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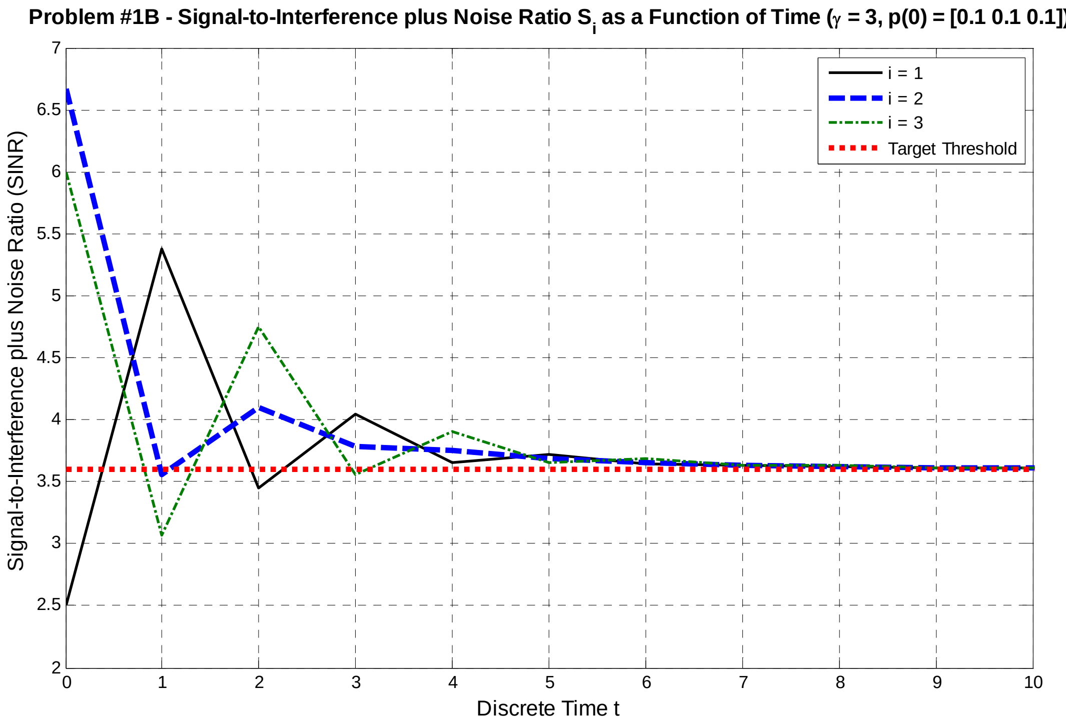
EE 263 – Linear Dynamical Systems

