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EE 398 – Image Compression

Problem Set V – Transform Coding

Problem #1 – Blockwise 8 × 8 Discrete Cosine Transform

For an 8 × 8 Discrete Cosine Transform (DCT) matrix, M = 8. We compute each entry as follows:

$$[A]_{mn} = \alpha_m \cos\left(\frac{m\pi(2n+1)}{2M}\right)$$

$$\alpha_m = \begin{cases} \sqrt{\frac{1}{M}} \text{ for } m = 0 \\ \sqrt{\frac{2}{M}} \text{ for } m > 0 \end{cases}$$

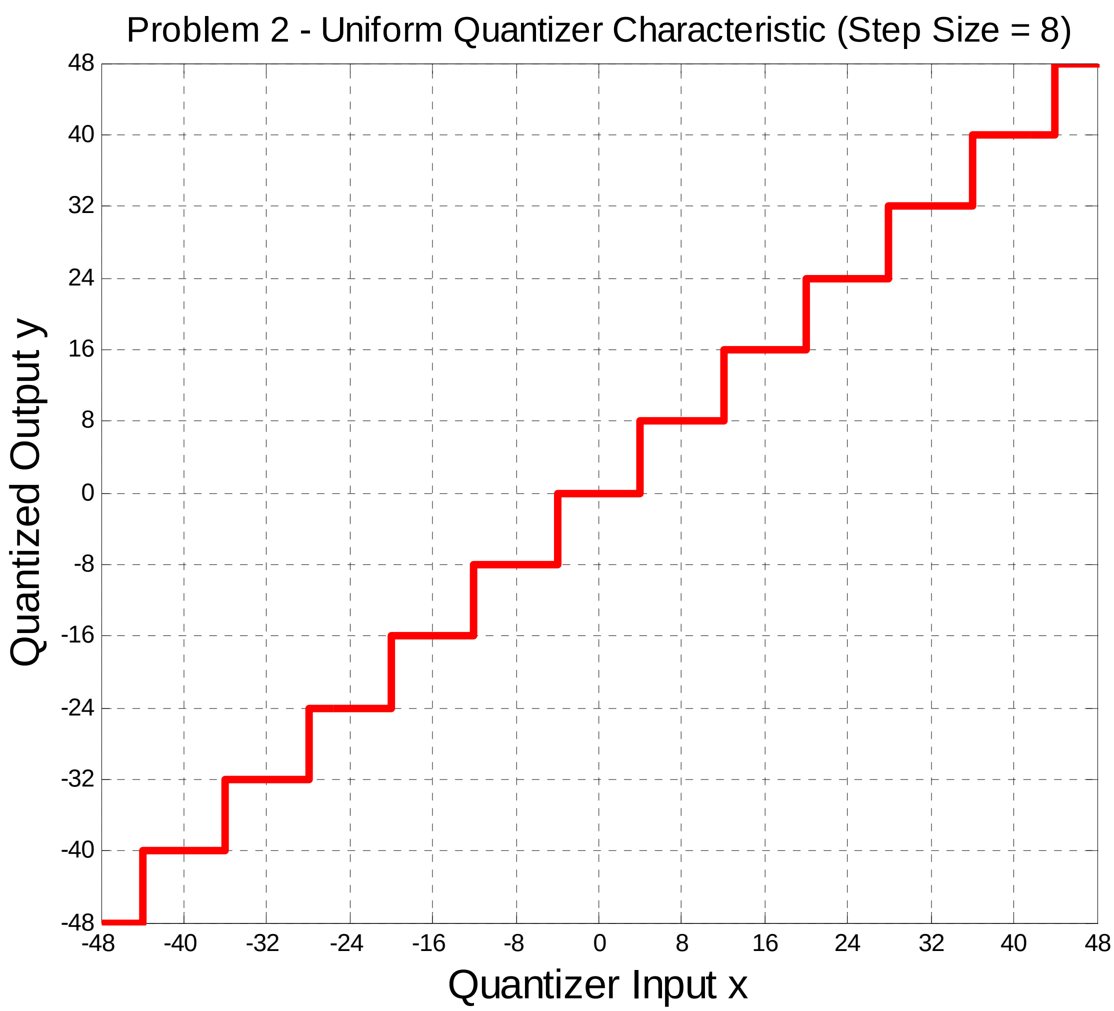
For M = 8, the transform matrix A has contents:

```
A =

    0.3536    0.3536    0.3536    0.3536    0.3536    0.3536    0.3536    0.3536

    0.4904    0.4157    0.2778    0.0975   -0.0975   -0.2778   -0.4157   -0.4904

    0.4619    0.1913   -0.1913   -0.4619   -0.4619   -0.1913    0.1913    0.4619

    0.4157   -0.0975   -0.4904   -0.2778    0.2778    0.4904    0.0975   -0.4157

    0.3536   -0.3536   -0.3536    0.3536    0.3536   -0.3536   -0.3536    0.3536

    0.2778   -0.4904    0.0975    0.4157   -0.4157   -0.0975    0.4904   -0.2778

    0.1913   -0.4619    0.4619   -0.1913   -0.1913    0.4619   -0.4619    0.1913

    0.0975   -0.2778    0.4157   -0.4904    0.4904   -0.4157    0.2778   -0.0975
```

