CVX Problem Set III – Convex Optimization Problems

Problem #1 – Optimal Activity Levels

$$x^\star = \begin{bmatrix} 4 \\ 22.5 \\ 31 \\ 1.5 \end{bmatrix} \qquad r^\star = \begin{bmatrix} 12 \\ 32.5 \\ 139 \\ 9 \end{bmatrix} \qquad \bar{p} = \begin{bmatrix} 3 \\ 1.4444 \\ 4.4839 \\ 6 \end{bmatrix} \qquad \boxed{\sum_{j=1}^{4} r_j^\star = 192.5}$$

On average, the third activity level x_3 yields the highest mean price but also generates the highest

revenue of all four activities; meanwhile, the fourth activity level sports, by far, the lowest activity

level since it consumes a copious amount of resources with its exorbitant average price. More than

72% of our ultimate maximized revenue arises from the third activity level, whereas the fourth

activity level generates less than 5% of our total revenue!

Problem #2 – Reformulating Constraints in CVX

(a.)   **The second entry (x–y) is neither concave nor convex!**
```
[x + 2*y, x - y] == 0;
```

(b.)   **The outer square is not monotonically increasing!**
```
(x + y)^4 <= x - y;
```

(c.)   **The reciprocal hyperbola can be concave or convex!**
```
inv_pos(x) + inv_pos(y) <= 1;
x >= 0;
y >= 0;
```

(d.)   **The function composition isn't necessarily increasing!**
```
norm([u v]) <= 3*x + y;
u >= x;
u >= 1;
v >= y;
v >= 2;
```

(e.)   **The quadratic cross-term can be concave or convex!**
```
x >= inv_pos(y); OR y >= inv_pos(x);
x >= 0;
y >= 0;
```

(f.)   **Convex over concave is not, in general, convex!**
```
quad_over_lin (x + y, x - y + 5) <= sqrt(y);
```

(g.)   **Odd powers are both concave (x < 0) & convex (x > 0)!**
```
pow_pos(x,3) + pow_pos(y,3) <= 1; OR norm([x y], 3) <= 1;
x >= 0;
y >= 0;
```

(h.)   **The quadratic cross-term can be concave or convex!**
```
quad_over_lin (x + z - 1, x) <= y - quad_over_lin (z, x);
OR quad_over_lin (x + z - 1, y) <= x - quad_over_lin (z, y);
x >= 0;
y >= 0;
```

Problem #3 – The Illumination Problem

Optimized Quantity	Equal Powers	Least-Squares	Regularized Lst-Squares	Chebyshev Approximation	Convex Optimization
$p_1^\star$	0.3454	1	0.5004	1	1
$p_2^\star$	0.3454	0	0.4777	0.1165	0.2023
$p_3^\star$	0.3454	1	0.0833	0	0
$p_4^\star$	0.3454	0	0.0002	0	0
$p_5^\star$	0.3454	0	0.4561	1	1
$p_6^\star$	0.3454	1	0.4354	0	0
$p_7^\star$	0.3454	0	0.4597	1	1
$p_8^\star$	0.3454	1	0.4307	0.0249	0.1882
$p_9^\star$	0.3454	0	0.4034	0	0
$p_{10}^\star$	0.3454	1	0.4526	1	1
$f_o(p^\star)$	0.467680	0.862784	0.443899	0.419824	0.357474