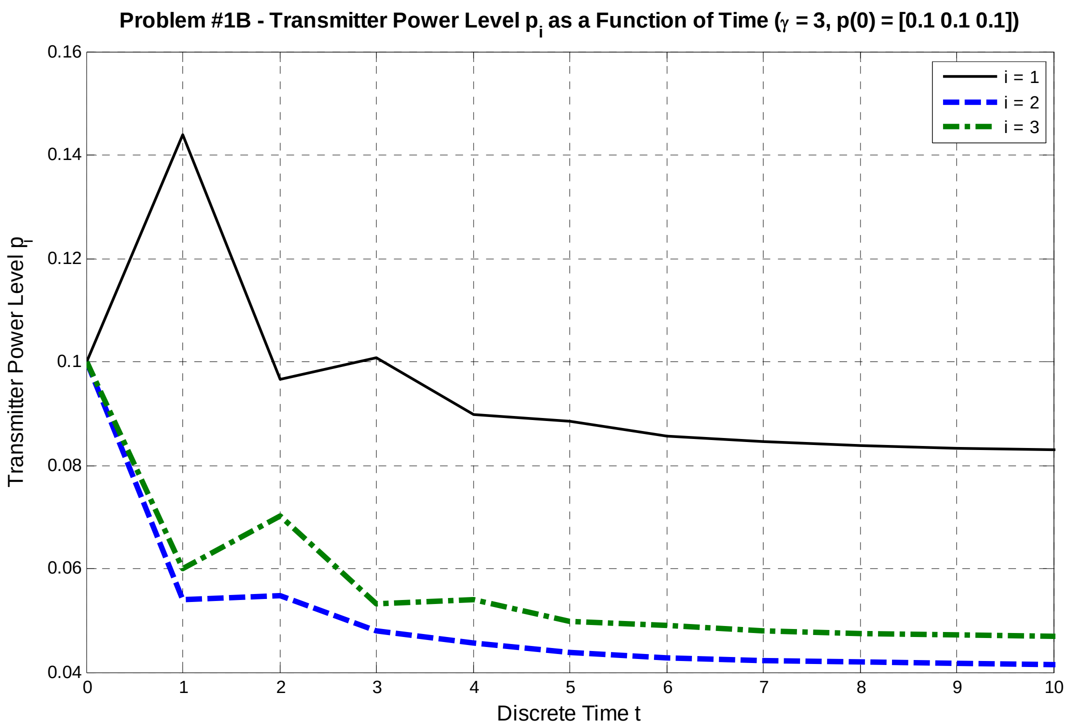
Problem Set I

Problem #1: A Simple Power Control Algorithm for a Wireless Network

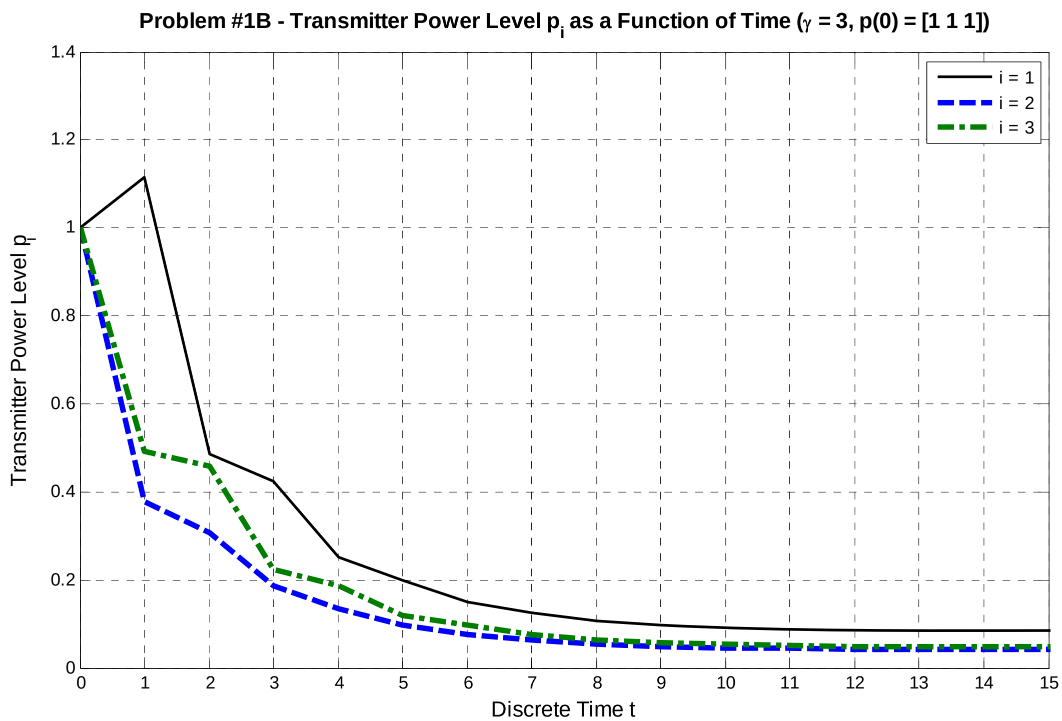
Setting $\gamma = 3$, we obtain the Signal-to-Interference plus Noise Ratio (SINR) at each receiver as a function of time:



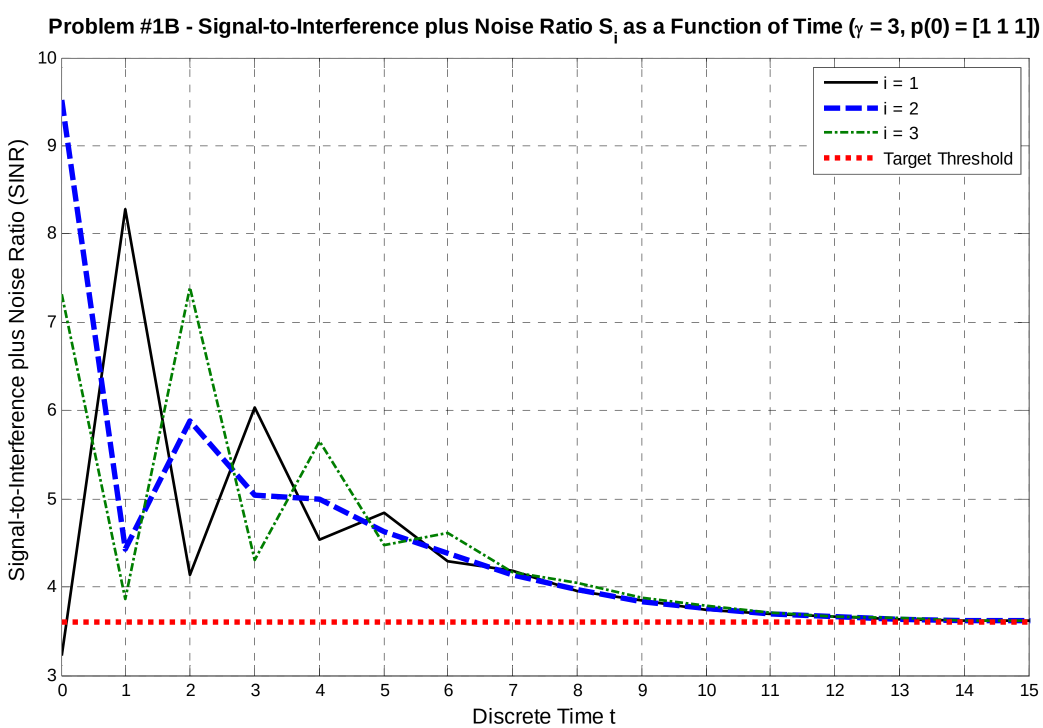
Initially, the second and third receivers receive unnecessarily high SINRs, while the first receiver operates well below threshold, but, fortunately, within ten seconds, all of the receivers boast an SINR of nearly exactly $\alpha\gamma = 3.6$, which is safely above threshold as desired. Meanwhile, the transmitter powers also converge:



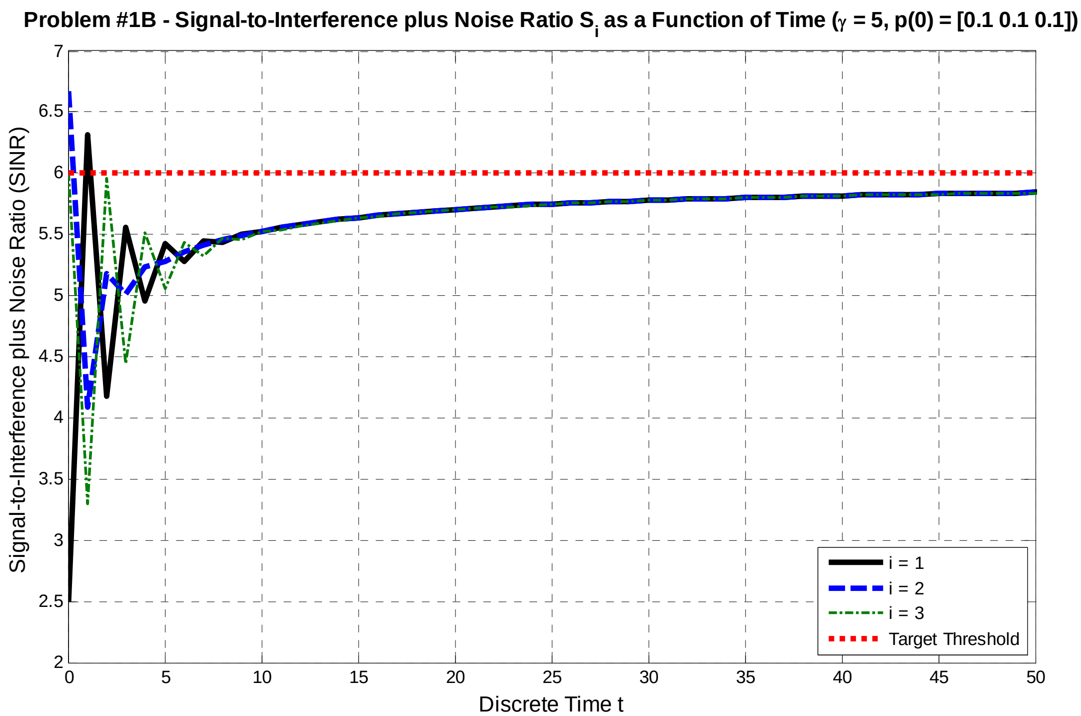
Even though the transmitter powers begin at the same value (by my design), the eventual transmission powers decay to smaller values. The first transmitter power encroaches 0.08, the second transmitter power approaches 0.05, and the third transmitter power converges toward 0.04. In brief, all of the transmitters behave tractably under this simple power control algorithm. Even as we heighten the initial transmit powers to [1 1 1], the individual transmit powers still converge:



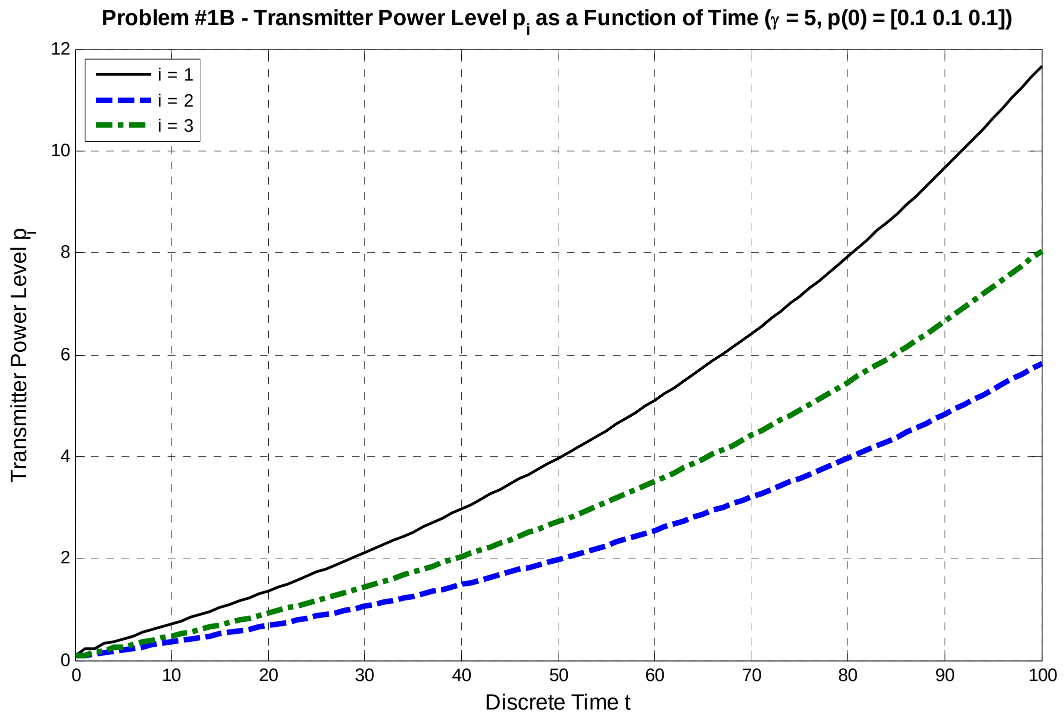
Even at this heightened initial power, all three transmission powers decay to reasonably low values (less than 0.1). Meanwhile, the three SINRs comfortably converge to the safety threshold, revealing that our power control algorithm satisfies the threshold condition with stable transmitted powers.



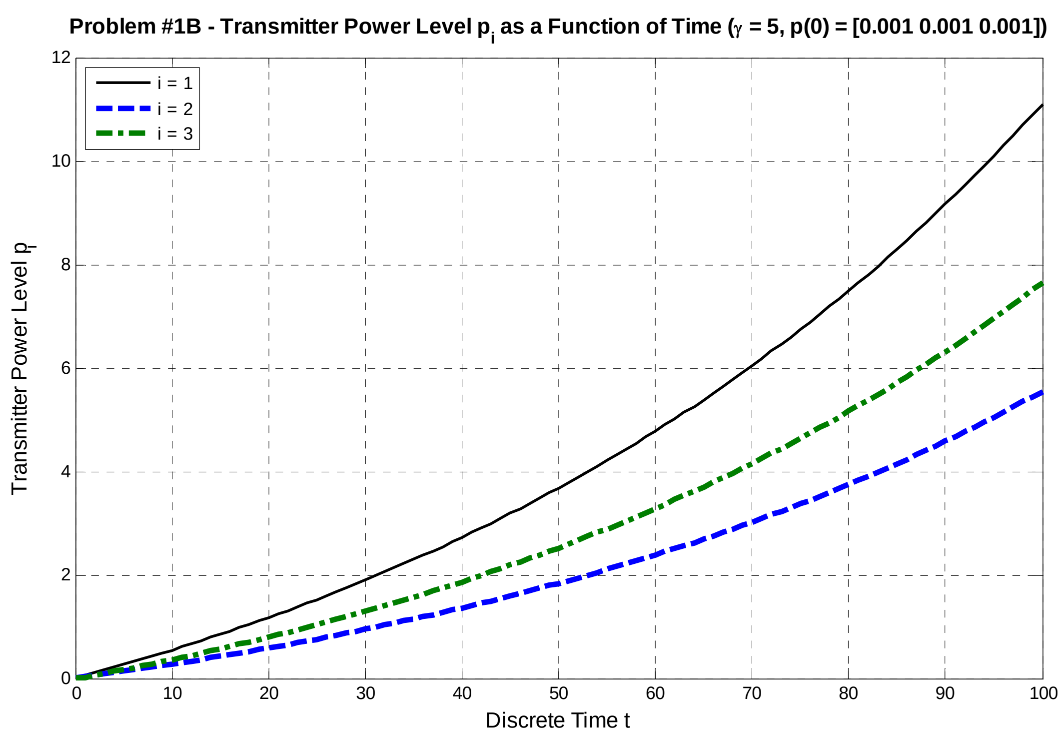
However, once we set $\gamma = 5$, essentially increasing the desired threshold and forcing higher transmit power, our control system collapses. For one, the SINR of each transmitter approaches the safety threshold from below, indicating that not a single SINR will ever reach the mandated safety threshold:



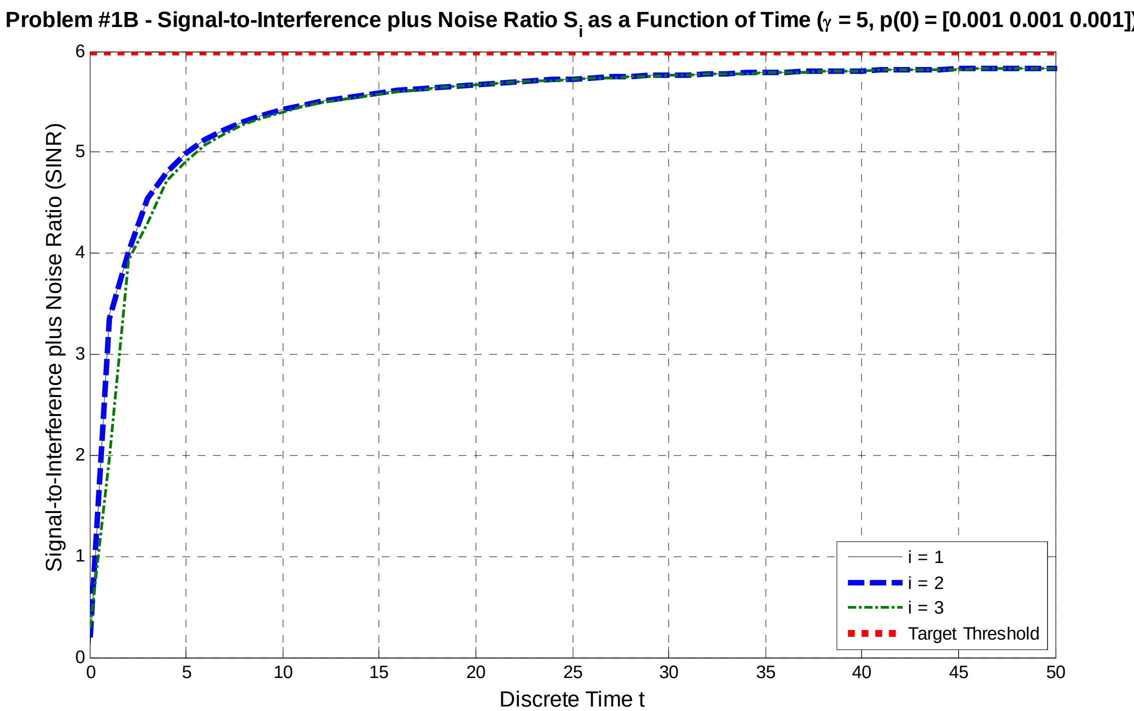
Furthermore, the transmission powers of our transmitters diverge, as the transmitters require more and more power to meet the SINR requirements as time progresses:



In an attempt to decrease the undesirably unstable transmission power evolution, we might decrease the initial transmit powers to p(0) = [0.001 0.001 0.001], assuming as we have that the transmitters are identical. However, even this lower initial power does not prevent divergence:



Studying also the SINR behavior with this new set of initial conditions, we also see, to our dismay, that the receiver SINR values never really attain the safety threshold of $\alpha\gamma = 6$. While the receiver SINRs do *approach* this value, the convergence occurs from *below*, so the actual SINR always falls below the safety threshold of 6, actually falling woefully short before $t = 20$ and therefore risking potential ruination of the first part ($t < 20$) of the communicated signal:



Thus, our algorithm simply cannot control the system to satisfy this higher threshold at $\gamma = 5$.