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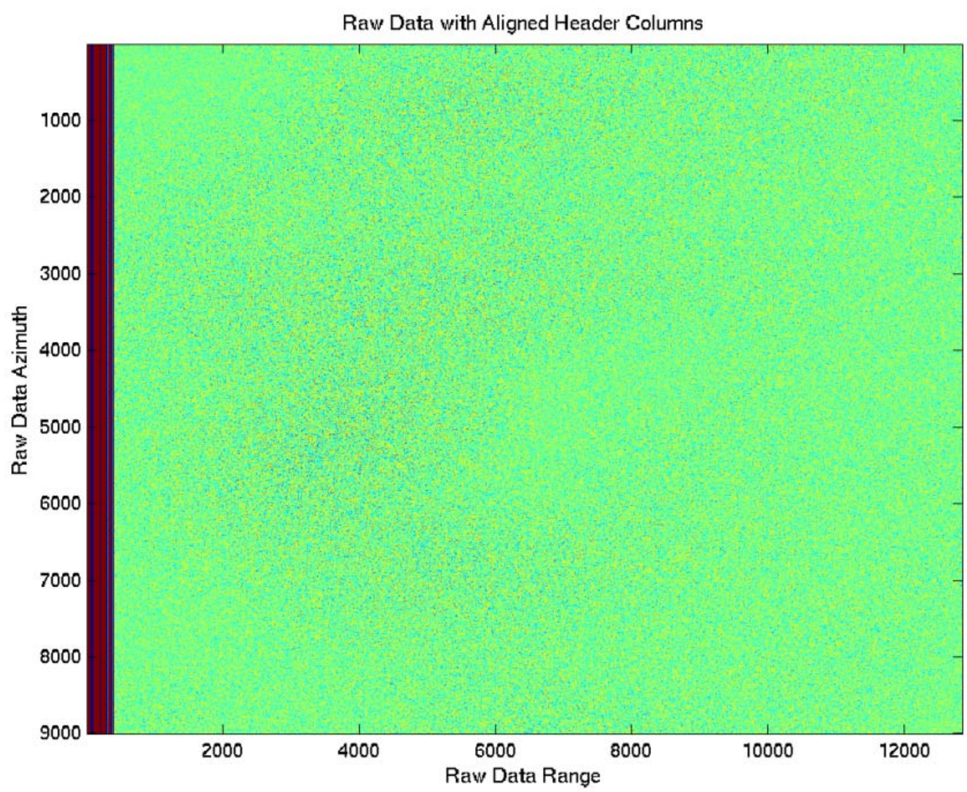
February 28, 2007
EE 355 – Imaging Radar & Its Applications

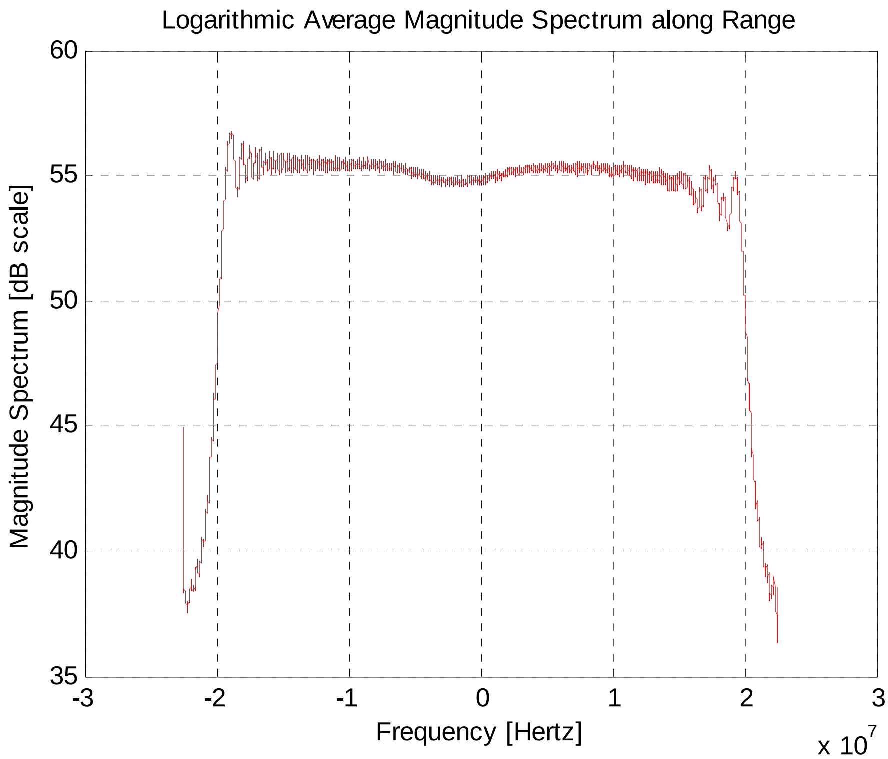
Problem Set VII
Synthetic Aperture Radar (SAR) Image Formation from Unknown Raw Data

Problem #1 – Image Parameter Determination

Trial and error quickly narrows the possibilities for data recording format; we read the alien data file under different assumptions, such as 'float' data, 'float32' data, 'float64' data, 'double' data, 'char' data, and various 'uint' recording formats. Upon loading the data as **'uint8'** – or **'char'** – format, we immediately remark that the data is intelligible, written as integers between 0 and 63, and therefore suggestive of six-bit quantization similar to the five-bit quantization we saw in the ERS data from Problem Set IV. To determine line length, we image the raw data seeking trends that might indicate header lines. Noticing several columns that contain uniform bands of values that distinctly protrude from surrounding data, either by their massive uniformity or by their unusual values (127 and 255 are impossible data values), we re-index our matrix until these unusual header values align into columns:



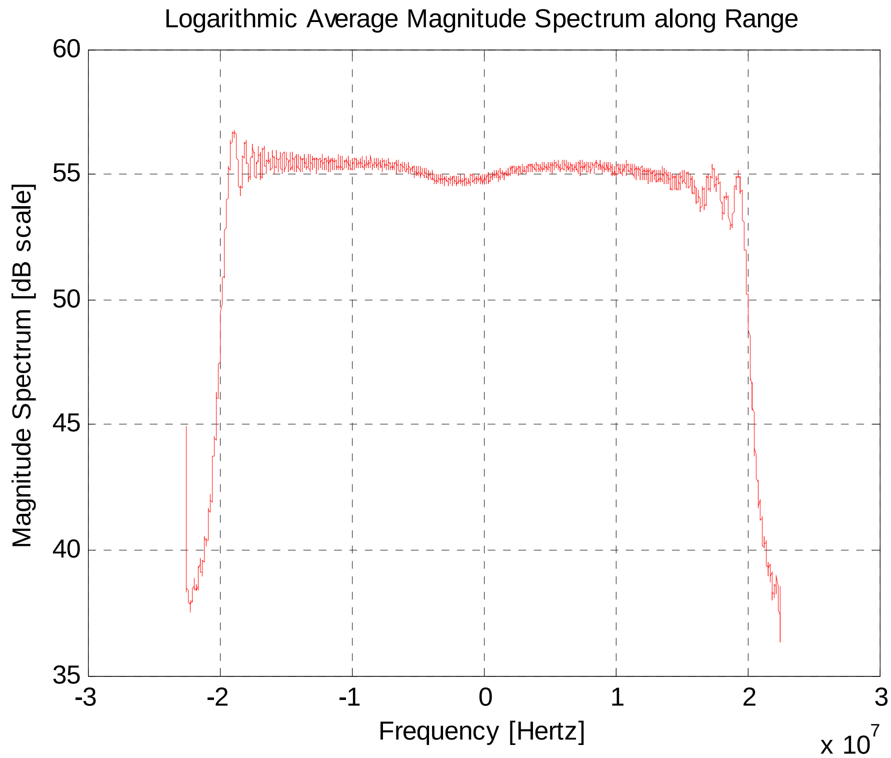
At that point, we know that our raw data initially contains **12,850 lines in the range direction**; upon dividing the entire file size by the size of a single entry (8 bits or 1 byte) and then by 12,850 lines, we conclude that the data comprises **9000 lines in the azimuth direction**. Reading the entire file confirms these line lengths. Finally, noticing the alternating speckle in the raw data, we surmise that the data alternates between real and imaginary, and plotting the range spectra confirm that the data, centered about zero frequency, has indeed been I/Q-sampled.



Thus, in preparation for range compression, we remove the **400 header lines** to reduce our data matrix to 12,450 range lines, subtract the average value (31.5) from each individual entry, and combine real/imaginary pairs of range values to reduce our matrix of actual data to **6,225 complex entries** along each azimuth line.

Problem #2 – Radar Parameter Determination

We return to the plot that revealed zero-centered spectra across the range. Upon averaging several of these range response spectra over 4096 azimuth lines, we see the following average magnitude spectrum:

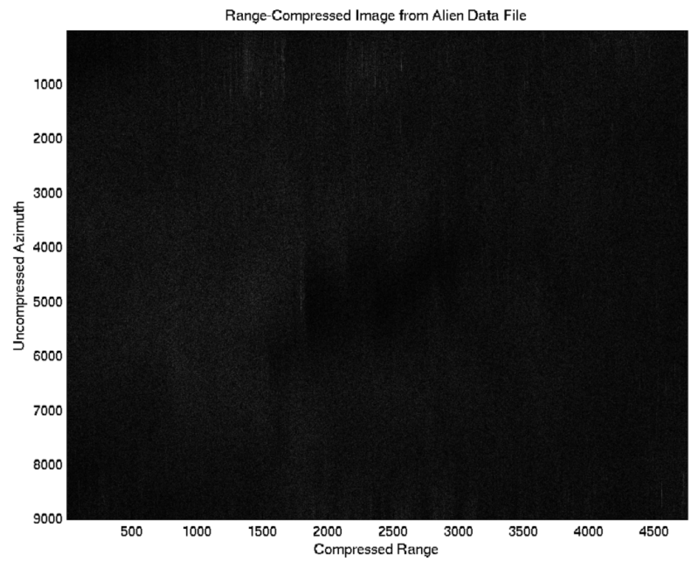


The width of this approximate rectangular pulse reveals the bandwidth of our range chirp to be about 38.833 MHz if we designate the 3-dB down point (around 52.6 dB compared to the 55.6 dB maximum) as the defining level. We know that the chirp slope from the range response is approximately -1.2 $\times 10^{12}$ Hz/sec, so we divide our graphically determined bandwidth by the absolute value of this slope to obtain an initial estimate for the pulse duration of ≈ 32.361 μsec. However, because these slope and pulse length estimates are only *approximate*, we must autofocus the range to refine their accuracy. Tapping the sub-aperture shift algorithm and keeping our bandwidth estimate constant, we generate a chirp of slope -1.2 $\times 10^{12}$ Hz/sec and duration 32.361 μsec. Correlating with the average range spectrum, splitting the resultant spectrum into positive and negative frequency halves, and subsequently correlating these sub-apertures, we detect a peak at the location indicating the pixel offset of our generated chirp; because the slope of our artificially generated reference chirp deviates slightly from the slope of the actual data range chirp, we see a slight difference in the two sub-aperture spectra, and that index difference leads directly to an estimate of our slope error, which we compute as $\Delta s = \frac{2\Delta t \cdot s^2}{bandwidth}$, where Δt indicates the time difference between our sub-aperture peaks in seconds. However, since we do not know the actual chirp slope, we must use our current estimate in the formula, forcing us to continue iterating the procedure until the error Δs is sufficiently small; we consider our estimate sufficiently refined when our slope estimate deviates by less than one thousandth of the slope. The output of the iterative sub-aperture process converges in the following way:

```
Mismatched Chirp with Estimated Reference Slope: -1.2e+12
----------------------------------------------------------------------
Pixel Offset = 8 pixels | Implied Slope Error = 3.29639e+09 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.1967e+12
----------------------------------------------------------------------
Pixel Offset = 6 pixels | Implied Slope Error = 2.45873e+09 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.19424e+12
----------------------------------------------------------------------
Pixel Offset = 5 pixels | Implied Slope Error = 2.04053e+09 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.1922e+12
----------------------------------------------------------------------
Pixel Offset = 4 pixels | Implied Slope Error = 1.62685e+09 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.19058e+12
----------------------------------------------------------------------
Pixel Offset = 3 pixels | Implied Slope Error = 1.21681e+09 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.18936e+12
----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 8.09548e+08 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.18855e+12
----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 4.04223e+08 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.18815e+12
----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 4.03948e+08 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.18774e+12
----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 4.03674e+08 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.18734e+12
----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 4.03399e+08 Hz/sec

Mismatched Chirp with Estimated Reference Slope: -1.18694e+12
----------------------------------------------------------------------
Pixel Offset = 0 pixels | Implied Slope Error = 0 Hz/sec

sEstimate = -1.186532514639469e+012 Hz/sec
```

Thus, our reference chirp autofocuses to a slope of approximately $\boldsymbol{s} \approx$ **-1.18653251463946** $\times$ $\mathbf{10^{12}}$ **Hz/sec**. After confirming that our average magnitude spectrum holds across all 9000 azimuth lines, we accept the bandwidth and divide it by the exact chirp slope to ascertain the range chirp pulse duration of approximately $\boldsymbol{\tau} \approx$ **32.72813810062299 μsec**. Applying these values to generate a true matched filter reference chirp of $\lfloor f_s \tau \rfloor \approx$

$\lfloor(44.997 \times 10^6 \text{ Hz})(32.7281381 \times 10^{-6} \text{ sec})\rfloor = 1473$ range chirp samples, we perform range compression and truncate the convolution wrap-around samples; the resultant range-compressed data file now contains only 4752 range bins, with the 1473 samples at the end removed because of their coincidence with chirp convolution wrap-around. To verify that we have performed range compression adequately, we observe that the resultant image now displays high resolution along the horizontal (range) direction:



Before we perform azimuth compression, we can determine several other radar parameters. First, noting that the alien spacecraft follows an approximately circular orbit around the Earth at an altitude of z = 213,325 m, we apply Newton's Second Law to write the equation of motion for an orbiting spacecraft:

$$F_{gravitational} = m_{craft} \cdot a_{centripetal}$$

$$\frac{GM_{Earth} m_{craft}}{r^2} = m_{craft} \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM_{earth}}{r}} \approx \sqrt{\frac{\left(6.6742 \times 10^{-11} \ \frac{m^3}{kg \cdot s^2}\right)(5.9742 \times 10^{24} \ kg)}{(6{,}378{,}000 \ m) + \ (213{,}325 \ m)}}$$

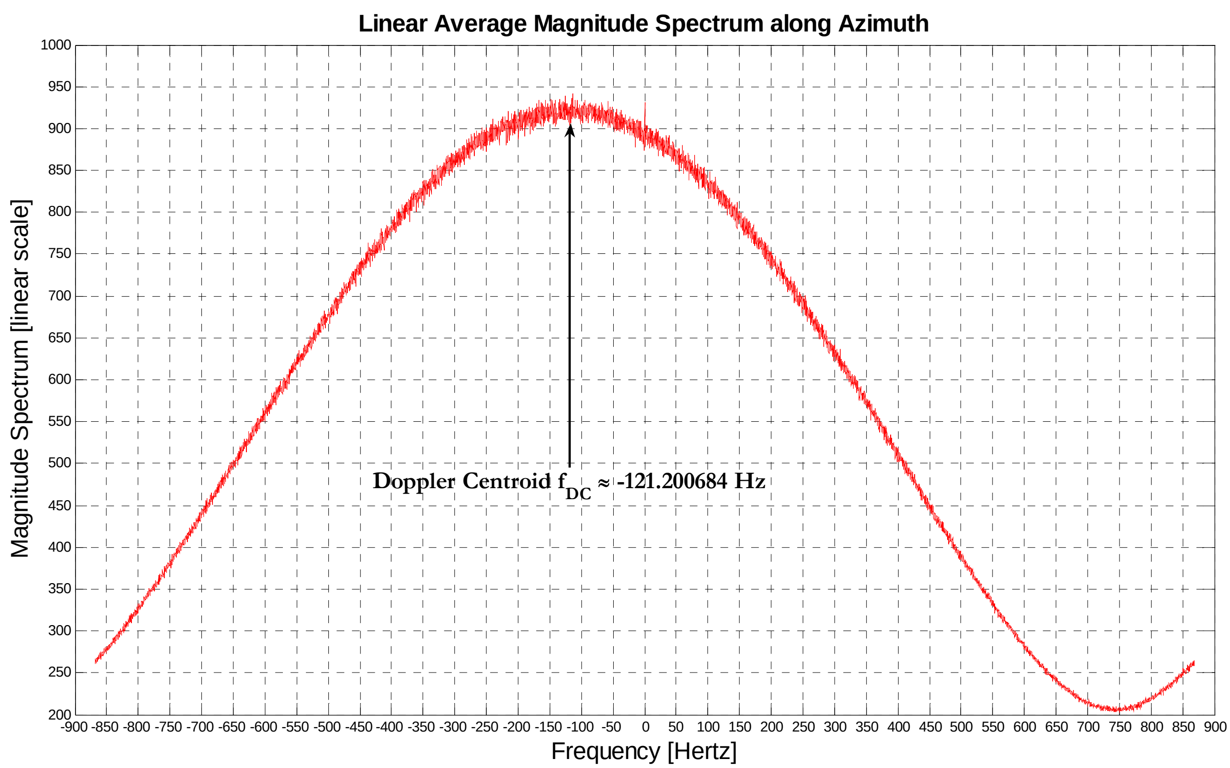
$$\boldsymbol{v \approx 7777.73468 \frac{m}{sec}}.$$

For most calculations along the azimuth, the effective velocity will prove most applicable:

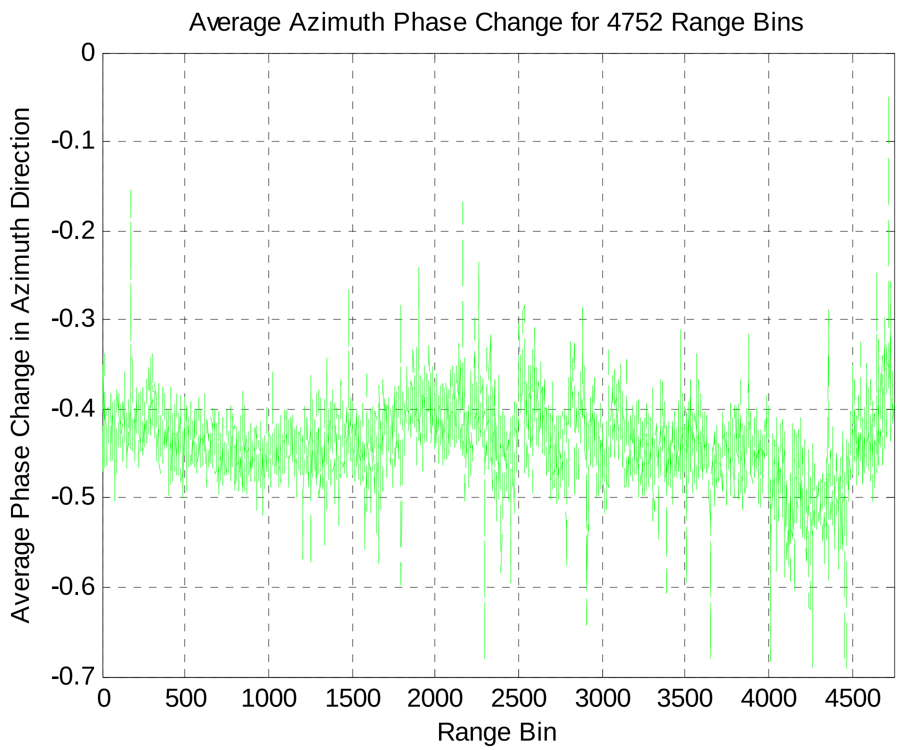
$$v_{eff} = v\sqrt{\frac{R_{Earth}}{z + R_{Earth}}} \approx \left(7777.73468\,\frac{m}{sec}\right)\sqrt{\frac{(6{,}378{,}000\ m)}{(213{,}325\ m) + (6{,}378{,}000\ m)}}$$

$$\boldsymbol{v_{eff} \approx 7650.8382\,\frac{m}{sec}}.$$

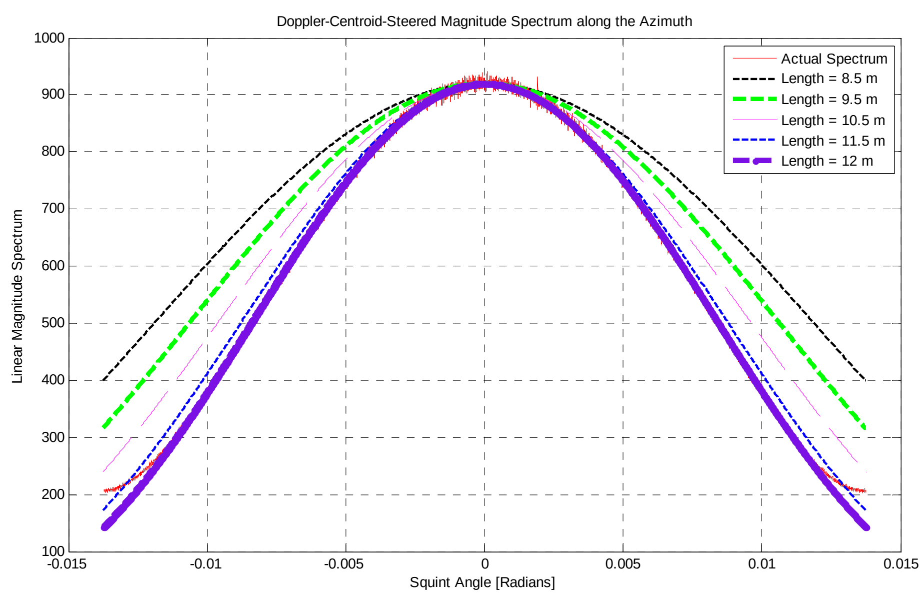
Even though we will not apply the unfocused SAR algorithm, we should apply the Fast-Fourier Transform (FFT) to each range bin to obtain the azimuth spectra. Averaging these azimuth spectra, we can then determine the Doppler centroid frequency by locating the frequency at which the spectral pattern peaks:



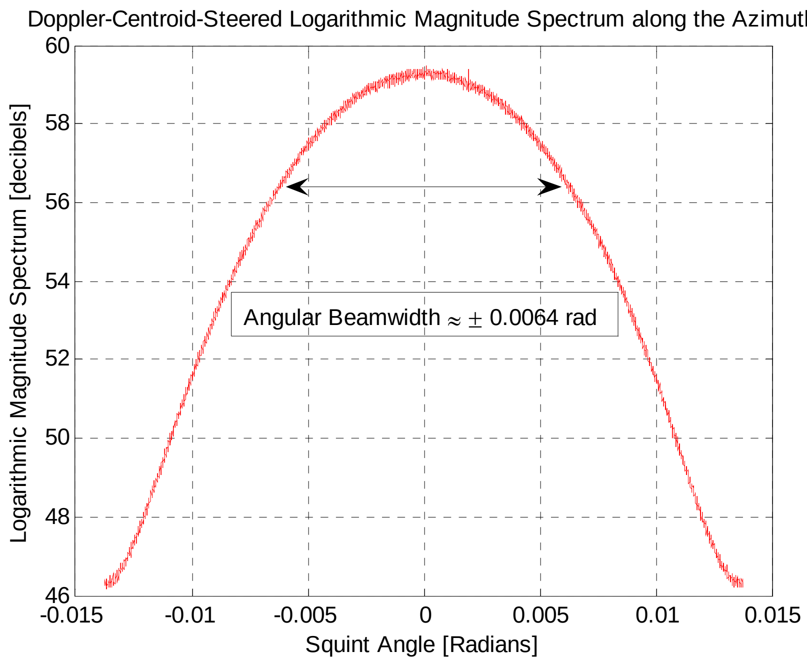
The frequency axis stretches from $-\frac{\text{PRF}}{2}$ to $\frac{\text{PRF}}{2}$, allowing us to approximate, for the purposes of azimuth autofocus, that our azimuth response bandwidth is approximately the Pulse Repetition Frequency (PRF), which we know to be 1736 Hz. We cannot estimate the Doppler centroid too precisely by mere graphical inspection, but we can also apply the average phase change algorithm to obtain a similar Doppler centroid estimate of $\boldsymbol{f_{DC} \approx -121.200710875496\ \text{Hz}}$, measuring the average phase changes along each range bin and then averaging across azimuth:



Finally, for completeness, we can approximate the antenna length by studying the steered azimuth magnitude spectrum. Because we know that the far-field spectral response on our rectangular aperture antenna is a squared sinc pattern, we attempt to fit such a model to our spectrum:



The value of antenna length in $sinc^2\left(\frac{\ell\theta_{sq}}{\lambda}\right)$ that best fits the actual steered spectrum is approximately 12 m, although we can obtain a similar estimate by observing the logarithmic spectrum 3-dB angular beamwidth, which stretches to approximately ± 0.0064 radians on either side of beam center.



Thus, the total 3-dB angular beamwidth is approximately 0.0128 radians, leading to an antenna length:

$$\theta_{3\text{ dB}} \approx 0.89\frac{\lambda}{\ell}$$

$$\ell \approx 0.89\frac{\lambda}{\theta_{3\text{ dB}}}$$

$$\ell \approx 16.845\text{ m.}$$

Both estimates share the same order of magnitude, but they differ just enough to warn us that this parameter may not be definitively calculable to a sufficiently precise degree for us to use in the azimuth SAR algorithm. Instead, we shall resort to autofocus methods, assuming that the azimuth bandwidth spans the entire PRF, from $-\frac{\text{PRF}}{2}$ to $\frac{\text{PRF}}{2}$. Under this assumption, we no longer need the antenna length to perform azimuth compression.

Problem #3 – Azimuth Compression

Before we proceed with azimuth compression, let us first autofocus along the azimuth to obtain the approximate range values across the swath. Tapping the sub-aperture shift algorithm much as we did for range compression, we begin by assuming a look angle around 45°, allowing us to estimate the range to beam center by solving the Law of Cosines for *r*: $R_E^2 = r^2 + (R_E + z)^2 + 2r(R_E + z)\cos 45°$. The estimate we obtain is ≈ 296,955.853889 m. Thus, we begin our autofocus process by assuming an approximate chirp rate

$$f_{rate} = -\frac{2v_{eff}^2}{\lambda \cdot r},$$

where *r* represents the estimated range to the central bin. To combat the noise that accompanies every individual azimuth spectrum in the integrated beamwidth, we average the azimuth spectra across every range bin in the beamwidth to obtain a relatively noise-clean chirp spectrum against which we can compare our estimated reference chirp function through matched filter correlation. However, as in range autofocus, our initial reference chirp is unmatched, both in chirp rate and consequently in chirp pulse duration, which we set by default to $\tau_{az} = \frac{PRF}{f_{rate}}$, assuming that the beamwidth covers our entire spectral width (PRF). Unlike the range autofocus process, we now center our chirp about a carrier frequency equal to the Doppler centroid, hence performing sub-aperture shift with two shifted sub-apertures. With each iteration of this sub-aperture shift algorithm, our estimate of the chirp rate approaches the true chirp rate, allowing us to simultaneously improve our estimate of the effective pulse length. Within a few iterations, our chirp rate estimate converges to the chirp rate within one thousandth of the PRF = 1736 Hz:

```
Mismatched Chirp with Estimated Chirp Rate: -1627.2
----------------------------------------------------------------------
Pixel Offset = 51 pixels | Implied Slope Error = 22.404 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1604.8
----------------------------------------------------------------------
Pixel Offset = 48 pixels | Implied Slope Error = 20.5094 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1584.29
----------------------------------------------------------------------
Pixel Offset = 40 pixels | Implied Slope Error = 16.6571 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1567.63
----------------------------------------------------------------------
Pixel Offset = 37 pixels | Implied Slope Error = 15.0856 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1552.55
----------------------------------------------------------------------
Pixel Offset = 34 pixels | Implied Slope Error = 13.5969 Hz/sec
```

```
Mismatched Chirp with Estimated Chirp Rate: -1538.95
----------------------------------------------------------------------
Pixel Offset = 30 pixels | Implied Slope Error = 11.788 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1527.16
----------------------------------------------------------------------
Pixel Offset = 27 pixels | Implied Slope Error = 10.4473 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1516.72
----------------------------------------------------------------------
Pixel Offset = 23 pixels | Implied Slope Error = 8.77822 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1507.94
----------------------------------------------------------------------
Pixel Offset = 20 pixels | Implied Slope Error = 7.54514 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1500.39
----------------------------------------------------------------------
Pixel Offset = 19 pixels | Implied Slope Error = 7.09633 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1493.3
----------------------------------------------------------------------
Pixel Offset = 17 pixels | Implied Slope Error = 6.28943 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1487.01
----------------------------------------------------------------------
Pixel Offset = 15 pixels | Implied Slope Error = 5.50285 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1481.5
----------------------------------------------------------------------
Pixel Offset = 14 pixels | Implied Slope Error = 5.09805 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1476.41
----------------------------------------------------------------------
Pixel Offset = 13 pixels | Implied Slope Error = 4.70138 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1471.7
----------------------------------------------------------------------
Pixel Offset = 12 pixels | Implied Slope Error = 4.31214 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1467.39
----------------------------------------------------------------------
Pixel Offset = 11 pixels | Implied Slope Error = 3.92967 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1463.46
----------------------------------------------------------------------
Pixel Offset = 10 pixels | Implied Slope Error = 3.55331 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1459.91
----------------------------------------------------------------------
Pixel Offset = 9 pixels | Implied Slope Error = 3.18247 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1456.73
----------------------------------------------------------------------
Pixel Offset = 8 pixels | Implied Slope Error = 2.81654 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1453.91
----------------------------------------------------------------------
Pixel Offset = 8 pixels | Implied Slope Error = 2.80566 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1451.1
----------------------------------------------------------------------
Pixel Offset = 7 pixels | Implied Slope Error = 2.44549 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1448.66
----------------------------------------------------------------------
Pixel Offset = 6 pixels | Implied Slope Error = 2.08908 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1446.57
----------------------------------------------------------------------
Pixel Offset = 6 pixels | Implied Slope Error = 2.08305 Hz/sec
```

```
Mismatched Chirp with Estimated Chirp Rate: -1444.49
-----------------------------------------------------------------------
Pixel Offset = 5 pixels | Implied Slope Error = 1.73088 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1442.76
-----------------------------------------------------------------------
Pixel Offset = 5 pixels | Implied Slope Error = 1.72674 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1441.03
-----------------------------------------------------------------------
Pixel Offset = 4 pixels | Implied Slope Error = 1.37809 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1439.65
-----------------------------------------------------------------------
Pixel Offset = 4 pixels | Implied Slope Error = 1.37545 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1438.28
-----------------------------------------------------------------------
Pixel Offset = 4 pixels | Implied Slope Error = 1.37282 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1436.9
-----------------------------------------------------------------------
Pixel Offset = 3 pixels | Implied Slope Error = 1.02765 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1435.88
-----------------------------------------------------------------------
Pixel Offset = 3 pixels | Implied Slope Error = 1.02618 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1434.85
-----------------------------------------------------------------------
Pixel Offset = 3 pixels | Implied Slope Error = 1.02472 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1433.82
-----------------------------------------------------------------------
Pixel Offset = 3 pixels | Implied Slope Error = 1.02325 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1432.8
-----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 0.681196 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1432.12
-----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 0.680549 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1431.44
-----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 0.679902 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1430.76
-----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 0.679256 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1430.08
-----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 0.678612 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1429.4
-----------------------------------------------------------------------
Pixel Offset = 2 pixels | Implied Slope Error = 0.677968 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1428.72
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.338662 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1428.38
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.338502 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1428.05
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.338341 Hz/sec
```

```
Mismatched Chirp with Estimated Chirp Rate: -1427.71
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.338181 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1427.37
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.338021 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1427.03
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.337861 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1426.69
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.337701 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1426.36
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.337541 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1426.02
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.337381 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1425.68
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.337222 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1425.34
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.337062 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1425.01
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.336903 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1424.67
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.336743 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1424.33
-----------------------------------------------------------------------
Pixel Offset = 1 pixels | Implied Slope Error = 0.336584 Hz/sec

Mismatched Chirp with Estimated Chirp Rate: -1424
-----------------------------------------------------------------------
Pixel Offset = 0 pixels | Implied Slope Error = 0 Hz/sec

fRate = -1423.996750121125 Hz

tAzimuth = 1.21910390585676 sec
```

Once we converge on a satisfactorily accurate chirp rate for the average azimuth spectrum $\boldsymbol{f_{rate} \approx -1423.996750121125}$ **Hz,** we can determine the approximate Doppler centroid range for the central range bin since we averaged azimuth spectra across the entire beamwidth. We compute $r_{DC} = -\frac{2v_{eff}^2}{\lambda f_{rate}} \approx -\frac{2(7650.8382)^2}{(0.24227)(-1423.99675)} \approx 340{,}649.125\ m$. However, what we seek is the range r_0 to the *first* bin, so we must determine the central range and subtract the appropriate slant range bin spacing from it, as follows:

$$r_0 = r_{center} - \Delta x_{slant} \cdot \frac{N_{range\ bins}}{2}$$

$$r_0 = r_{DC}\sqrt{1-\left(\frac{\lambda f_{DC}}{2v_{eff}}\right)^2} - \frac{c}{2f_s}\frac{N_{range\ bins}}{2}$$

$$r_0 = (340{,}649.125\ m)\sqrt{1-\left(\frac{(0.24227\ m)(-121.2\ Hz)}{2\left(7650.8382\,\frac{m}{\sec}\right)}\right)^2} - \frac{\left(299{,}792{,}458\,\frac{m}{\sec}\right)}{2(44.997\times 10^6\ Hz)}\frac{(4752)}{2}$$

$$\boldsymbol{r_0 \approx 331{,}416.2717151514\ \mathrm{m}}.$$

Notice that this value depends quite sensitively on our computation of the effective velocity, since the range at the central bin varies as the *square* of the effective velocity: $r_{DC} = -\frac{2v_{eff}^2}{\lambda f_{rate}}$. Hence, our autofocus process essentially corrects for the error in one estimate by compensating in the other; both the effective velocity *and* the first bin slant range likely have errors, but, because we assume that we know the spacecraft velocity better than we know the range, we keep the effective velocity constant and allow autofocus to correct for errors in both measurements through the range value. Thus, even if we were incorrect to assume a nearly circular orbit during our velocity computation, our autofocused chirp rate will correct for the deviation by yielding a range distance that also differs from its true value. In brief, without knowing either the effective velocity or first bin slant range with total accuracy, we can ascertain neither, but we can nevertheless focus our image since the sub-aperture shift autofocused chirp rate , which depends on the ratio of our uncertainties, will still converge to $f_{rate} = -\frac{2\boldsymbol{v_{eff}^2}}{\boldsymbol{\lambda\cdot r}}$, allowing one of our unknown parameters to vary and correct the other. Therefore, even though our effective velocity and slant range estimates are approximate, the chirp rate produced from autofocus and the slant range increment spacing $\Delta x_{slant} = \frac{c}{2f_s}$ separating adjacent bins will still yield a viable image. Knowing the approximate range, we estimate the incidence angle with the Law of Cosines and the look angle with the Law of Sines. Note that the look angle is nearly 45°, validating our initial assumption:

$$\theta_{inc} = \pi - cos^{-1}\left(\frac{r_{center}^2 + R_{Earth}^2 - (R_{Earth}+z)^2}{2r_{center}R_{Earth}}\right) \approx 51.261574°.$$

$$\theta_{look} = sin^{-1}\left(\frac{R_{Earth}}{z+R_{Earth}} sin(\pi - \theta_i)\right) \approx 43.2449°.$$

Problem #4 – Range Migration, Focused SAR, and Additional Looks

Before we finalize our image with azimuth matched filter compression, we correct for range migration in the image. Even though little migration exists in this particular data set, we note that focusing along the azimuth in the SAR algorithm exacerbates any migration that might otherwise appear relatively unnoticeable; because the chirp rate in each range bin depends heavily on the range value as $f_{rate} = -\frac{2v_{eff}^2}{\lambda \cdot r}$, an incorrect range resulting from migration could derail our chirp rate estimate enough to blur our target and spread the bin energy, as we see in any slight chirp mismatch dating back to earlier assignments. Much like we discovered during our exploration of PSLR in range chirp mismatch (and providing the incentive for sub-aperture shift), slight discrepancies in reference chirp slope result in matched filter outputs with drastically and noticeably reduced PSLRs and ISLRs; the sharpness of the main lobe dulls, and the comparably high side lobes have spread the backscatter energy across several range and azimuth bins. Thus, whereas range curvature appears merely as slants and shadows in unfocused SAR images, the strong dependence of chirp rate and center frequency on range position makes slight range migration errors much more noticeable in focused SAR images. Thus, in light of such sensitivity to proper range calculations, we transform along the azimuth *before* matched filtering to rectify range migration.

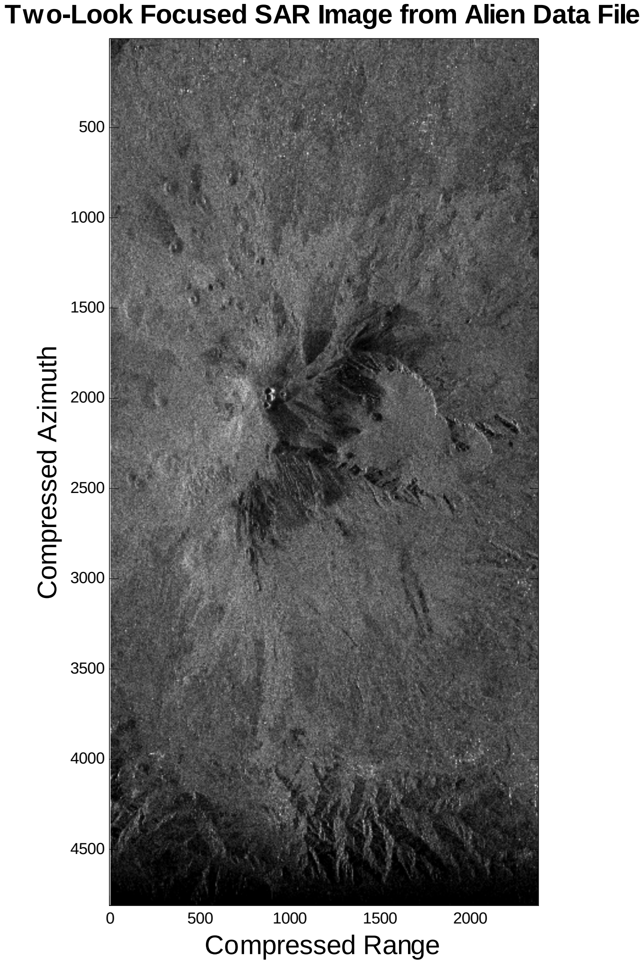
We employ the cut-and-paste algorithm, but other, more sophisticated interpolation schemes also exist. My cut-and-paste algorithm loops through each range bin and increments the range variable as $r = r_0 + (i-1)\delta_r$, where *i* is the range bin index, and $\delta_r = \frac{c}{2f_s}$ represents the range resolution. We increment the frequency or azimuth coordinate as $f = \frac{j-1}{2048\ azimuth\ lines} \cdot PRF$, where *j* is the azimuth line index. Upon shifting any possible aliased frequencies into a single PRF-length Doppler-centered spectrum so that the azimuth coordinates fall within $\pm\frac{\text{PRF}}{2}$ of the Doppler centroid f_{DC}, we compute the range migration as $dr = \frac{f^2 r \lambda^2}{8v_{eff}^2}$. Finally, in order to convert the necessary range migration correction into a range bin or pixel offset, we compute the integral quantity $\Delta r = round\left(\frac{dr}{\delta_r}\right) = round\left(\frac{1}{\delta_r}\frac{f^2\lambda^2 r}{8v_{eff}^2}\right)$ and sample the range data with the appropriate (r, f)-dependent offset for each individual pixel.

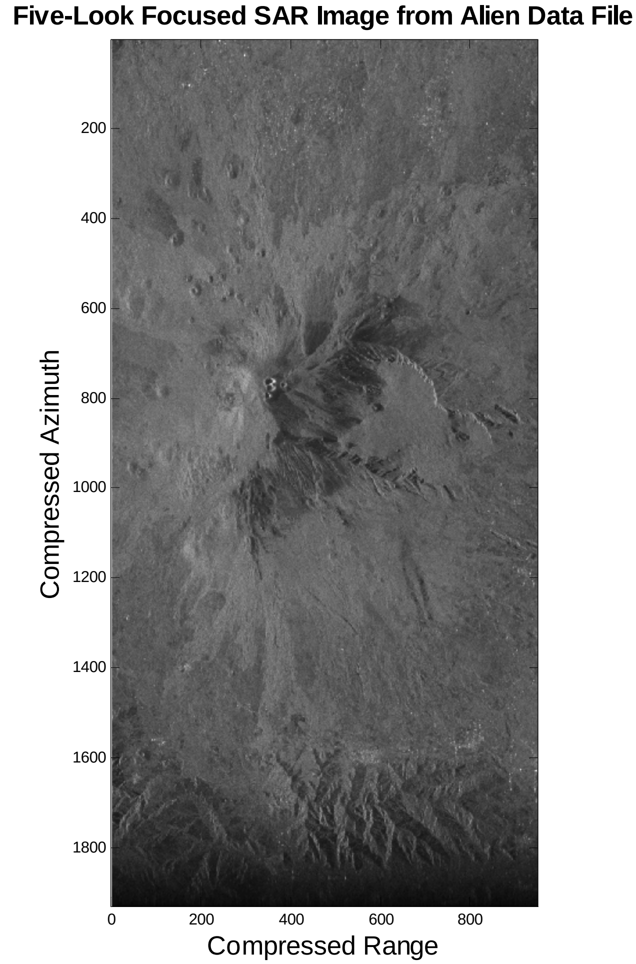
Finally, we revisit each range bin to perform azimuth matched filtering, tapping the slant range we determined earlier. In each range bin, we compute the corresponding $f_{rate} = -\frac{2v_{eff}^2}{\lambda \cdot r}$ using the incremented range value $r_i = r_0 + (i-1) \cdot \frac{c}{2f_s}$, leading to an effective azimuth chirp pulse duration of $\tau_{az} = \frac{PRF}{f_{rate}}$. As with all azimuth responses, we center our chirp on a carrier frequency equal to the Doppler centroid frequency $f_{DC} \approx -121.2007$ Hz. However, before we can place any given patch into our image matrix, we must ensure that every azimuth line contains only valid data; in other words, we must remove convolution wrap-around from azimuth matched filtering. As in the range, to ensure that every patch contains only valid data, we subtract from each patch the number of samples equal to the longest azimuth chirp pulse:

$$max\{N_{azimuth\ chirp}\} = \left\lceil \frac{PRF}{min|f_{rate}|} \times PRF \right\rceil = \left\lceil \frac{PRF^2\ \lambda \left(r_0 + N_{range\ bins} \times \frac{c}{2f_s}\right)}{2\ v_{eff}^2} \right\rceil \approx 1752 \text{ azimuth chirp samples.}$$

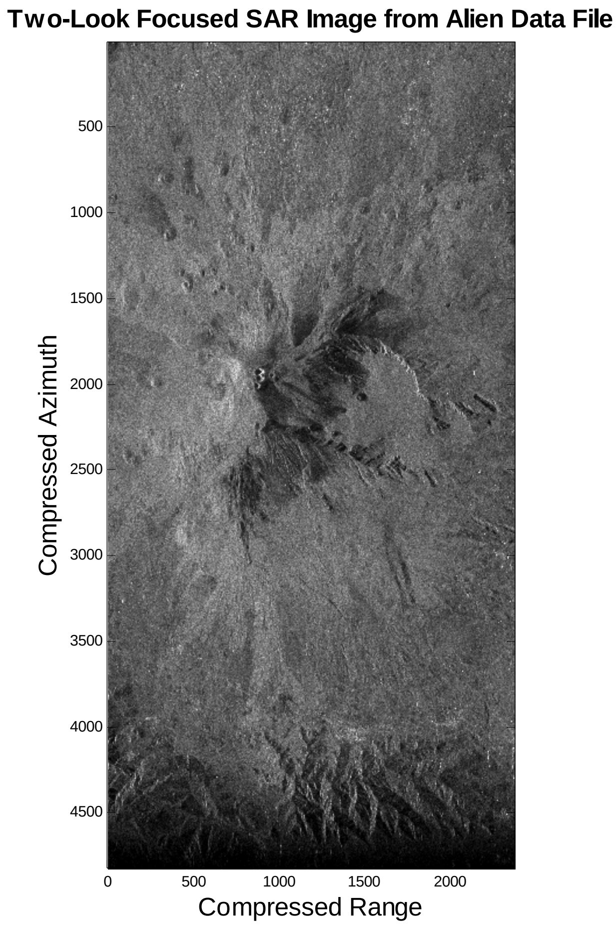
Normally, we would need the azimuth beamwidth to determine the appropriate pulse length to remove, but since we assume that our beamwidth spans the entire azimuth spectrum ($f_{DC} \pm$ PRF), we simply remove the number of samples from the longest (most distant) chirp. The resulting of this matched filtering yields a focused SAR image.

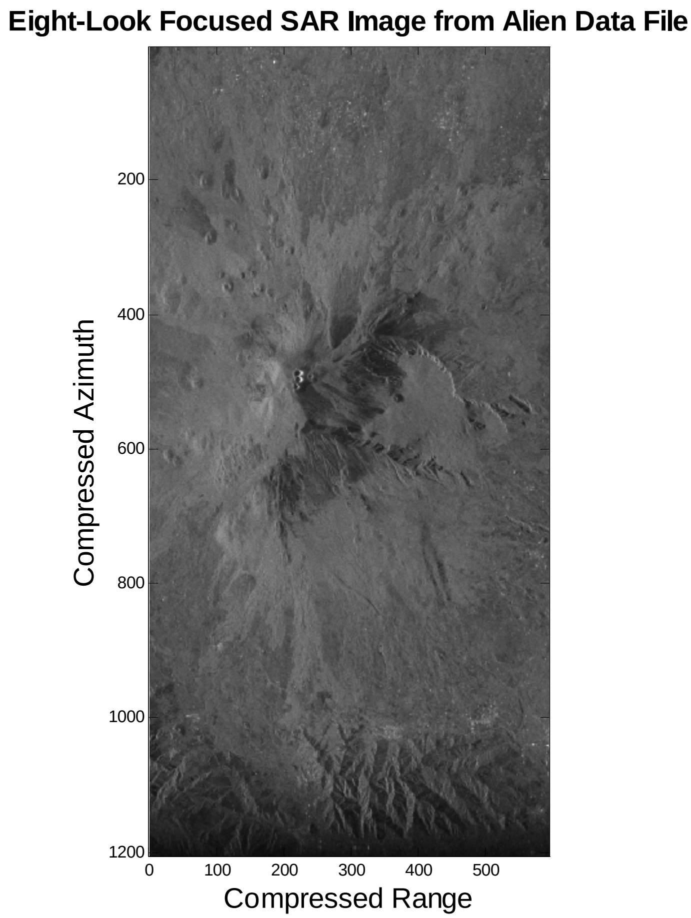
However, to purge our azimuth-compressed image of the speckle noise that plagues our picture with undesirable graininess, we must obtain two or more looks in the azimuth direction to average – and therefore smoothen – the noise power across several lines. However, we note that our azimuth ground pixel spacing $\Delta x_{azimuth} = c \cdot \frac{PRF}{f_{rate}} \approx 4.335261 \frac{m}{pixel}$ approximately matches our range ground bin spacing $\Delta x_{range} = \frac{c}{2f_s \sin\theta_i} \approx 4.270818 \frac{m}{pixel}$ with only one azimuth look and one range look, so, if we choose to acquire additional azimuth looks, we must also acquire an equal number of range looks to maintain approximately square pixels in a visually pleasing image. Thus, taking multiple looks along both the range *and* azimuth directions, we can exchange the extreme precision of two looks for the blurrier but less noisy display (of four looks). Both range migration-corrected images clearly display a mountain in high SAR resolution:





This dichotomy exists because the additional looks not only reduce speckle noise but also average a larger number of pixels, effectively degrading resolution by blurring the combined pixels into a larger ground resolution cell. Thus, if we do not mind (or cannot notice) slight blurriness, then the additional looks will actually benefit us by removing the unwanted grains of noise and by reducing the size of our image, allowing us to store it more efficiently. However, if we take more than five looks, the combination of more and more pixels eventually makes the blurriness intolerable, as the lack of resolution and point recognition soon becomes visibly evident. Compare, for example, the two-look and eight-look images, displayed on a lighter scale to enhance contrast:





Notice, in particular, that the speckle noise is completely absent in the eight-look image from the massive averaging and combination of resolution cells, but also notice that our visual acuity of point-like features also degenerates with the speckle noise. Thus, if we want to reduce the noise that plagues our single-look and double-look images, we must surrender resolution as we take more looks. The best compromise between low noise and high resolution appears to result from four looks, although aesthetic judgment may vary with the viewer; beauty lies in the eye of beholder. My personal favorite, the four-look image, completely corrected for range migration (with no PRF ambiguity), occupies the next page.

Four-Look Focused SAR Image from Alien Data File