Problem #2 – Uniform Quantizer

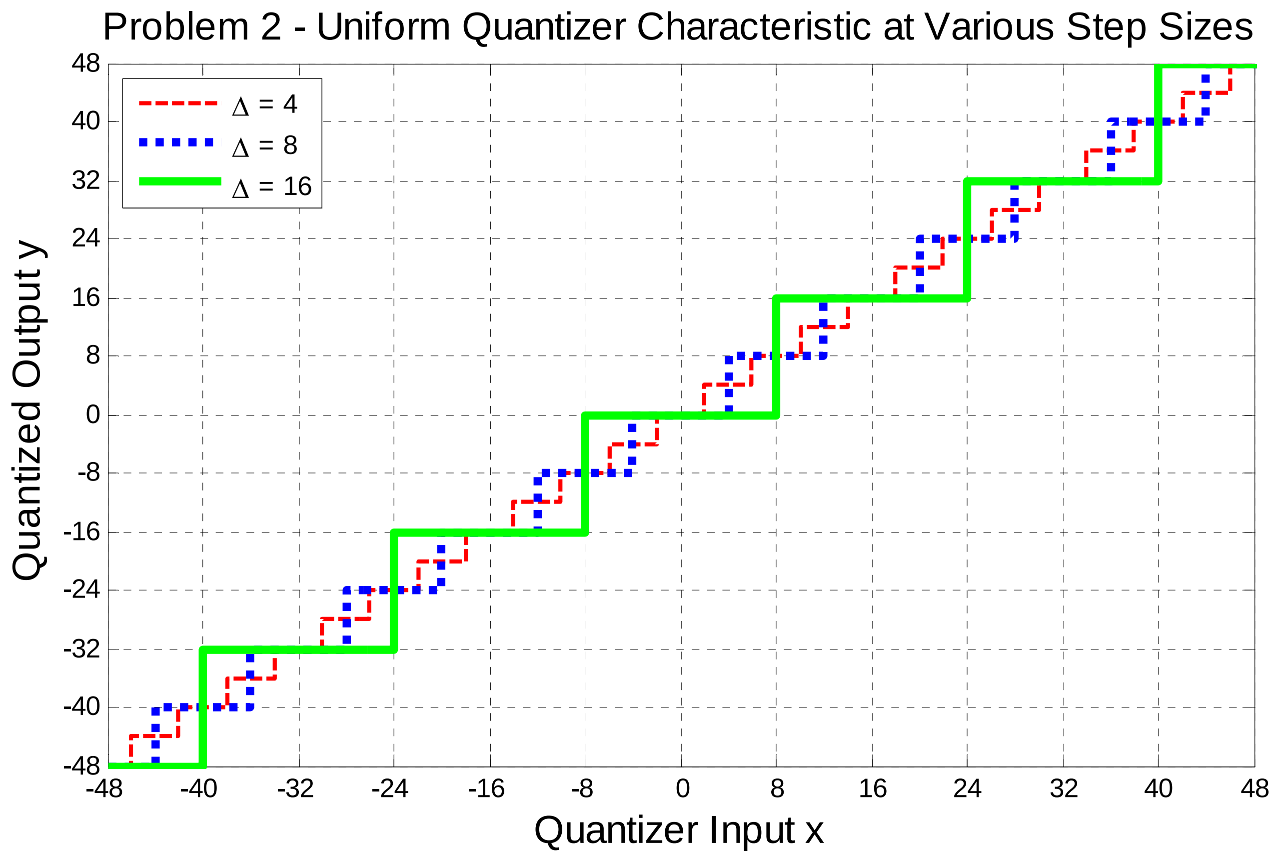
We can quantize the coefficients using a mid-tread uniform quantizer without threshold characteristic. Configuring our quantization to step by chunks of a specified size, we essentially round all values in a single step-sized interval to assume the representative central interval value. To implement this type of quantizer, we can simply divide a value by the desired step size and round off any residue before multiplying back the step size: `quantized = round(input/delta) * delta;`

