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EE 364 – Convex Optimization

CVX Problem Set V – Duality

Problem #3 – Log-Optimal Investment Strategy

Even though CVX fails to support the logarithm, we can reformulate our logarithmic objective function if the event probabilities are uniformly distributed:

$$\text{Minimize } \sum_{i=1}^{m} \pi_i \ln(p_i^T x)$$

$$\text{Minimize } \sum_{i=1}^{m} \left(\frac{1}{m}\right) \ln(p_i^T x)$$

$$\text{Minimize } \sum_{i=1}^{m} \ln(p_i^T x)^{\frac{1}{m}}$$

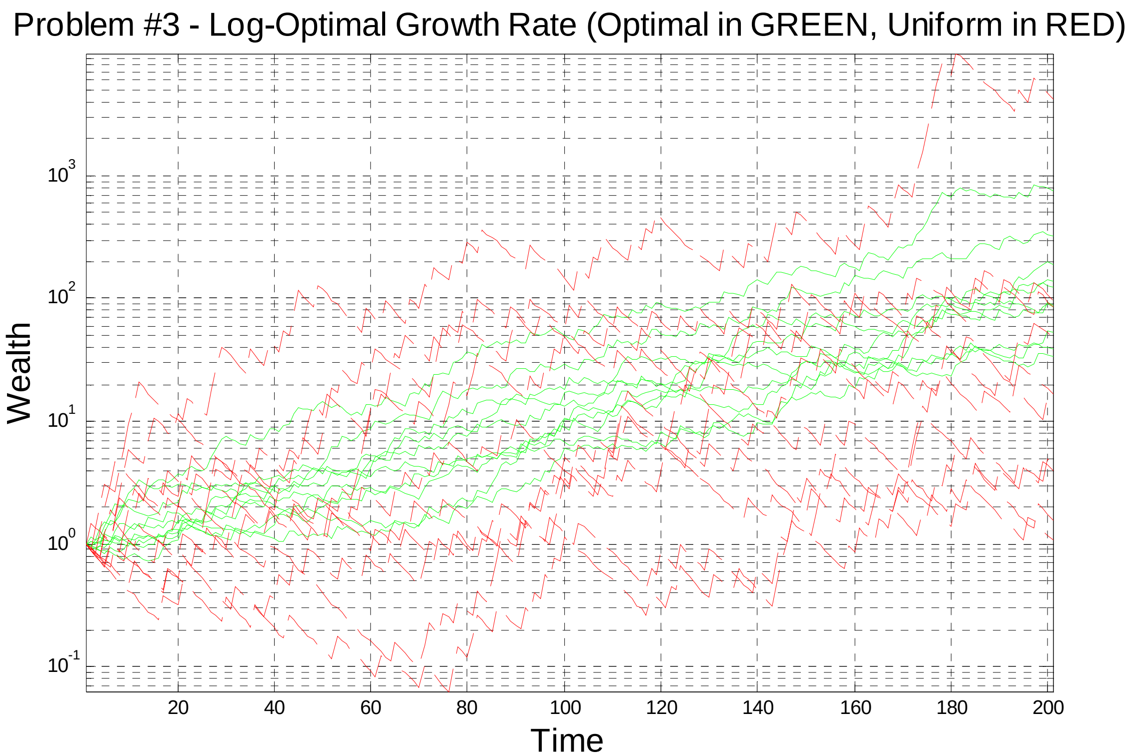
$$\text{Minimize } \ln\left(\prod_{i=1}^{m} \{p_i^T x\}^{\frac{1}{m}}\right)$$

Because the logarithmic function is monotonically increasing, we can reformulate our log-product minimization as the equivalent minimization of its argument:

$$\text{Minimize } \prod_{i=1}^{m} \{p_i^T x\}^{\frac{1}{m}}$$

$$\text{Minimize } \left(\prod_{i=1}^{m} p_i^T x\right)^{\frac{1}{m}}$$

$$\text{Minimize } \mathit{Geometric\ Mean}\ \{p_i^T x\}$$



Optimizing according to the convex optimization problem delineated in Problem 4.60, we obtain the following optimal investment plan:

$$x_{opt} = \begin{bmatrix} 0.0580 \\ 0.4000 \\ 0.2923 \\ 0.2497 \\ 0.0000 \end{bmatrix}$$

Application of this strategy to the market achieves an optimal long-term growth rate of **0.0231**, more than twice the uniform allocation growth rate of **0.0114**.

The optimal strategy solution reveals that we invest the majority of our current wealth in the second asset, which represents a compromise between the high risk and reward of asset 1 and the conservatively certain low reward of asset 5. Because the fifth element of our optimal solution is virtually zero, we eschew the safe asset, most likely due to its extremely listless growth (only 1% per period!). Meanwhile, the high-risk plan offers 3.5 times growth, but only 20% of the time, with loss of 50% in the remaining 80%. Thus, our strategy strives for the intermediate solutions, with a more even distribution of growth at higher than 1% rates.

Problem #6 – Heuristic Suboptimal Solution for Boolean LP

We obtain the following solution to the relaxed LP:

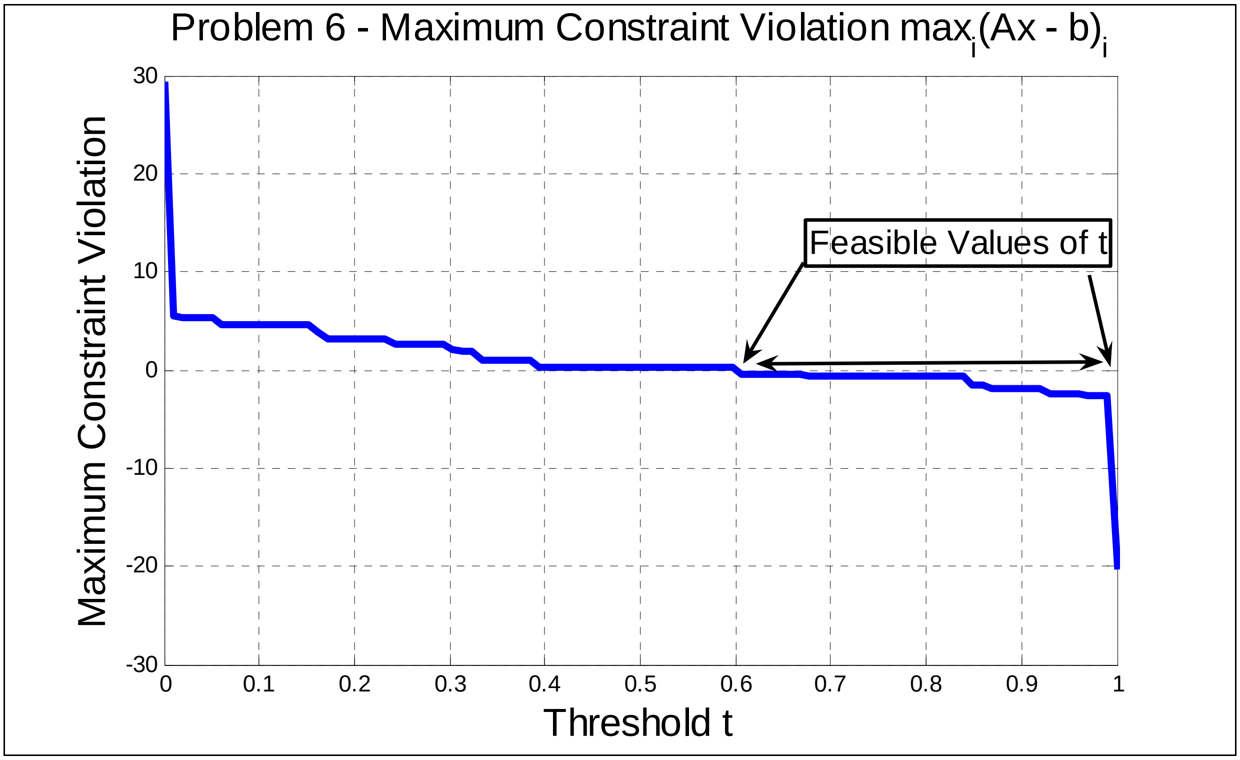
```
x =

    0.0000
    1.0000
    0.0000
    0.0000
    1.0000
    0.9198
    0.0000
    0.0000
    1.0000
    0.0000
    0.0000
    0.0000
    1.0000
    1.0000
    1.0000
    0.8434
    0.3050
    0.6691
    1.0000
    0.0000
    0.0000
    0.0000
    0.1714
    0.8653
    0.0000
    0.0000
    0.7634
    0.0089
    1.0000
    0.0000
    1.0000
    0.0000
    1.0000
```

```
    1.0000
    1.0000
    0.0000
    0.0000
    0.0000
    0.0185
    0.0000
    1.0000
    1.0000
    1.0000
    0.0000
    0.0000
    1.0000
    0.3285
    0.0000
    1.0000
    0.0000
    0.0000
    0.0000
    1.0000
    1.0000
    0.0000
    0.0508
    0.9607
    0.0000
    0.0000
    1.0000
    1.0000
    1.0000
    1.0000
    0.0000
    1.0000
    1.0000
    1.0000
    1.0000
```

```
    0.0000
    1.0000
    0.0000
    0.0000
    0.2378
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    1.0000
    1.0000
    0.0000
    0.1578
    1.0000
    1.0000
    1.0000
    0.0000
    1.0000
    0.3000
    1.0000
    1.0000
    0.0000
    1.0000
    0.3851
    1.0000
    0.0000
    0.6006
    0.0000
    0.0000
    0.0000
    0.0000
```

Most of the optimizing values of our solution vector are zeros or ones, indicating that our solution is nearly feasible for even the unrelaxed problem. Thus, rounding should work well, since thresholding the few decimal values that permeate the true relaxed optimum should not alter our objective function by a drastic amount. Indeed, when we explore a variety of rounding thresholds, we find that a threshold of approximately $t = 0.6061$ yields feasibility in our approximation of the optimal solution. We obtain an associated upper bound to our optimal value $p^\star$ of approximately $p^\star \leq -32.4450$ and an optimally established lower bound of about $p^\star \geq -33.1672$. The gap between bounds is only $U - L \approx 0.72222...$.



We can judge the feasibility of our thresholded optimization problem by checking the sign of our violation function; if the maximum violation is negative, then our thresheld point is feasible in the original problem, because the first linear inequality constraint is negative, as prescribed. Similarly, our objective is then valid over the same values of *t*. We choose the lowest, at $t \approx 0.61$.