Christopher Tsai
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EE 355 – Imaging Radar and Its Applications

Problem Set V
A Focused SAR Processor

Problem #1 – Range Compression and Resolution

$Chirp\ rate: s = 4.189166\ \times 10^{11}\ Hz/s$

$Pulse\ duration: \tau_p = 37.12\ \mu s$

$Sampling\ frequency: f_s = 18.96\ \times 10^6\ Hz$

$Chirp\ bandwidth\ B = f_s \tau_p = 1.555 \times 10^7\ Hz$

$Pulse\ repetition\ frequency: PRF = 1679.9\ Hz$

$Minimum\ range: r_0 = 830{,}000$ m

$Radius\ of\ Earth: R_E = 6{,}378{,}000$ m

$Spacecraft\ altitude: z = 790{,}000$ m

$Azimuth\ velocity: v_x = 7550\ m/s$

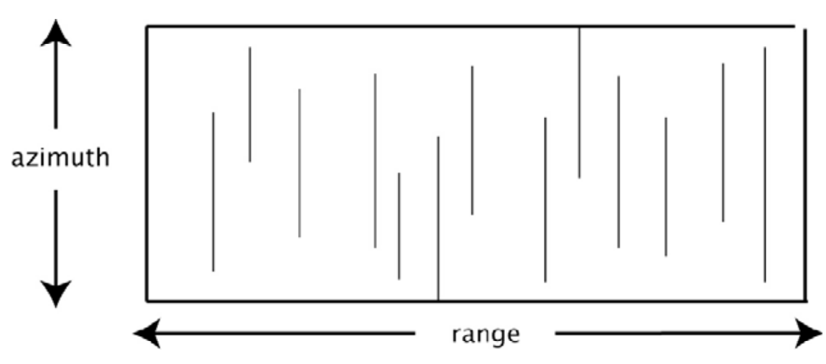
$Wavelength: \lambda = 0.0566\ m$

$Antenna\ length: l = 10\ m$

$$Pulse\ spacing = \frac{v_x}{PRF} \approx 4.4943 \frac{m}{pulse}$$

$$Azimuth\ beamwidth = \frac{\lambda r_0}{l} \approx 4697.8\ m$$

After removing the header (412 bytes) and combining adjacent pairs of the remaining 9806 raw data columns to form complex numbers, we obtain 4903 range bins:



Even though we can extract azimuth spectra from the raw data, we first compress the range by correlating each range column with a matched filter chirp. Due to discrete-time convolution, the edge samples of the resultant range-compressed impulse response will irrecoverably convolute data from the other end of the signal; because this wrap-around effect corrupts the information in the signal, we remove the final segment of our range compression corresponding to the overlap. The number of these corrupted samples matches the number of samples in our reference chirp: $\mathrm{N_{chirp\ samps}} = \lfloor \mathrm{f_s \tau_p} \rfloor = 703\ \text{samples}.$ The number of valid range bins in our final image therefore amounts to

$$\mathrm{N_{range}} = \mathrm{N_{complex}} - \mathrm{N_{chirp\ samps}} = \frac{\mathrm{N_{raw}} - \mathrm{N_{header}}}{2} - \mathrm{N_{chirp\ samps}} = \frac{(10218) - (412)}{2} - 703$$

$$\boxed{\mathrm{N_{range}} = 4200\ \text{range bins}}$$

In order to perform the Fast Fourier Transform (FFT) algorithms in the C programming language, we must ensure that we set our signal length to a power of two. While higher powers of two are preferable (with zero-padding to fill spaces without real data), we can also transform for range compression with shorter FFT lengths if we are willing to lose some information. Since our base signal contains 4200 meaningful range samples (actually 4903 at the time of correlation transformation), we can round down to a minimum of

$$\boxed{\min|\mathrm{N_{FFT}}| = 2^{\lfloor \log_2 4903 \rfloor} = 2^{12} = 4096\ \text{samples.}}$$

If we adopt a minimalist approach, we can probably also perform compression with only

$$\min|\mathrm{N_{FFT}}| = 2^{11} = 2048\ \text{samples},$$

but more typically, we round to the next *highest* power of two so we do not lose data. In our range compression algorithm, for example, we set an FFT length of $\mathrm{N_{FFT}} = 2^{\lceil \log_2 4903 \rceil} = 8192\ \text{samples.}$

The effective spacecraft velocity is

$$\boxed{Effective\ velocity: v_{eff} = v_x \sqrt{\frac{R_E}{R_E + z}} \approx 7121.8072 \frac{m}{s}}$$

The central range from the spacecraft to the center of the swath is

$$r_c = r_0 + \frac{N_{range}}{2} \frac{c}{2 f_s} \approx 846{,}602.6065\ m.$$

$$Look\ angle: \theta_c = \cos^{-1}\left(\frac{r_C^2+(R_E+z)^2-R_E^2}{2r_C(R_E+z)}\right) \approx 0.34589\ rad \approx 19.818°.$$

$$Central\ angle: \beta_c = \sin^{-1}\left(\frac{r_C}{R_E}\sin(\theta_c)\right) \approx 0.0450179\ rad \approx 2.5793°.$$

$$Incidence\ angle: i_c = \theta_c + \beta_c \approx 0.390908\ rad \approx 22.3974°.$$

$$\boxed{Slant\ range\ resolution: \delta_{slant\ range} = \frac{c}{2B} \approx 9.6396\,\frac{m}{resel}}$$

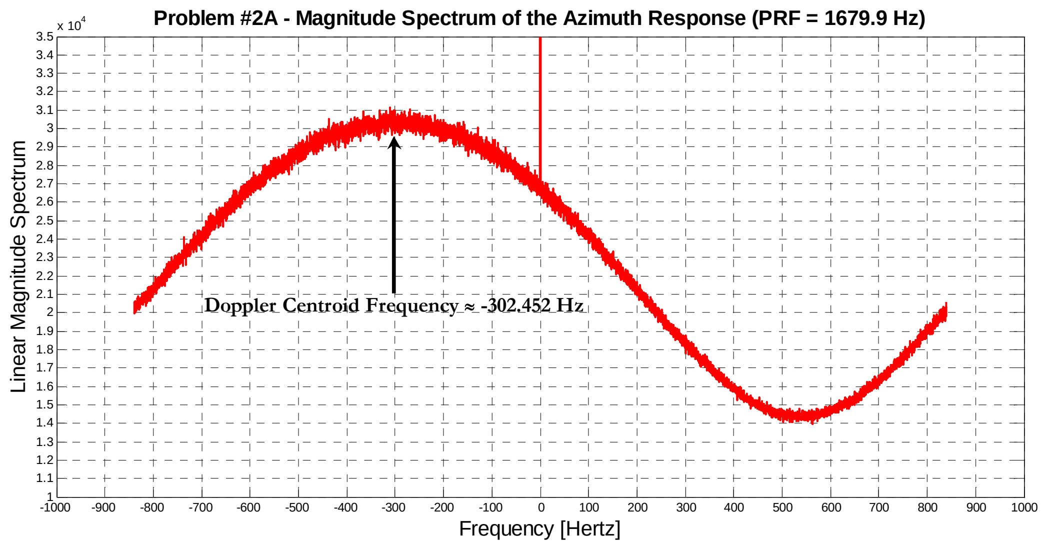
$$\boxed{Ground\ range\ resolution: \delta_{ground\ range} = \frac{\delta_{slant\ range}}{\sin(i_c)} \approx 25.299\,\frac{m}{resel}}$$

$$\boxed{Slant\ range\ bin\ spacing: \Delta x_{slant\ range} = \frac{c}{2f_s} \approx 7.906\,\frac{m}{pixel}}$$

$$\boxed{Ground\ range\ bin\ spacing: \Delta x_{ground\ range} = \frac{\Delta x_{slant\ range}}{\sin(i_c)} \approx 20.749\,\frac{m}{pixel}}$$

Problem #2 – A Focused SAR Processor Multilook Image

As we discovered in Problem Set IV, we can determine the Doppler centroid frequency by observing the average magnitude spectrum along the azimuth and ascertain this numerical value with the average phase change algorithm. The same average magnitude spectrum arises from the data:



The first 8192 azimuth lines reveal a Doppler centroid of approximately -302.452026 Hz.

Setting the azimuth beamwidth to 80% of the actual available azimuth beamwidth to restrict the reference function below 1024 samples, we generate 2048-sample patches out of 10,100 lines of azimuth data from the European Remote-Sensing Satellite (ERS) file. As with the unfocused Synthetic Aperture Radar (SAR) processor, we overlay the patches with deliberate overlap between two adjacent sets, but, unlike the unfocused SAR processor, we now perform azimuth compression through correlation with a matched azimuth chirp. Since we receive return spectra that have quadratic phase, matching the return with a chirp of effective pulse length given by azimuth beamwidth illumination time and slope given by the quadratic phase coefficient will yield a compressed azimuth spectrum, from which we then truncate reference chirp-corrupted values much as we did following range compression. The process is entirely analogous to range compression, with the additional requirement that we must overlay adjacent patches with the knowledge that our truncation will produce gaps in the data for which our patch overlaps must then compensate.

The following parameters prove fundamental to azimuth compression, so our program displays them:

```
Effective velocity = 7121.807129 m/s
Range chirp bandwidth = 15550184.000000 Hz
Range center = 846602.625000 m
Range = 863205.187500 m
Doppler centroid range = 863205.812500 m
Incidence angle = 22.397387 degrees
Slant pixel spacing = 7.906003 m/pixel
Slant range resolution = 9.639617 m/resel
Ground pixel spacing = 20.749128 m/pixel
Ground range resolution = 25.298958 m/resel
Azimuth pixel spacing = 3.998988 m/pixel
Number of azimuth looks = 5
Number of azimuth samples = 921
Length of valid azimuth data = 1125
Number of azimuth lines in the image = 2025
Number of range bins in the image = 4200
Number of lines per patch = 2048
Number of patches = 9
```

We set the final partial patch to zero since our patches partition the majority of the data, so zeros will contribute insignificantly to the overall image.

Of particular interest to squaring our pixels in the final SAR-processed image is the azimuth resolution and pixel spacing, which we compute as

$$\boxed{\textit{Ground azimuth resolution}: \delta_{ground\ azimuth} = \frac{l}{80\% \times 2} = 6.25\,\frac{m}{resel}}$$

$$\boxed{\textit{Ground azimuth point spacing}: \Delta x_{\text{ground azimuth}} = \frac{R_E}{R_E + z}\frac{v_x}{PRF} \approx 3.99899\,\frac{m}{pixel}}$$

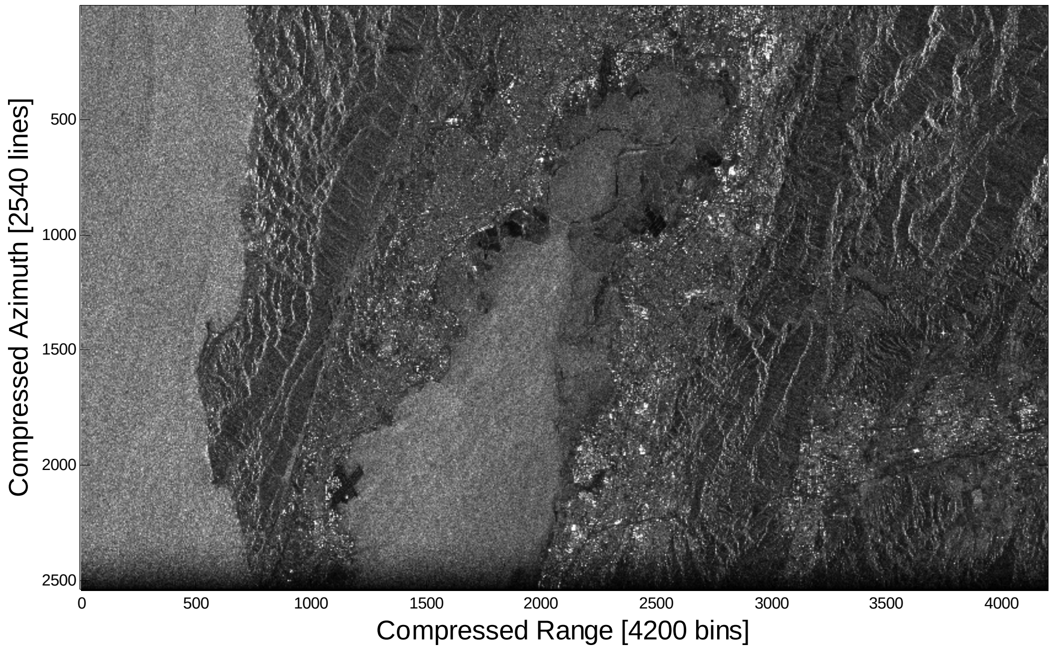
In order to square the image pixels as much as possible, we try to attain near-unity aspect ratio between the ground range bin spacing and the ground azimuth point spacing:

$$N_{\text{looks azimuth}}\, \Delta x_{\text{ground azimuth}} = N_{\text{looks range}}\, \Delta x_{\text{ground range}}$$