For a quantization level step size of 8 and inputs of various quantities, we can calculate and plot the uniform quantizer output characteristic, which we display below:



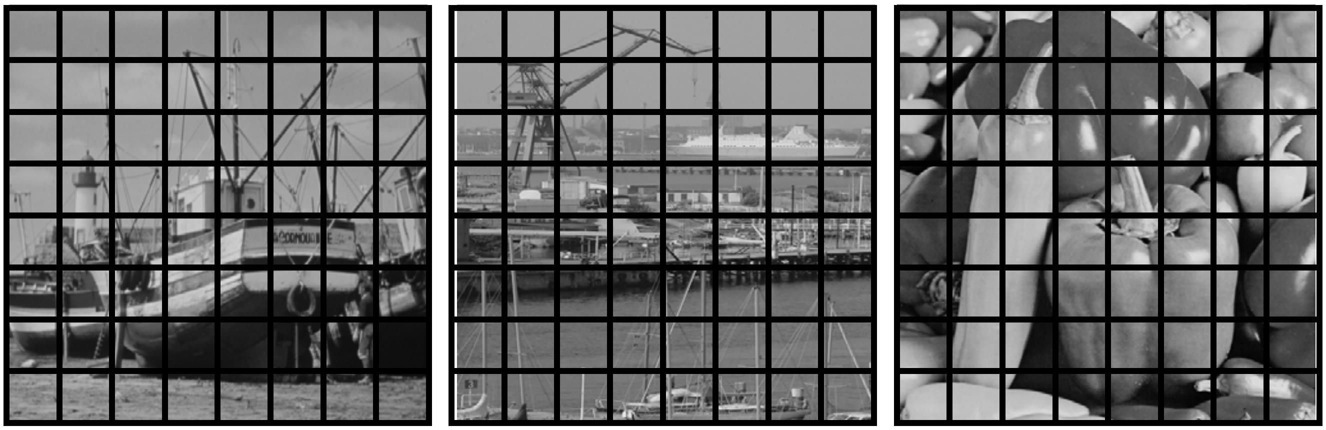
Notice how the steps are centered about multiples of `delta` = 8 along the input axis.

Varying the step size, we can increase or decrease the resolution or coarseness of our quantizer. At all sizes, the intervals are centered about multiples of the step size:



Problem #3 – Distortion and Bit-Rate Estimation

Prior to uniform quantization (at a variety of step sizes), we block our images and stack the blocks:



We encode coefficients in the same cell across all blocks with the same variable-length code, but we encode different cells differently throughout the block. Thus, in order to realize the optimal code for each cell, we must iterate across the 64 pixels in a block, computing the block entropy to determine the rate we can achieve with a given quantization step size. Varying the quantizer step size over the range: $\Delta \in \{1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128,\ 256,\ 512\}$, our blockwise DCT coefficient quantization suffers more and more distortion as we quantize more and more coarsely: