Christopher Tsai
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EE 398 – Image Compression

Problem Set VI – The Wavelet Transform

Problem #1 – Reversible Haar Wavelet Transform

[Shifted] Analysis Filter:

n	$h_0[n]$	$h_1[n]$
1	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
2	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

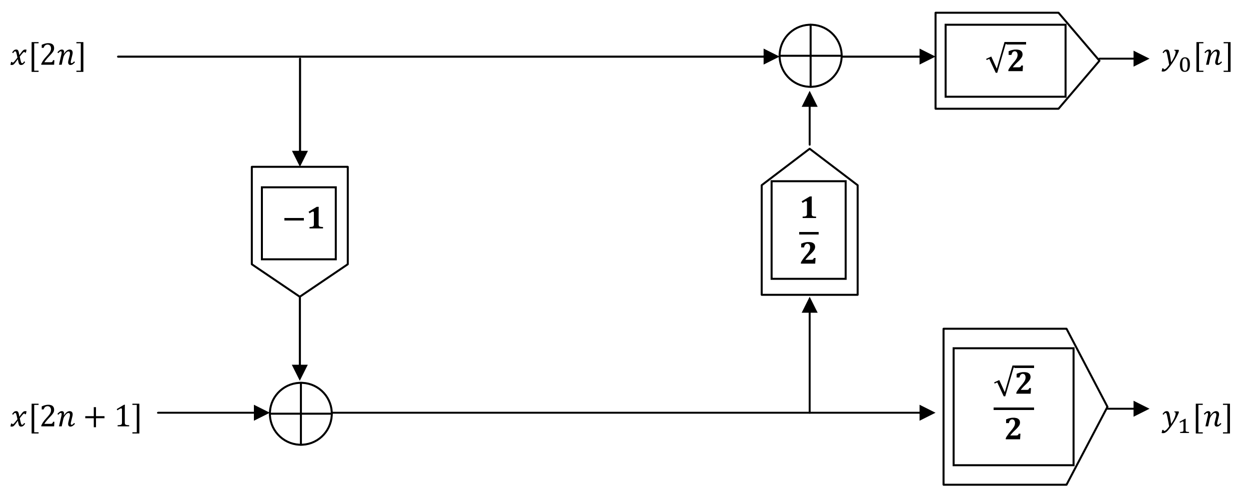
[Shifted] Synthesis Filter:

$$g_0[n] = h_0[-n] = h_0[n]$$

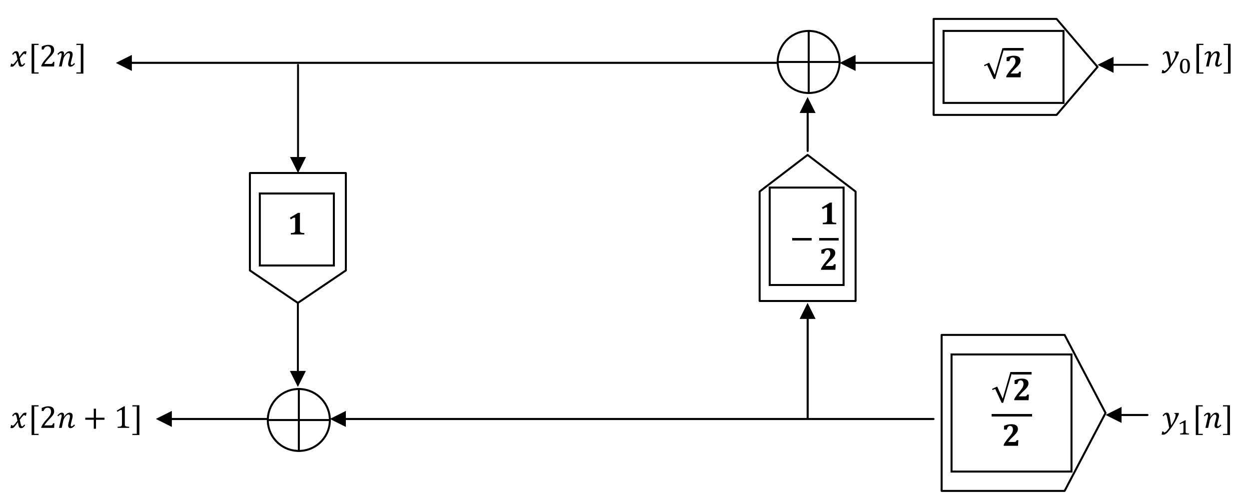
$$g_1[n] = h_1[-n] = -h_1[n]$$

n	$g_0[n]$	$g_1[n]$
1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
2	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$

When modified to a reversible transform, the Haar wavelet transform becomes the S-transform. We implement it in a lifting structure. For **analysis**, our diagram flows rightward:

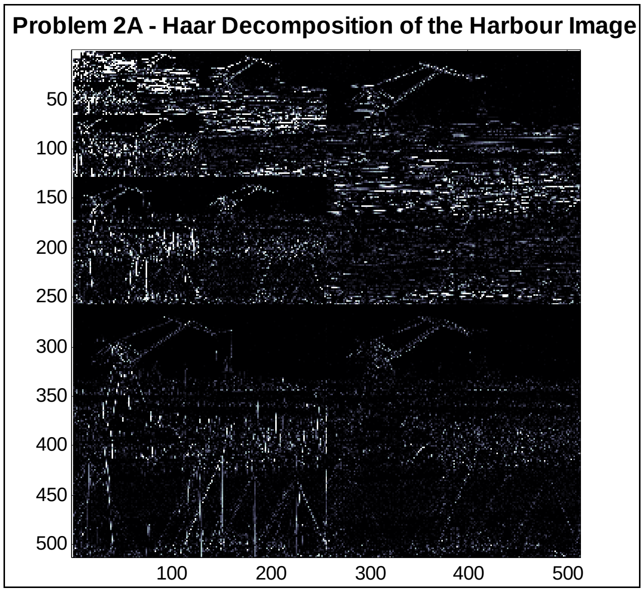


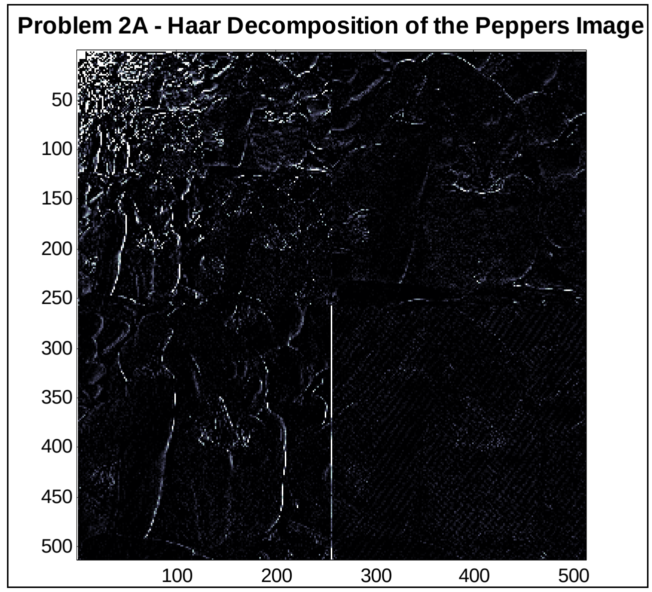
Likewise, we can simply reverse the block diagram to perform **synthesis**:



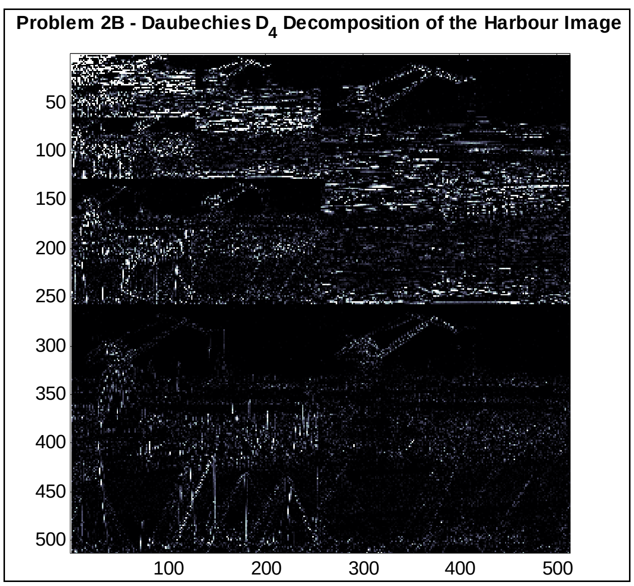
Problem #2 – Coding Gain of the Wavelet Transform

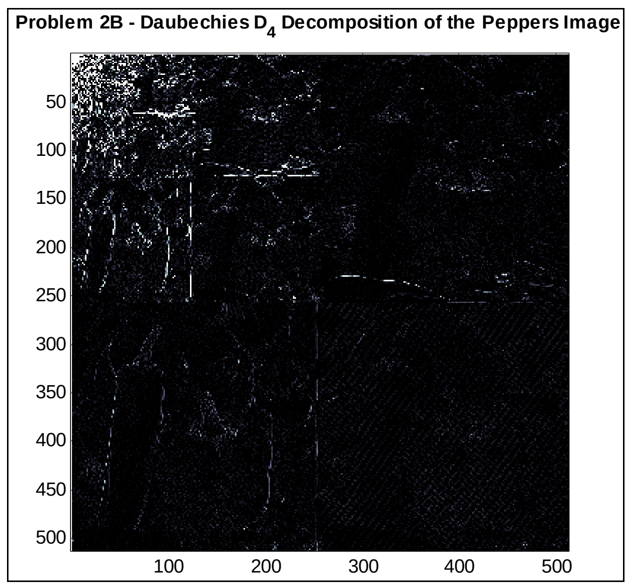
We first employ the Haar wavelet in our transform, resulting in the following coefficient images:





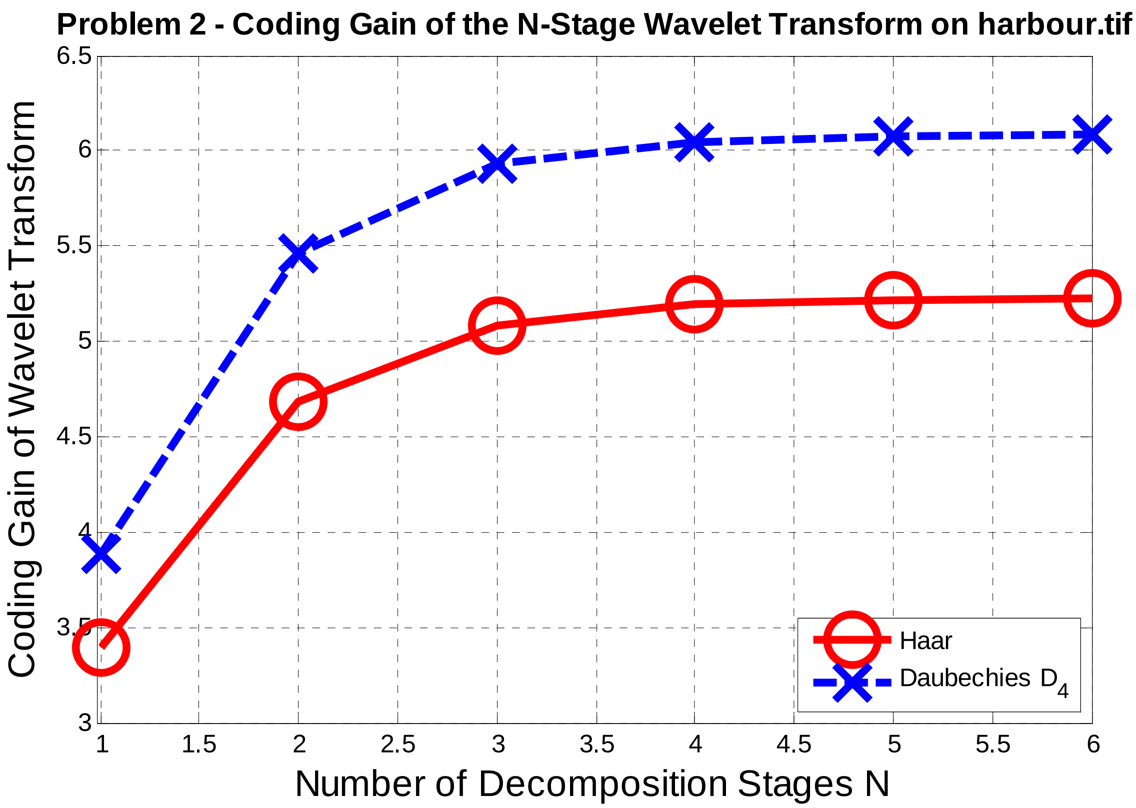
Similarly, we can decompose 'harbour.tif' and 'peppers.tif' with the Daubechies D_4 wavelet:





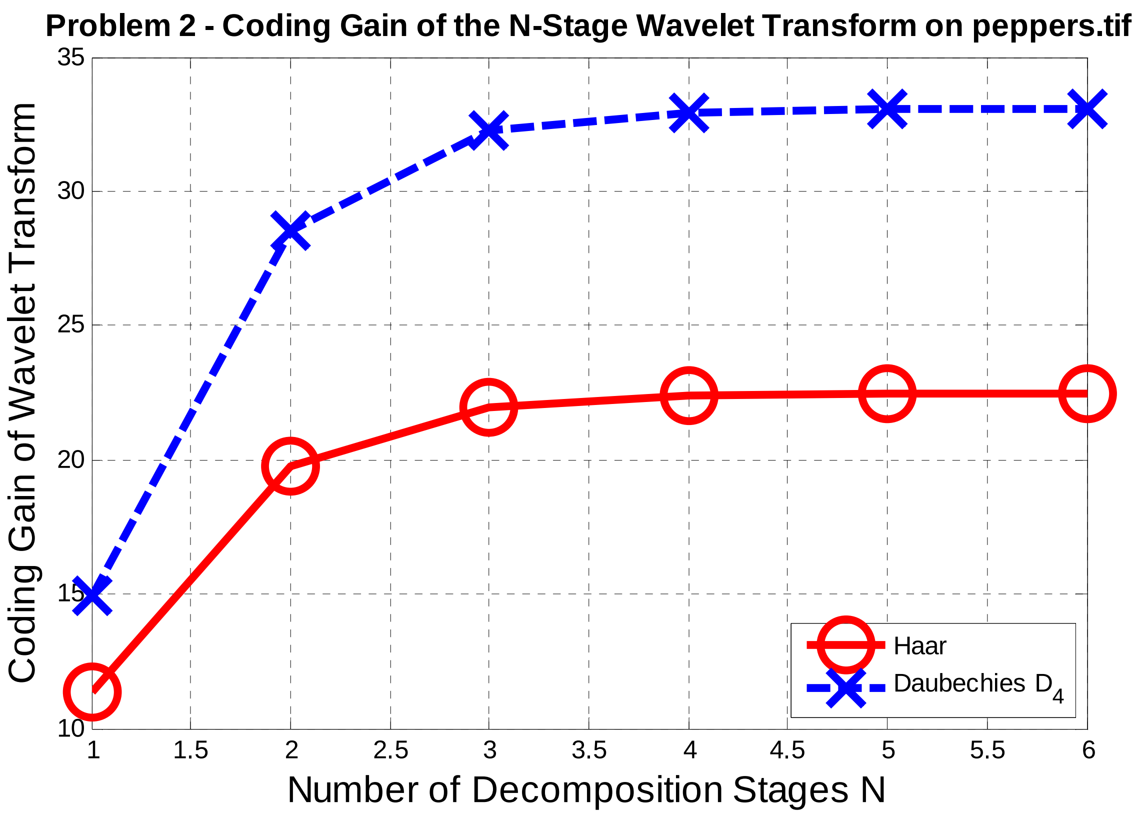
The coefficient images resemble the original compressed image more and more as we decompose the image, since the downsampled images contain larger pixel transitions and hence greater detail. However, the horizontal and vertical edge images also exhibit much detail. The harbour image contains significant horizontal edges, whereas the peppers image comprises predominantly vertical edges.

The two images offer different levels of coding gain. For the harbour image, wavelet decomposition with the Haar and Daubechies D_4 wavelets yields commensurate outcomes, with the Daubechies wavelet unsurprisingly outperforming the Haar (D_1) wavelet; as the longer, more complicated FIR filter, the D_4 wavelet decomposition filters achieve lower coefficient variance and hence higher coding gain:



For the peppers image, on the other hand, the discrepancy grows even greater; the Daubechies D_4 wavelet completely dominates the Haar (D_1) wavelet, especially when the additional stages accumulate and capitalize on the reach of the longer Daubechies filter. We tabulate the coding gains on the following page:

	N = 1	**N = 2**	**N = 3**	**N = 4**	**N = 5**	**N = 6**
Haar D_1 (harbour.tif)	**3.3934**	**4.6879**	**5.0851**	**5.1915**	**5.2170**	**5.2231**
Daubechies D_4 (harbour.tif)	**3.8826**	**5.4610**	**5.9283**	**6.0490**	**6.0782**	**6.0856**
Haar D_1 (peppers.tif)	**11.3412**	**19.7423**	**21.9829**	**22.4042**	**22.4741**	**22.4793**
Daubechies D_4 (peppers.tif)	**14.9244**	**28.5589**	**32.2687**	**32.9316**	**33.0376**	**33.0487**



The additional coefficients and the resulting high-order consideration and transformation clearly yield superior wavelet transforms from a coding gain perspective. Despite the complication of a longer FIR filter, the spatial prescience and variance reduction more than compensate for the increased length.