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February 14, 2008
EE 262 – 2D Imaging

Problem Set VI – Imaging the Earth's Subsurface

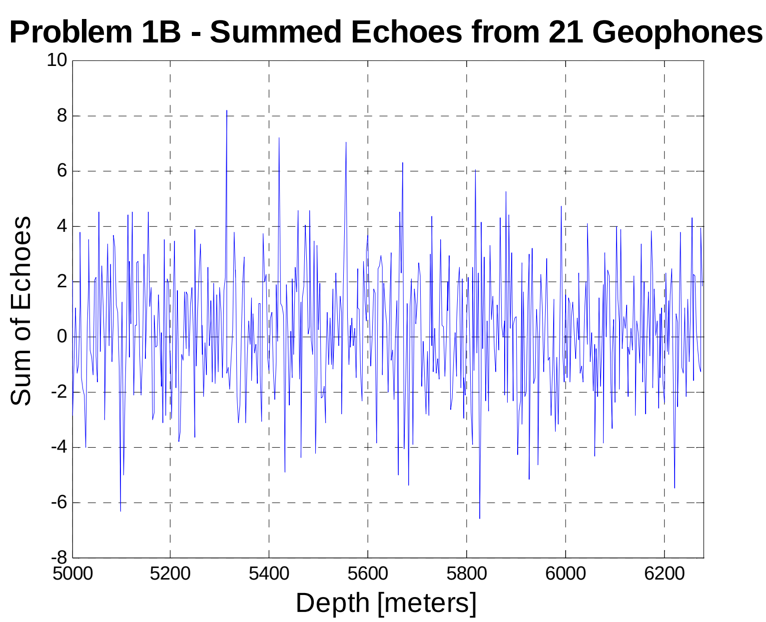
Problem #1 – Geophone Echo Stacking

(a.) The time delay to the first sample of provided geophone data is t_0 = 2 seconds, so we begin computing depth only after 2 seconds have elapsed, incrementing by small time steps thereafter. If we known the velocity of sound in the rocks (5000 m/s), we can then convert our time increments to depth *D* [in meters]:

$$D = \frac{vt}{2} = \frac{v + (t_0 + n \cdot \Delta t)}{2}$$

The depth corresponding to the first sample is therefore $\boldsymbol{D_0 = \frac{vt_0}{2} = 5000\ \text{meters}}$. Because our data comprises 512 samples, we increment time by 511 steps to obtain the depth of the last sample, which we compute to be $\boldsymbol{D_f = \frac{vt_f}{2} = \frac{v + (t_0 + 511 \cdot \Delta t)}{2} = 6277.5\ \text{meters}}$.

(b.) If we stack the echoes from all 21 geophones into a single signal, then we obtain the following signal:



The individual geophone measurements contain too much noise for the sum to reveal distinctly identifiable layers. In other words, the noise peaks cloud our vision of the geophone peaks.

(c.) However, we can improve our measurement sum by properly positioning each geophone's measurement in the stack. In other words, for each line of data we observe, we can approximate its depth according to its arrival time and average the geophone echoes across varying depths rather than line-by-line of our discretized data. For each of our lines, we extract the measurement from

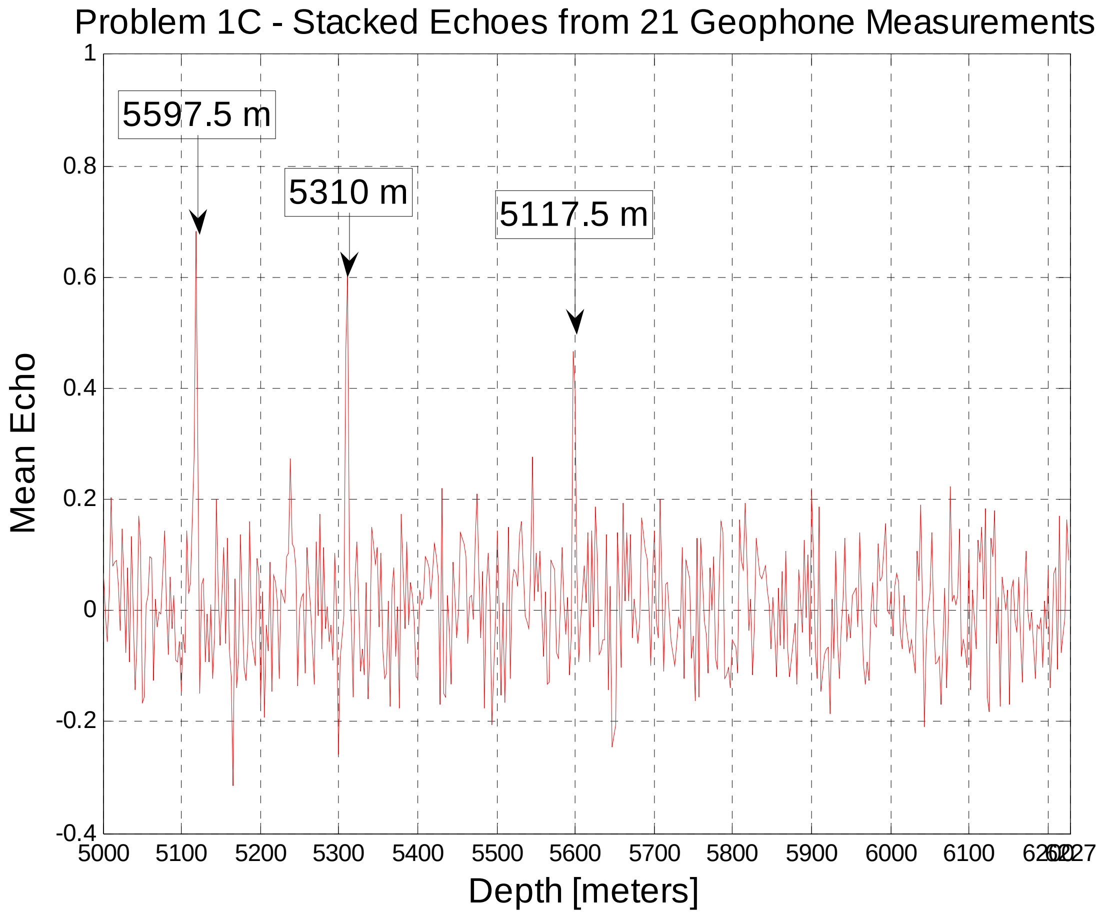
$$index = \left\lfloor \frac{\sqrt{4d^2 + s_i^2} - vt_0}{v\Delta t} + 1 \right\rfloor$$

and average measurements from each geophone $s_i \in [0, 1600]$ as we move from one geophone to the next, incrementing the geophone by spacing of 80 m between each of our 21 measurements.

We use the computed depth vector to consummate our calculations:

$$d_i = \frac{\sqrt{(vt_i)^2 - s_i^2}}{2} \text{ incremented by } \Delta d = \frac{v\Delta t}{2}$$

After compiling our depth vector, we compute a set of indices as outlined above for each of our 21 geophones (each with its own surface position s_i), and average the evaluated measurements to obtain an average with improved signal-to-noise ratio. Effectively, we calculate the depth of each sample and compensate for the hyperbolic trajectory of the sample points in a depth-cognizant *stack* of measurements, in which samples acquired from the same depth add constructively to separate themselves from the noise. The technique proves successful, as our next graph evinces:



From this plot, we can unambiguously discern three distinct peaks despite the noise in the measurements. Unlike our averaged intensity from the previous plot, our new measurement sum has positioned each measurement according to geophone spacing and estimated depth, thereby overcoming the noise with several measurements at the same key depths. Due to our explicit knowledge of geometry, we have modified each time trace so that several echoes coincide at layers of interest, allowing their sum to enhance one another since we sum them at the same depth. We can now identify **three** layers, located at the depths:

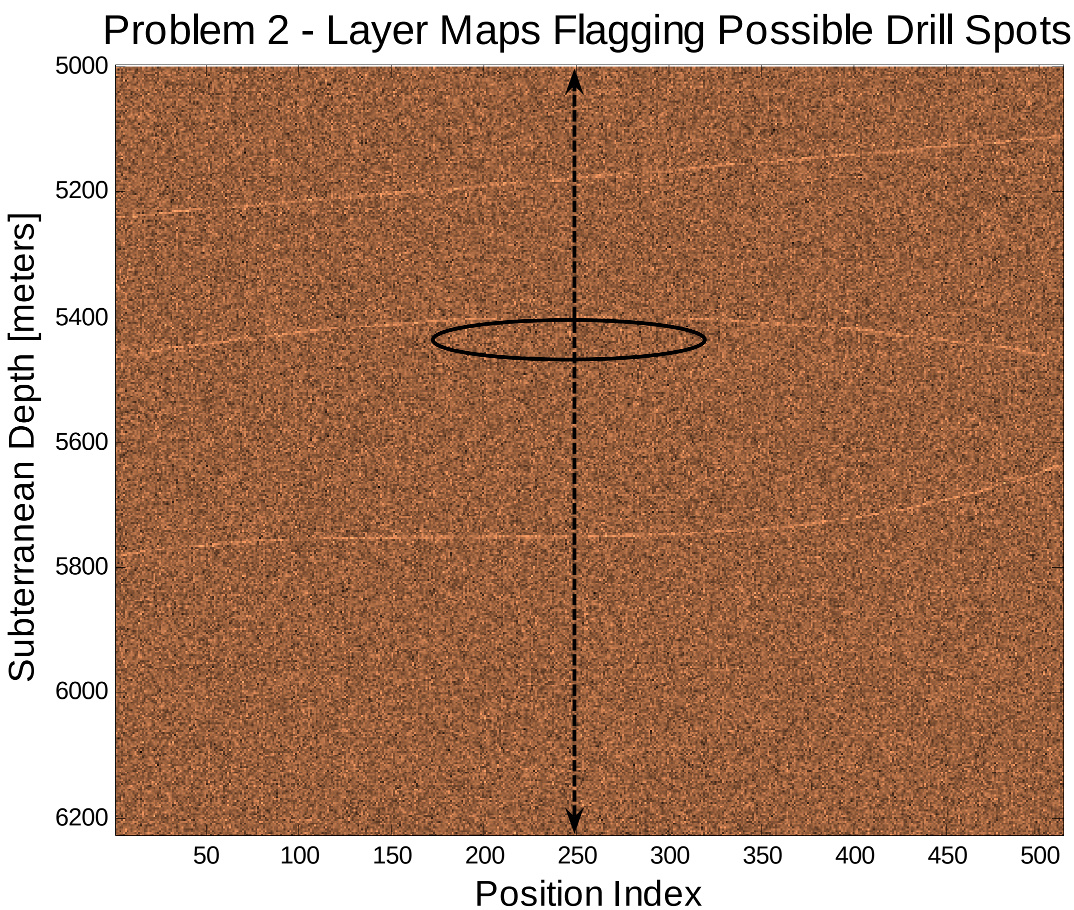
$$\boldsymbol{d_1 = 5117.5\ m}$$

$$\boldsymbol{d_2 = 5310\ m}$$

$$\boldsymbol{d_3 = 5597.5\ m}$$

Problem #2 – More Geophone Echoes

Repeatedly exploiting the stacking technique detailed in Problem 1, we image the depth as a function of position. For each incremental position, we cycle through the 21 geophones, placing each of its measurements at the appropriate depth and accruing an array of measurements at various depths. Concatenating these measurements at the various positions, we obtain the image:



By the portentously concave curvature of the second layer, we suspect that oil resides at the location elliptically encompassed in the image above. Because layers often buckle under tectonic forces, oil tends to migrate upward with a propensity to accumulate under rocks in large concentrations. By targeting our drilling at the surface directly above these concave bulges of concentrated oil, we maximize our chances at locating a reservoir. The downward curvature of the intermediate layer adumbrates the existence of oil around a depth of approximately 5454 meters, so we drill directly above this bulge, around position 250.

Problem #3 – Vibroseis Frequencies

We can manufacture a narrow pulse by accumulating an array of sinusoids at vibroseis frequencies ranging from 1 Hz to 10 Hz in 0.1 Hz increments. Thus, by summing 91 sinusoids of the form: $s_i = \cos(2\pi f_i t)$ for $f_i \in [1\ Hz : 0.01\ Hz : 10\ Hz]$ from $-5\ sec < t < 5\ sec$, we obtain the synthesized pulse waveform pictured below: