)$ and $VC(v_j)$ have a side(or at least a point) in common. If $VC(v_i)$ and $VC(v_j)$ only share a single point, the edge $e = (v_i, v_j)$ is referred to as a degenerate edge. It is a well-known fact that Delaunay Triangulation is the dual graph of Voronoi diagram. An example of a Voronoi Diagram and its corresponding Delaunay Triangulation is represented in Fig. 1.

3) Vertex Region: Consider a graph $G = (V, E)$ on the set of points V, where for each $v_i \in V$, $N(v_i)$ stands for the neighbor set of v_i. For each $v_i \in V$, we define the Vertex Region of v_i in graph G, $VR_G(v_i)$, as the set of all points in the plane that are closer to v_i than to any other point in $N(v_i)$. Fig. 2 shows the Vertex Region for a node in a sample graph along with the Voronoi diagram of the same set of nodes.

4) Non-Degenerate Delaunay Graph (NDDG): Having a set of points V in the plane, we construct a Graph $G = (V, E)$ on V in a way that E only contains the set of all non-degenerate edges of $DT(V)$. We call this graph NDDG of the set of points.

Remark 1. *For each* $v_i \in V$, *we have* $N(v_i) \subset V$ *and thus,* $VC(v_i) \subset VR_G(v_i)$.

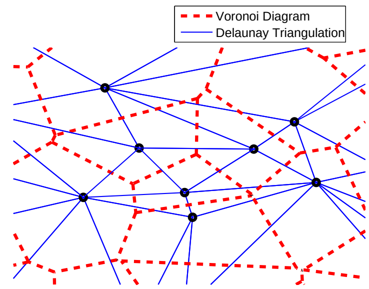

Fig. 1. Voronoi Diagram and Delaunay Triangulation

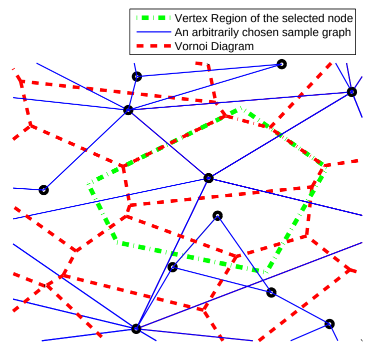

Fig. 2. Vertex Region Definition

III. Theorem and Proof

Before going through the proof of the main theorem, We will prove another theorem that will be later used in the main proof. The following theorem has also been used in [7] by the authors.

Theorem 1. *Graph* $G = (V, E)$ *on the nodes* V *in the Euclidean plane supports greedy routing* iff *for every node* $v_i \in V$, $VR_G(v_i) = VC(v_i)$.

Proof: For the sufficiency condition, suppose that the graph $G = (V, E)$ satisfies the condition $VR_G(v_i) = VC(v_i)$ for each node $v_i \in V$. We can then prove that G supports greedy routing by contradiction: assuming that G does not support greedy routing, there exists a point $v_i \in V$ where a packet might get stuck and cannot be forwarded any further while v_i is not the closest point in V to the packet destination. Assuming that v_i is not the closest point in V to the packet destination, the packet destination should be located outside of the $VC(v_i)$. As $VR_G(v_i) = VC(v_i)$, the destination point should be also located out of $VR_G(v_i)$. Therefore, there should be a point $v_j \in N(v_i)$ that is closer to the packet destination than v_i to which the packet will be forwarded. This is in contradiction with the assumption of the packet getting stuck in v_i.

For the necessity condition, it is assumed that G supports greedy routing. We can then prove that $VR_G(v_i) = VC(v_i)$ for every $v_i \in V$. According to Remark 1, for each $v_i \in V$, $VC(v_i) \subset VR_G(v_i)$. Therefore, it suffices to show that $VR_G(v_i) \subset VC(v_i)$, as well. This can be proven by contradiction. Suppose that there exists a point $x \in VR_G(v_i)$ that does not belong to $VC(v_i)$. Since $x \notin VC(v_i)$ and Voronoi Cells partition the plane, x is in the Voronoi Cell of another node

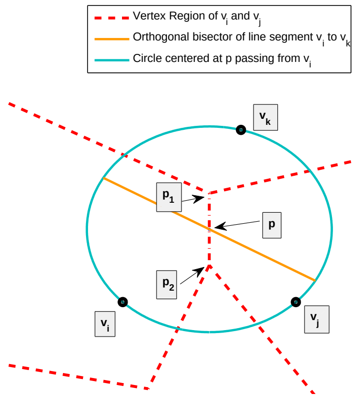

Fig. 3. Proof of Theorem2

v_j. Therefore, v_i is not the closest member of V to x. On the other hand $x \in VR_G(v_i)$, and hence v_i has no closer neighbor to x than itself. Thus, if a packet destined to x reaches v_i or starts in v_i, it cannot be delivered to v_j. Therefore G does not support greedy routing and this is in contradiction with the initial assumption. ■

Theorem 2. *Graph $G = (V, E)$ supports greedy routing* iff *E contains all of the non-degenerate edges of $DT(V)$ as its subset.*

Proof: It is obvious that if E contains all of non-degenerate edges of $DT(V)$ as its subset, we will have $VR_G(v_i) = VC(v_i)$ for every $v_i \in V$ and thus, using Theorem 1, it is concluded that G supports greedy routing.
We prove the necessity condition by contradiction. Suppose that G supports Greedy routing and there exists a non-degenerate edge $e = (v_i, v_j)$ in $DT(V)$ that does not belong to E. As e is a non-degenerate edge of $DT(V)$, $VC(v_i)$ and $VC(v_j)$ should have at least two different points p_1 and p_2 in common. Therefore $p_1, p_2 \in VC(v_i) \cap VC(v_j)$ and $p_1 \neq p_2$. Assume that p is the middle point of the line connecting p_1 to p_2. Since each Voronoi Cell is a convex set, $p \in VC(v_i) \cap VC(v_j)$, and thus p is a boundary point of $VC(v_i)$. As $p \in VC(v_i)$, it can be concluded that $p \in VR_G(v_i)$ according to Remark 1. Thus, p is either an interior or a boundary point of $VR_G(v_i)$. We show that p is an interior point of $VR_G(v_i)$, and thus $VR_G(v_i) \neq VC(v_i)$ which, according to Theorem 1, is in contradiction with the assumption that G supports greedy routing.
If p is assumed to be a boundary point of $VR_G(v_i)$, v_i has a neighbor v_k such that $d(p, v_i) = d(p, v_k)$, where d exhibits the Euclidean distance. Therefore, v_k is on the circle centered at p and passing v_i (Fig. 3). As $v_k \neq v_j$, the orthogonal bisector of the line segment connecting v_i to v_k crosses the orthogonal bisector of the line segment from v_i to v_j (the line passing p_1 and p_2), as demonstrated in Fig. 3. Therefore, points p_1 and p_2 belong to different half-planes constructed by the orthogonal bisector of the line segment from v_i to v_k. Hence, one of these points is strictly closer to v_k than v_i. However, this is in contradiction with the assumption that $p_1, p_2 \in VC(v_i)$. ■

Theorem 2 completely specifies the characteristics of greedy supporting geometric graphs. According to Theorem 2, any graph G on the point set V supports greedy routing *iff* G contains NDDG of V as its sub-graph. Therefore, NDDG of V is the sparsest greedy supporting graph on V. It should be noted that, for all practical purposes in real-world network routing, NDDG will be the same as Delaunay Triangulation. The only exception occurs when the Delaunay Triangulation graph contains a number of degenerate edges, which happens if and only if there are more than 3 points of V on the same circle. In this condition, the unique definition of Delaunay Triangulation fails and some other possible Delaunay Triangulations (without degenerate edges) might exist for the same set of points. However, for each set of points V, NDDG of V is always the common part of all possible Delaunay Triangulations for V.

IV. Conclusion

In this article, we have proven that the use of NDDG as substrate in greedy routing based networks is a necessary and sufficient condition in order to guarantee nearest-to-destination delivery and to avoid getting stuck in a local minimum. The arguments and results obtained here can be applied to any arbitrary greedy routing scheme (such as MFR, NFP, etc.) as our reasoning is only based on the fact that the algorithm wants to find a neighbor as the next hop which is closer to the destination. Closeness can be defined in terms of any arbitrary criterion, but the algorithm's criterion is not considered in our proof and does not affect our conclusions. Greedy routing is now a commonly used algorithm in a variety of applications. However, in many cases, the substrate over which the algorithm has been applied is not DT. In this work we showed that any substrate that does not contain DT as its sub-graph is prone to the risk of getting stuck in local minima. Our findings also imply that when checking the reliability of greedy routing on a given graph, it is sufficient to simply check whether or not it contains the DT graph. This significantly reduces time and effort when designing routing substrates and node configurations.

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