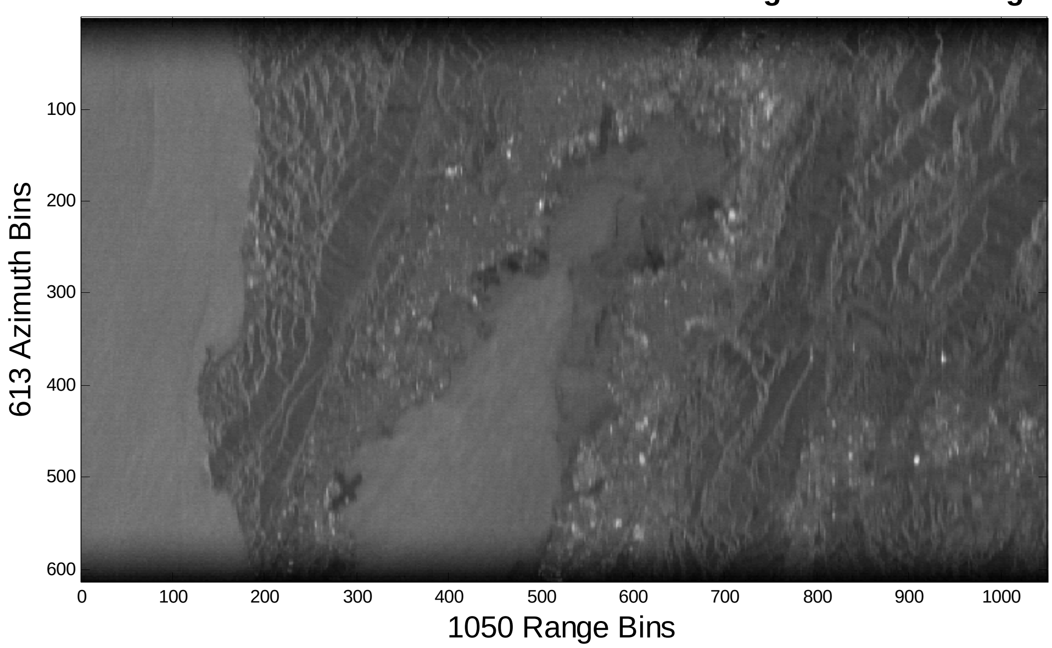
$$N_{\text{looks azimuth}} = \frac{\Delta x_{\text{ground range}}}{\Delta x_{\text{ground azimuth}}} N_{\text{looks range}} \approx 5.1866 \text{ looks}$$

Rounding to the nearest integral number of looks, we see that **five azimuth looks** will yield the squarest pixels. Instead of arithmetically averaging these five adjacent azimuth pixels in the normal straightforward manner, we accumulate their square magnitudes instead to accentuate contrast between bright and dark points. Finally, to smoothen the result and re-establish proper scaling, we square root the sum of our look pixels. The final image, with approximately square ground pixels, lucidly displays the San Francisco bay area with much finer resolution than its unfocused SAR counterpart, which we display below for comparison:

Problem #2B - Multi-Patch Focused SAR Image from ERS Data with Five Azimuth Looks

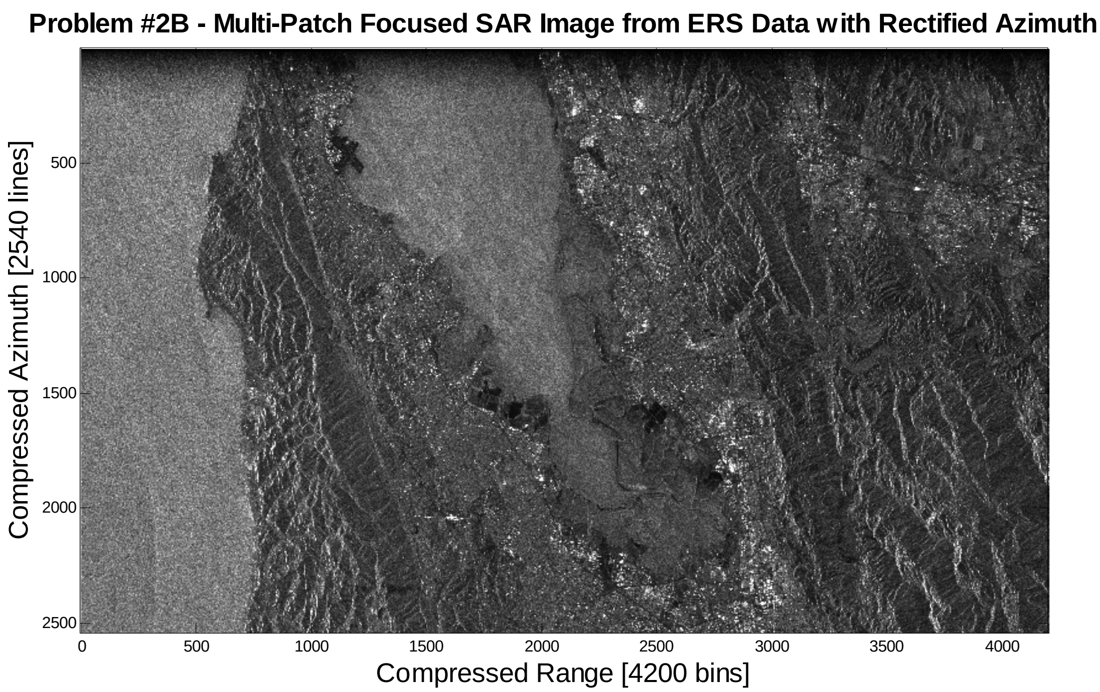


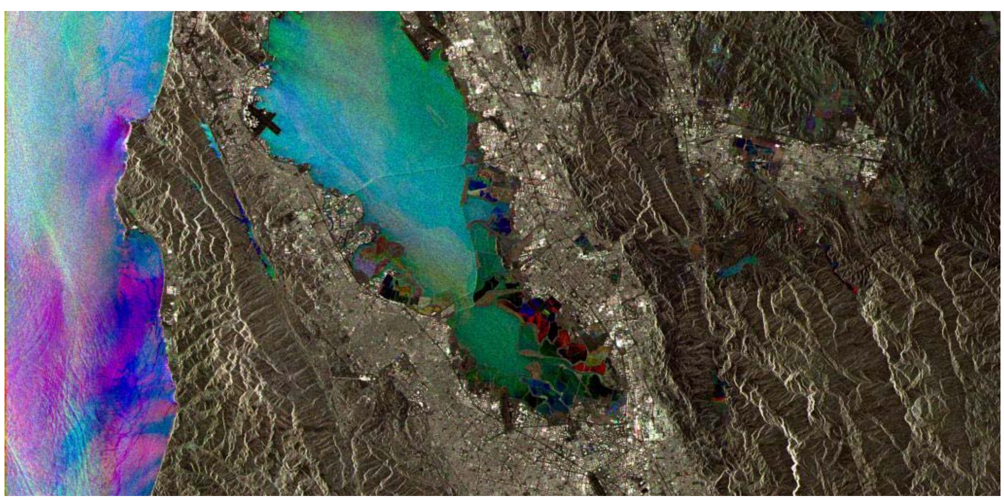
Problem #2D - 157-Patch Full ERS Unfocused SAR Image with Four Range Looks



Clearly, not only the point-wise resolution but also the contrast ratio improves in the focused SAR image, in which the exact locations of bright lights are much more precisely defined, and streets are distinguishable.

As we noted in the unfocused SAR image, the usual orientation of this region appears as the mirror image of our display, so we vertically invert the image and compare to the professionally generated radar image:





Our focused SAR processed image now boasts comparable resolution to the professionally processed ERS image; unlike the unfocused SAR image, we can recognize ridge lines along the hills and mountains as well as road lines and city streets. Furthermore, we distinguish fields and salt ponds whose outlines blurred together in the unfocused SAR, giving literal meaning to our **focused** SAR image. All in all, we can generally identify smaller features in the focused SAR image with the finer resolution and stronger grayscale contrast.