Christopher Tsai
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EE 362 – Applied Vision and Image Systems

Tutorial I – Image Formation

Problem #1 – Viewing Geometry

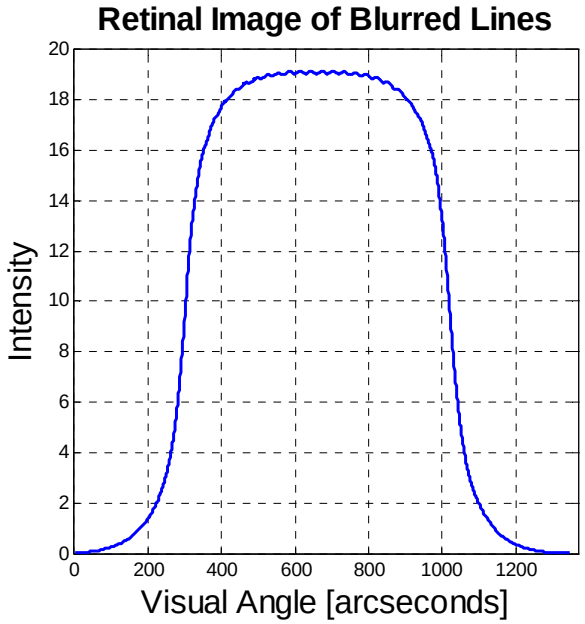
```
function DPI = space2dpi (dotSpacing, viewingDistance)
%
% DPI = SPACE2DPI (DOTSPACING, VIEWINGDISTANCE)
%
% Computes dots-per-inch (dpi) from dot spacing and viewing distance
%
% ------------------------------------------------------------------------
%
% INPUTS:
% -------
% dotSpacing - dot spacing [arcseconds]
% viewingDistance - distance from observer to paper [inches]
%
% OUTPUTS:
% --------
% DPI - dots per inch
%
% ------------------------------------------------------------------------

% Determining the viewing angle in arcseconds:
angleArcsec = 3600*atand(1/viewingDistance);

% Computing DPI from the arcsecond angle:
DPI = angleArcsec/dotSpacing;
```

```
function dotSpacing = dpi2space (DPI, viewingDistance)
%
% DOTSPACING = DPI2SPACE (DPI, VIEWINGDISTANCE)
%
% Computes dot spacing from dots-per-inch (DPI) and viewing distance
%
% ------------------------------------------------------------------------
%
% INPUTS:
% -------
% DPI - dots per inch
% viewingDistance - distance from observer to paper [inches]
%
% OUTPUTS:
% --------
% dotSpacing - dot spacing [arcseconds]
%
% ------------------------------------------------------------------------

% Determining the viewing angle in arcseconds:
angleArcsec = 3600*atand(1/viewingDistance);

% Computing DPI from the arcsecond angle:
dotSpacing = angleArcsec/DPI;
```

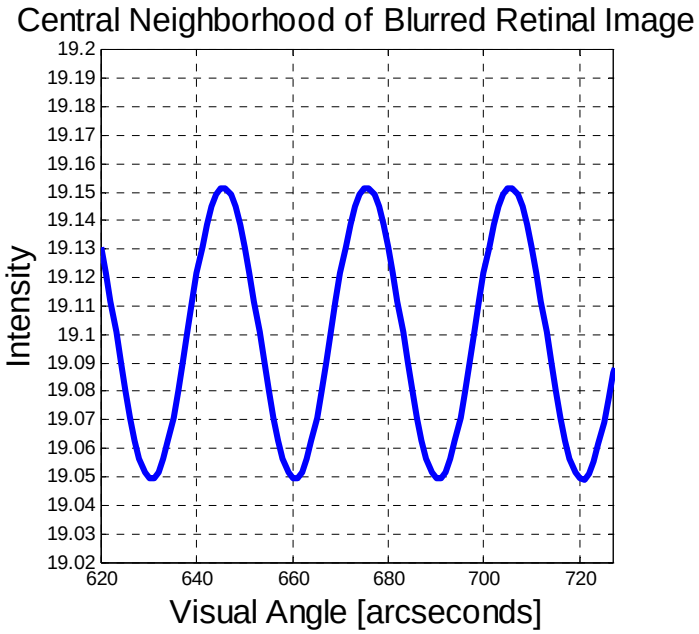
At a viewing distance of 12 inches, one needs a DPI of approximately **2858.185014** dots-per-inch!
At a viewing distance of 36 inches, one needs a DPI of approximately **954.684163** dots-per-inch!

Problem #2 – Line Spread Calculations

First, we peruse the retinal image of the blurred lines:

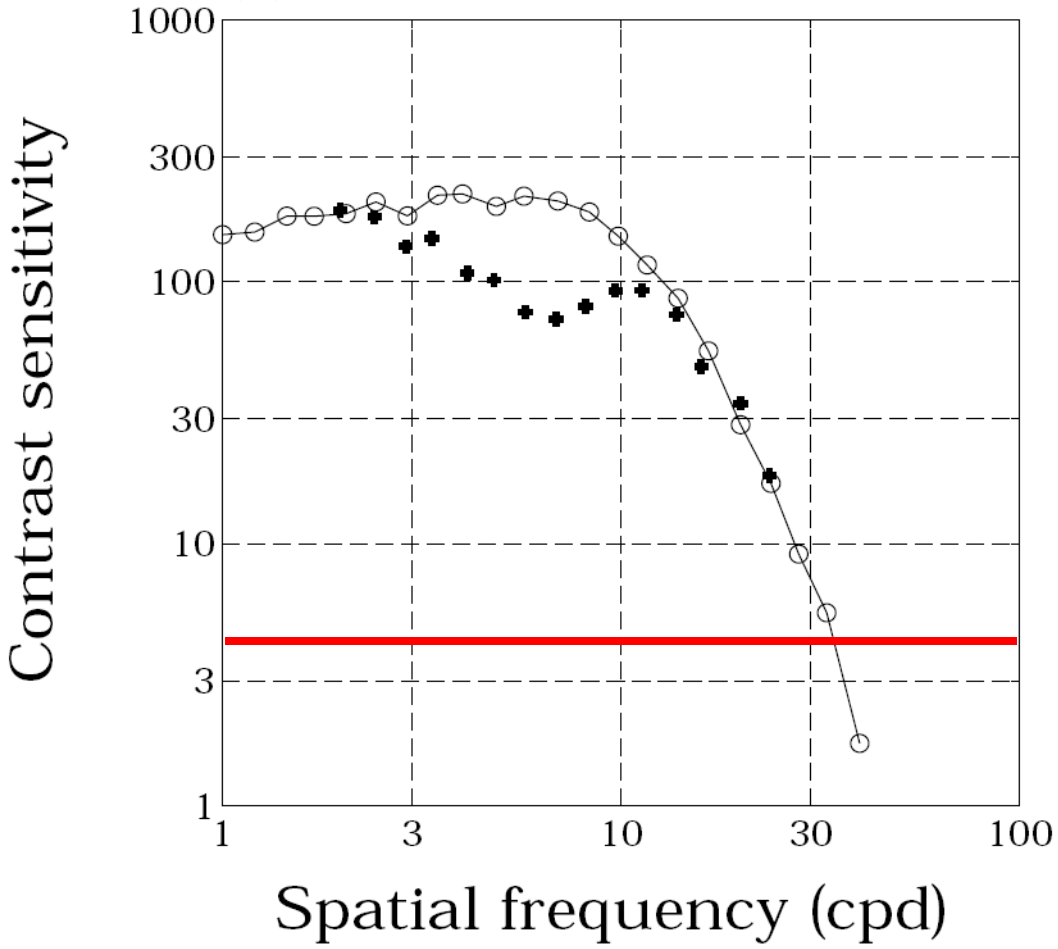


Modulated onto the central region is a sinusoidally varying pattern that we can magnify by zooming to center:



The local contrast of this sinusoidal pattern is $Contrast = \frac{\mathbf{max-min}}{\mathbf{mean}} \approx \frac{\mathbf{19.151-19.049}}{\mathbf{19.097}} \approx \boxed{\mathbf{0.5368\ \%}}$.

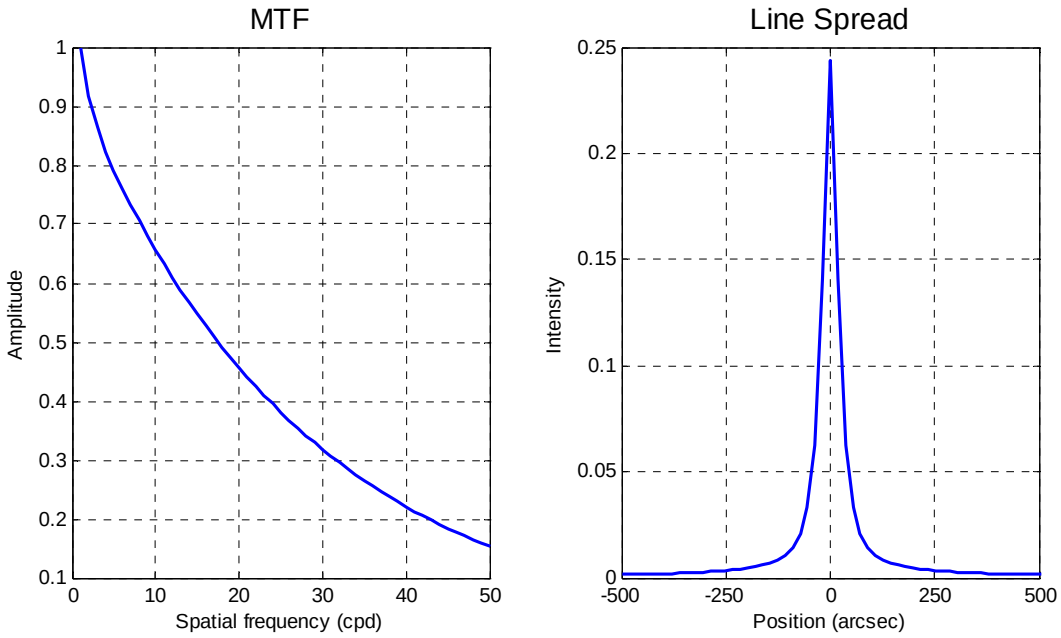
The principal local spatial frequency of this modulated wave is approximately 120 cycles per degree. According to Robson (1966), human sensitivity to contrast varies with spatial frequency. The horizontal (x) axis in this picture has a scale of cycles-per-degree, so our 120 cycles per degree value falls off the right side of the chart; however, noting that even a pattern at 40 cycles-per-degree falls below the $\log\left(\frac{100\%}{0.5368\%}\right) \approx 5.2273$ contrast sensitivity threshold bolded in the graph below, we conclude that moving to the right of the chart (to higher frequencies > 40 cpd) will result only in even lower contrast sensitivity, well below the contrast in the pattern. Thus, the high spatial frequency at the center of the retinal image falls below resolvability, as human sensitivity to contrast plummets below the pattern threshold for all spatial frequencies greater than approximately 8 cpd.



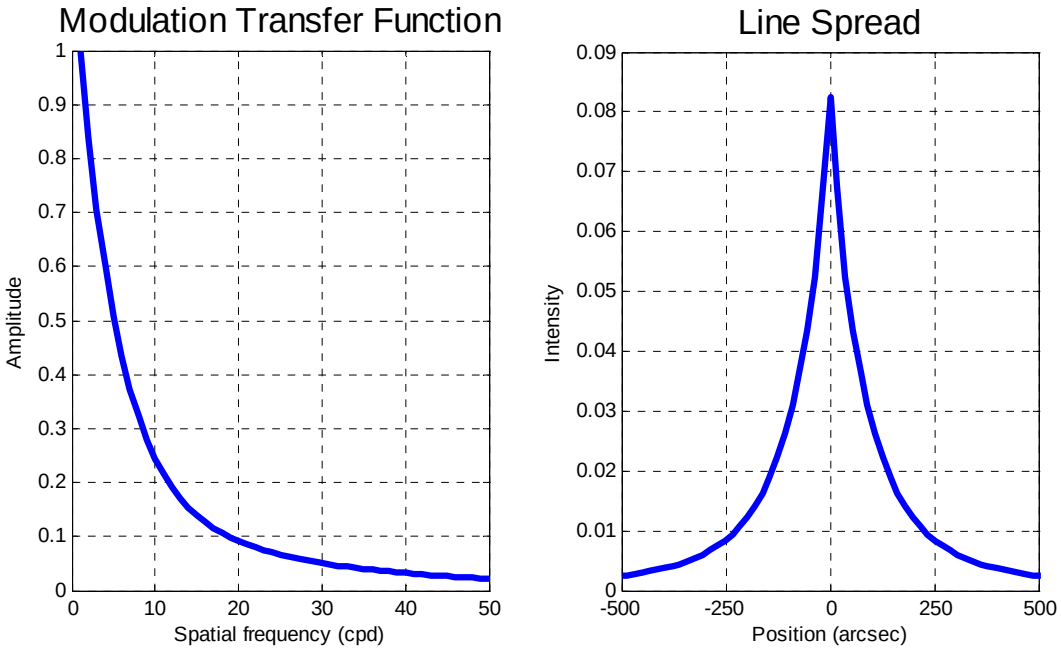
In order to identify this particular pattern, the human eye would need a contrast sensitivity of at least $\log\left(\frac{100\%}{0.5368\%}\right) \approx 5.2273$, but the human eye is unable to achieve such sensitivity at the pattern's high spatial frequency of 120 cpd.

Problem #3 – Modern Linespreads, Pointspreads, and MTFs

As simulated in the tutorial script, the 1.5-mm diameter pupil yields a line spread function and modulation transfer function displayed in the following double plot:



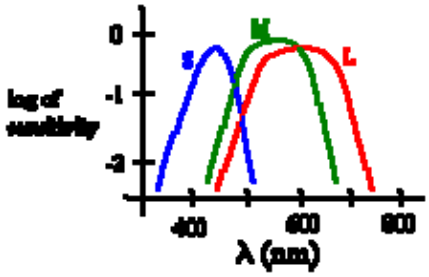
However, if we widen the diameter of the pupil to 8 mm, then the functions change:

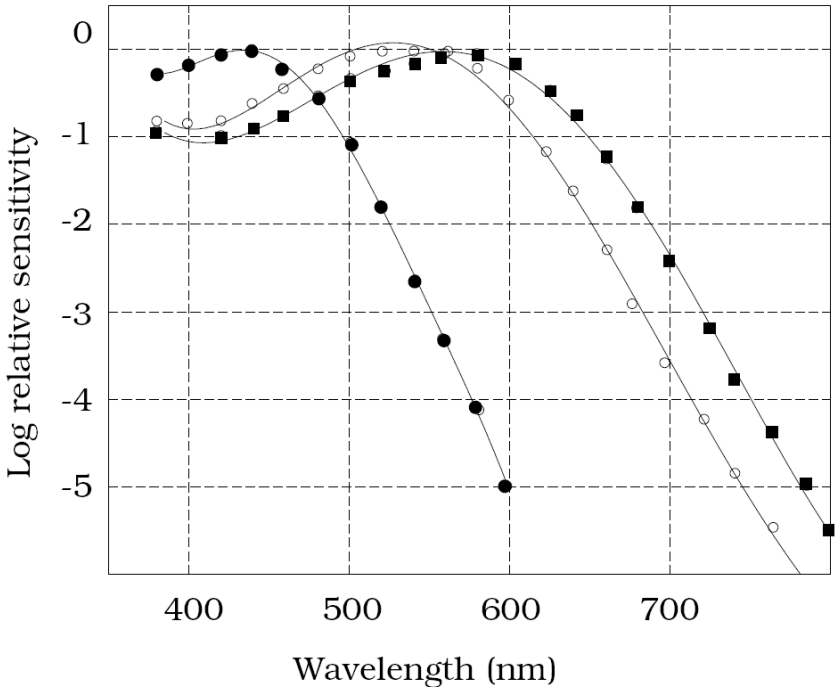


The wider pupil boasts a linespread with a wider but weaker mainlobe, in that the peak intensity has decreased, but the overall function is higher (nonzero) over a wider visual range. Meanwhile, the wider pupil's modulation transfer function filters away the high frequencies even more sharply, with a lower cutoff frequency; intuitively, this sharper and shorter cutoff explains the blurring that we experience when we widen our eyes. When we open our eyes wider, we can physically extend our range of vision slightly and collect more light overall, but the price we pay is decreased focus, or, in the frequency domain, a loss of high-frequency detail and edge perception. Despite the fact that we can still see the general image – the filter is not ideal, and only *attenuates* (rather than removing) higher frequencies – the amount of high-frequency detail perceptible cannot compare to that viewed through the smaller pupil. In essence, widening our eyes exchanges sharp focus and high-frequency detail for a higher signal-to-noise ratio.

All in all, we cannot categorically declare one image *better* than the other in absolute terms; neither image is the optimum, but they each have relative merits. The widened eye collects more light distributed across a larger physical range, but the smaller pupil captures finer detail, admitting higher frequencies of light than its gaping counterpart. However, if we decided to compare images based on sharpness, precision, or representational accuracy, then the smaller pupil forms the better image; though diffraction-limited, the small pupil can resolve greater detail, passing higher spatial frequencies and sampling from the eye's sharpest point – the fovea. Meanwhile, the widened pupil, though abundant in light, nevertheless exposes light to the sides of the eye, where aberrations begin to occur; hence, the frequency response attenuates higher frequency detail, leaving a blurrier, smoothened, lowpass filtered image.

Problem #4 – Chromatic Aberration: Simulation





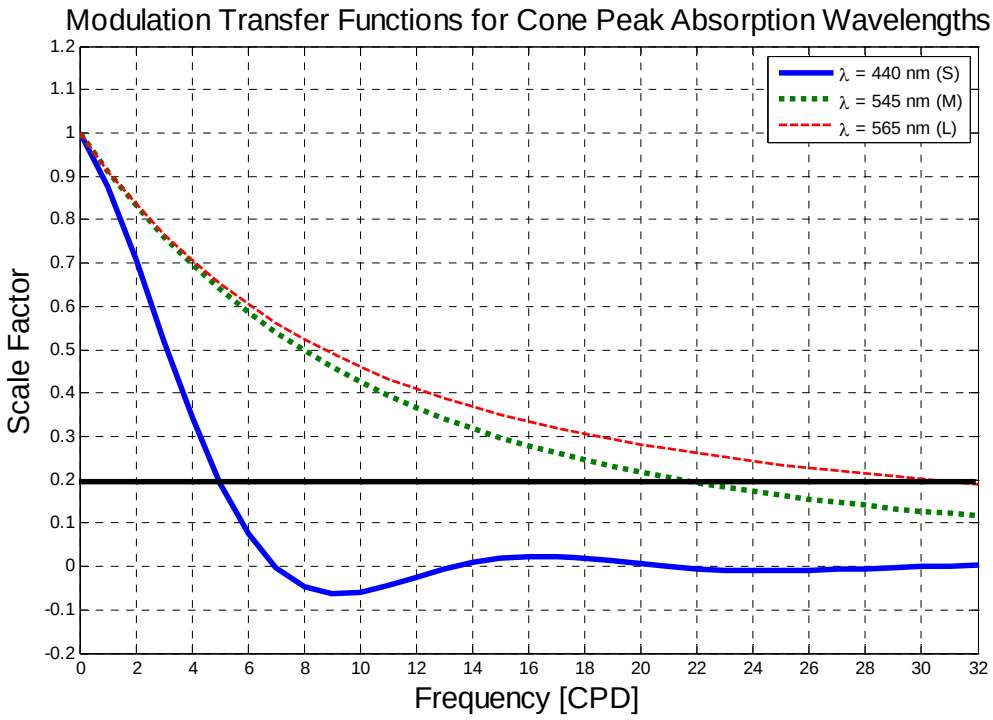
Estimated from page 93 of Brian Wandell's *Foundations of Vision*:

The S-cones exhibit a peak absorption wavelength of approximately 440 nm.

The M-cones absorb wavelengths up to about 545 nm.

The L-cones boast a peak absorption wavelength of around 565 nm.

Assuming that we cannot detect spatial frequencies when the modulation transfer function lies below 0.2, we can estimate the maximum spatial frequency that each cone type can detect:



The solid black line reveals the frequency above which our cones can no longer absorb.

The S-cones stop absorbing photons at frequencies of approximately 5 cycles per degree.

The M-cones cease absorption above about 21 cycles per degree.

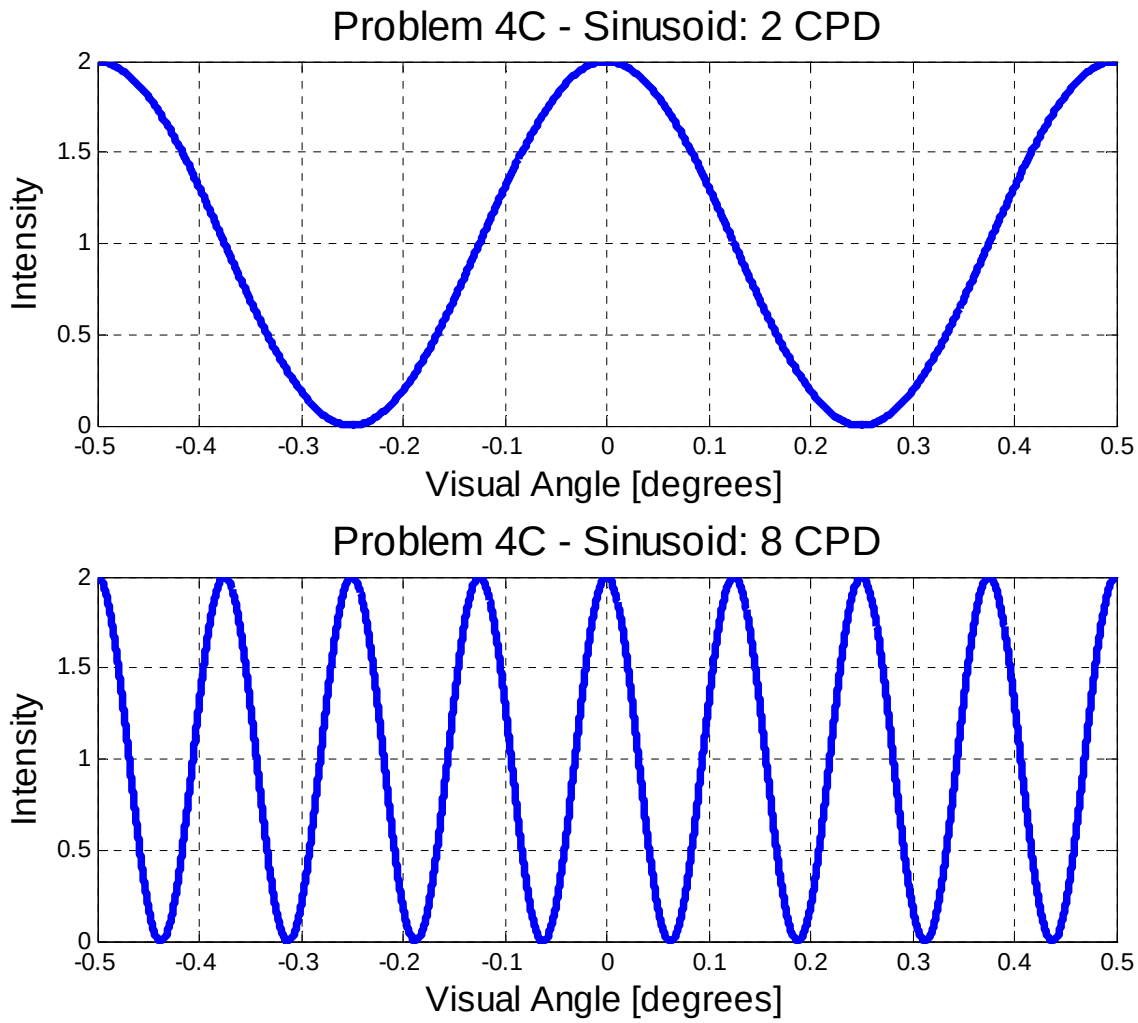
The L-cones absorb light at a maximum spatial frequency around 30 cycles per degree.

Given the highest spatial frequency incident on the retina, we must have enough cones to sample the central degree of the visual field at twice the maximum frequency according to the Nyquist criterion. Thus, with a maximum absorption frequency of approximately 5 cycles per degree, the central degree of the visual field must contain 10 S-cones. Similarly, with a maximum absorption frequency of 21 cycles per degree, the central degree must comprise at least 42 M-cones. Likewise, needing to capture a maximum spatial frequency of 30 cycles per degree, the central degree requires around 60 L-cones.

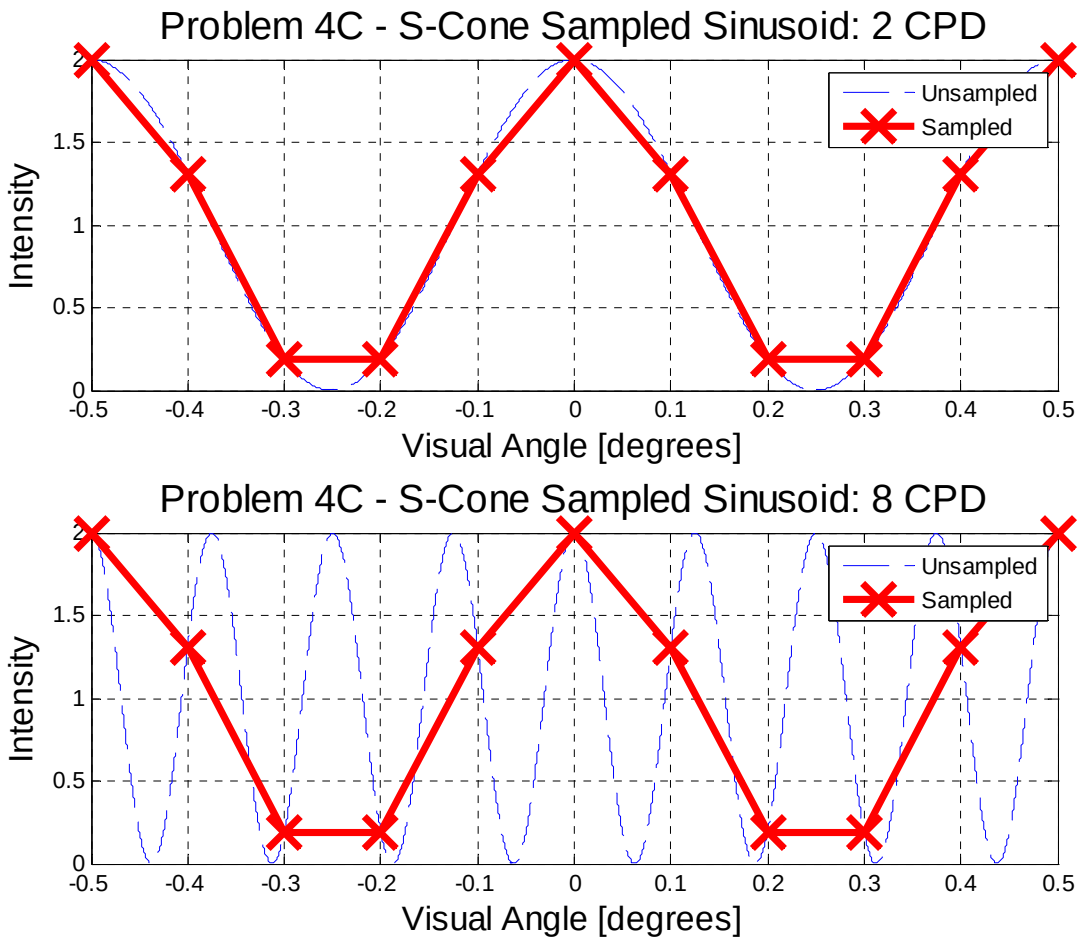
As summarized in Brian Wandell's *Foundations of Vision*, the central degree of visual angle contains approximately 5 – 7 S-cones, falling short but remaining close to our calculated requirement of 10 S-cones. Although their exact spatial arrangement remains recondite, the M and L cones

nevertheless pack tightly together with an approximate separation of 30 arcseconds. Thus, in the central one degree of visual angle, the fovea possesses 60 arcminutes – or 3600 arcseconds – of these densely packed cones. Thus, assuming an equal, uniform distribution of these cones between L-cones and M-cones, we conclude that, within these 3600 arcseconds, one can find an L or M cone every 30 arcseconds, leading to 120 such cones: 60 M-cones, and 60 L-cones. The number of M-cones well exceeds the computed Nyquist criterion, while the L-cone count matches the calculated requirement.

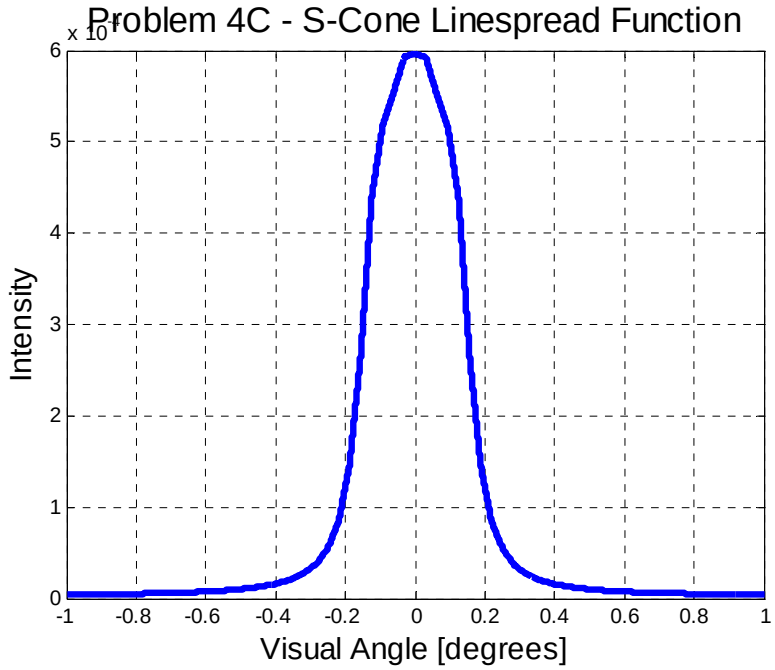
To simulate the sampling of signals at various frequencies, we generate two sinusoids, at 2 cycles per degree and 8 cycles per degree:



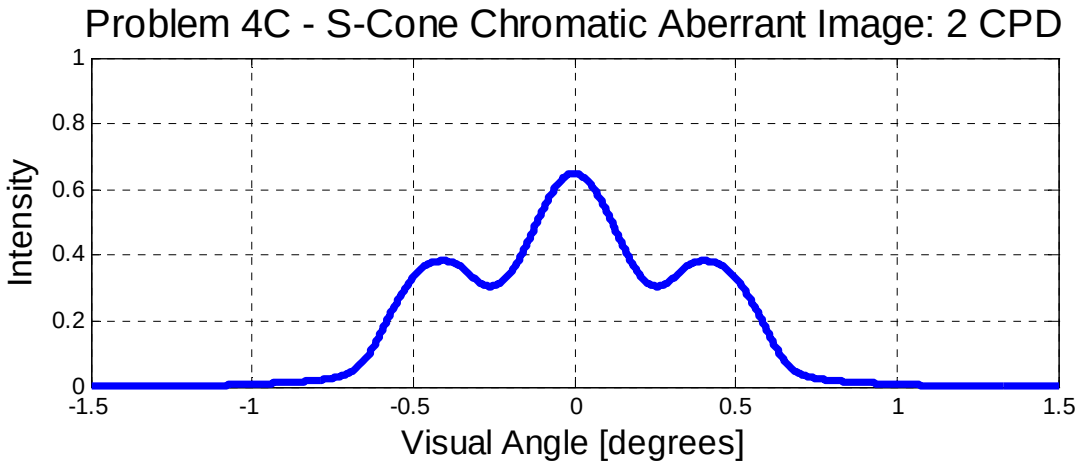
However, if the S-cones sample these signals at 10 cycles per degree, the high-frequency sinusoid will alias into the lower-frequency sinusoid because our cone spacing fails to satisfy the Nyquist criterion:

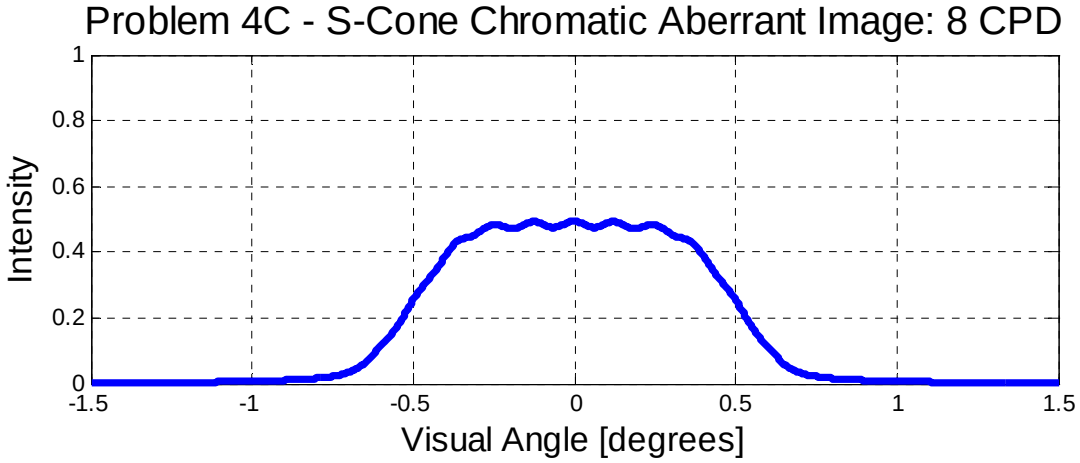


Because the light varies at such a high frequency that many oscillations occur between S cone samplers on the mosaic, the cones interpret the high-frequency 8-CPD sinusoid as a lower-frequency sinusoid. In this case, the two colors, despite their radical difference in frequency, appear identical and are irresolvable to the S-cones. In reality, however, the retinal image blossoms from convolution with the line spread function, whose wavelength-dependent chromatic aberration alters the actual image formation at the retina. We plot the line spread function below:

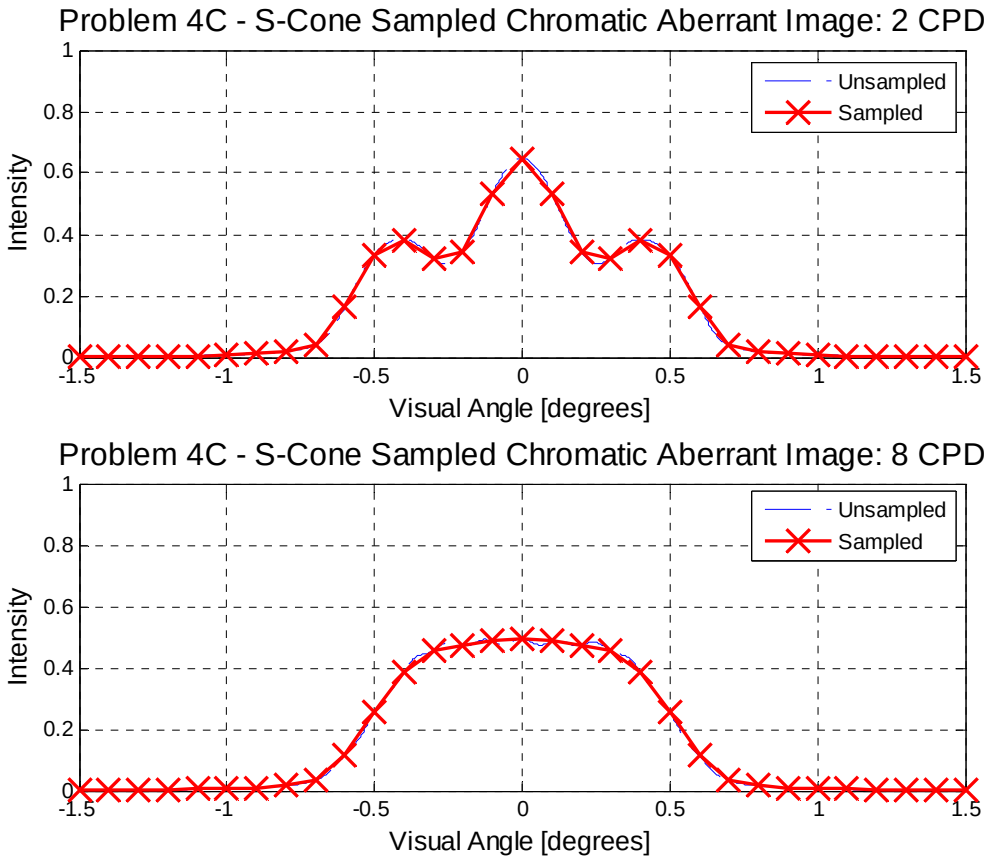


Following convolution with the S-cone line spread function, the retina receives the following signal:





As we quickly observe, the two frequencies – and hence, the two colors – yield dramatically different responses. The S-cones, suited to sampling the lower-frequency sinusoid, actually generate a pattern with noticeable contrast and variation in the central degree, though they are weakened when juxtaposed to the original sinusoidal patterns. On the other hand, the ripples barely protrude from the higher-frequency output, which is rather equally distributed across the central degree. The modulation transfer function has attenuated most of the high-frequency variations that originally defined the 8-CPD sinusoid (by a factor between 0 and -0.1, as shown in the MTF plot), so that the rapid fluctuations have virtually dissipated; meanwhile, the low-frequency characteristic of the S-cone modulation transfer function preserves the 2-CPD sinusoid with an attenuation of only 0.7 and a distinctly oscillatory profile, as the sample values attest:



Whereas an eye without chromatic aberration and its linespread convolution envisions the same intensity at both frequencies, the chromatically aberrant eye perceives two distinct retinal images.