Christopher Tsai
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EE 106 – Planetary Exploration

Problem Set I
Observing the Sky: The Birth of Astronomy

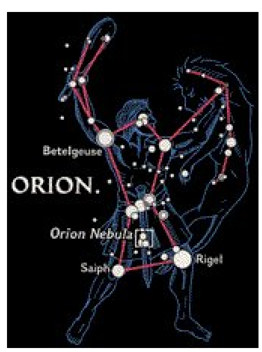
Problem 1.1

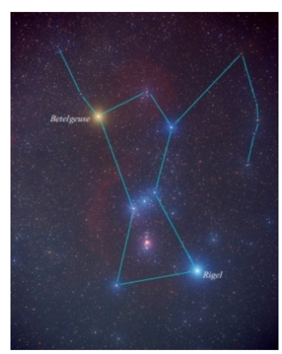
The ideal terrestrial locations from which one can observe all the stars over the course of a year lie along the Earth's **equator**, which, due to the Earth's rotation, will face each visible star for half of each day; under the dominance of sunlight, certain stars may not be visible during daytime hours, but they will likely enter view during another part of the day. Observers at non-equatorial locations might miss certain stars that happen to rise and set along the opposite hemisphere.

Because both poles remain fixed on the surface of Earth during rotation, polar observers can never see more than **one half** the sky from either pole. The North Pole views only $\frac{1}{2}$ of the sky.

Problem 1.3

Modern astronomers define a *constellation* as "one of 88 sectors into which we divide the sky" (Fraknoi 25). In other words, a constellation defines a named grouping of stars (and other nearby objects) occupying a certain region in the sky; the set of all 88 constellations partition the sky.



An astronomer who claims to have seen a comet in Orion is saying that she witnessed a comet moving through the sector of the sky that we have designated as *Orion*; she must have seen a comet in the region of sky that contains, among other entities, the stars that form the shape of the hunter Orion. The comet may or may not have actually passed through the stars themselves; her statement reveals only that she saw the comet in the sector of sky named after Orion.

Problem 1.5

1.) Galileo discovered and argued that rest is not the natural state of objects; rather, objects at rest continue to rest, while objects in motion continue to move, until external forces interact with the object; in other words, Galileo discovered that *forces* are required to either move a still object or stop a moving object.
2.) Galileo discovered that, regardless of their shape and size, objects rolling down a ramp or free-falling in the air accelerate uniformly.
3.) With the aid of his telescope, Galileo found that the Milky Way comprises a multitude of individual stars, which previously seemed like only indistinct blurs.
4.) Examining the planets, Galileo located Jupiter's four moons and discovered that they orbited Jupiter with periods spanning two to seventeen days.
5.) Galileo discovered that the planet Venus changes phase much like the Earth's moon; thus, Venus must orbit the sun, since she changes orientation with respect to her light source.
6.) Examining Earth's moon closely, Galileo observed imperfections in the moon's surface, such as craters, mountain ranges, valleys, and flat dark areas thought to be water.

Problem 1.10

Even though the Earth rotates about its own axis as it revolves around the sun, ancient astronomers could not witness stellar parallax because stars were too far away from Earth's surface. In other words, the amount of apparent shift resulting from the motion of Earth was immeasurably small, beyond all terrestrial observers' ability to perceive; because parallax grows less and less noticeable for objects farther and farther from the moving observer, the distance from Earth to the stars made any slight side-to-side motions of the stars impossible for ancients to perceive.

Problem 1.16

If Eratosthenes had found that, in Alexandria at noon on the first day of summer, the line to the sun makes an angle of 30° with the vertical, then, assuming that the line from Syene to the sun is nearly vertical, Eratosthenes would have claimed that the Earth's surface curved by approximately 30° (1/12 of a full circle) between the two cities. Consequently, he would have concluded that Alexandria must lie 1/12 of Earth's circumference north of Syene. Since the physical distance separating Alexandria and Syene was 5000 stadia, Eratosthenes would have computed Earth's circumference to be (12)·(5000 stadia) = $\boxed{\textbf{60,000 stadia}}$. If Eratosthenes standardized his measurement using a stadium of 1/6 km, then his estimate of circumference would be $\boxed{\approx\textbf{10,000 km}}$.

Problem 1.17

If Eratosthenes' results for the Earth's circumference (250,000 stadia) were quite accurate, and if the diameter of the Earth is 12,740 km, then the length of his stadium is approximately

$$\boldsymbol{l_{stadium}} = \frac{\boldsymbol{\pi \cdot d\ [km]}}{\boldsymbol{C_{Earth}\ [stadia]}} \approx \frac{\boldsymbol{\pi \cdot 12740\ \mathrm{km}}}{\boldsymbol{250{,}000\ \mathrm{stadia}}} \approx \boxed{\textbf{0.1600956 km}}.$$

Problem 1.19

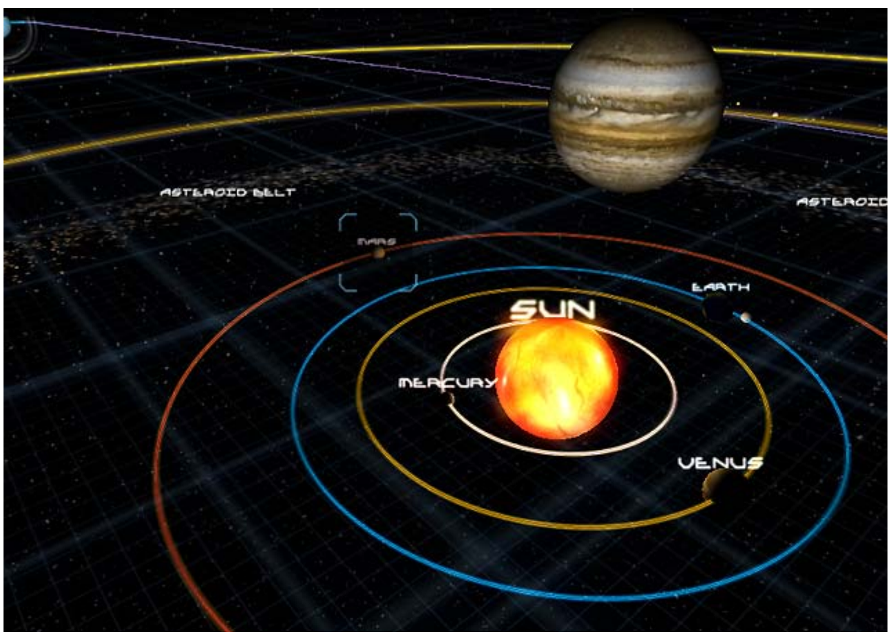
Assuming that the strange planet on which we have landed is approximately spherical like Earth, we begin our journey at one of the two poles, since the only locations from which one can see stars circling parallel to the horizon are the poles. Because our next location sees stars rising straight up in the east and setting straight down in the west, our 8000-mile trek must bring us from a pole to a point on the planet's equator; only observers at equatorial points can witness stars rising and setting perpendicularly to the horizon. As a result, our 8000-mile journey must have spanned exactly one-fourth of the planet's circumference, since we start at a pole (±90° latitude) and end at the equator (0° latitude). Thus, the strange planet's full circumference must be four times the distance that we walked: $\boxed{\boldsymbol{C \approx 32{,}000\ \textbf{miles}}}$.

Problem 2(a.) – Modern Constellations

There are **eighty-eight (88)** modern constellations, each of which contains tens, hundreds, or even thousands of stars of various brightnesses. Thus, because the International Astronomical Union (IAU) partitioned the sky into eighty-eight constellations based only on locality of stars, no deliberate limit exists to the number of stars in a constellation; while the number of stars in each constellation is finite, the IAU made its division without considering relative star counts.

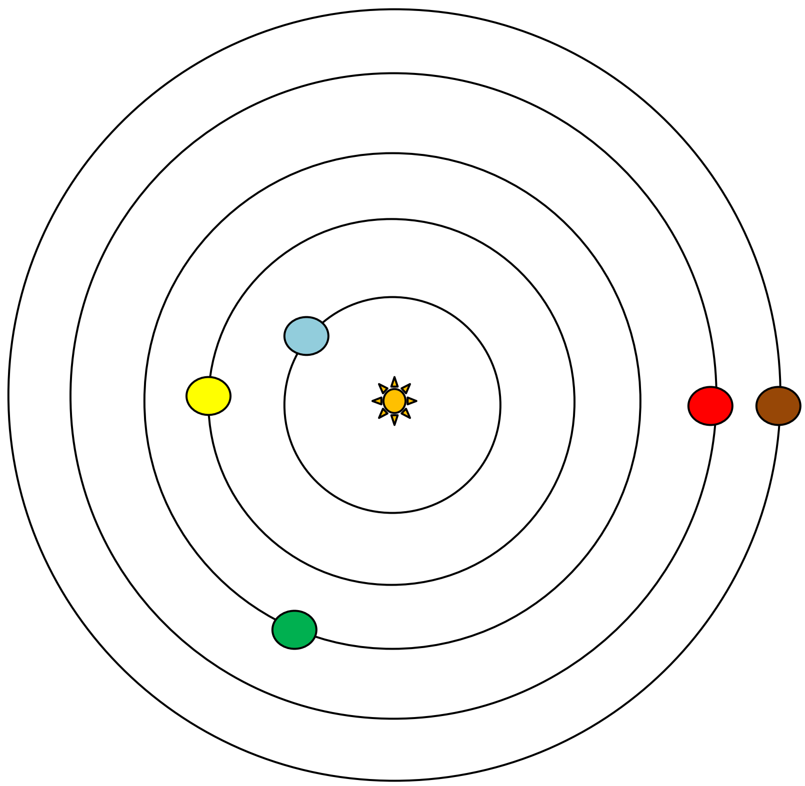
The group of constellations through which the Sun, Moon, and planets pass is called the **zodiac**. Even though the original astrologers classified twelve constellations in the zodiac, the recent addition of Ophiucius increases the number of constellations in the zodiac to **thirteen**: Capricornus, Aquarius, Pisces, Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpius, Ophiucius, and Sagittarius.

Problem 2(b.) – Alignment with the Sun



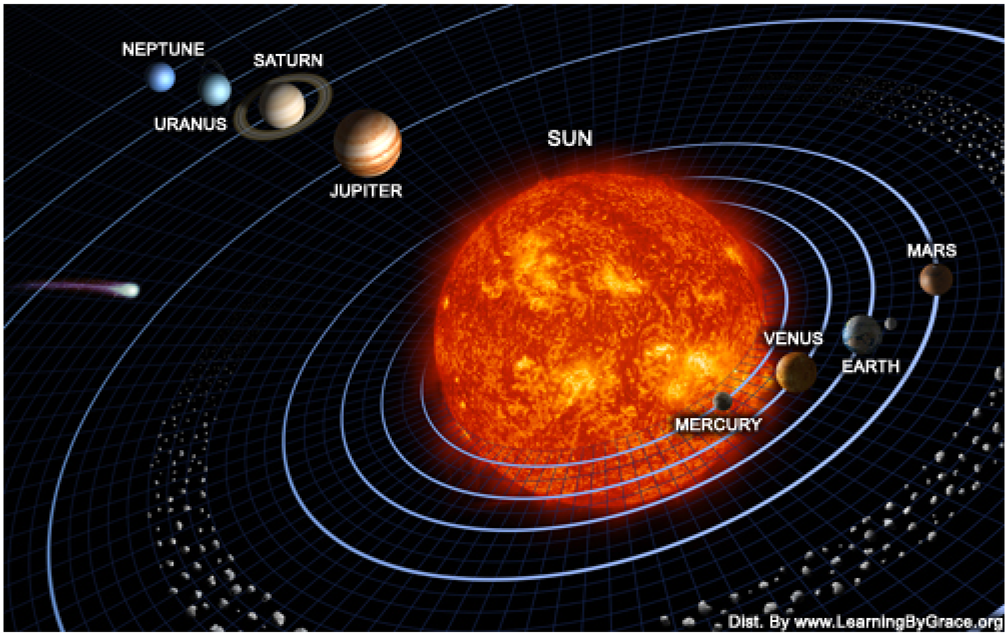
Neither **Venus nor Mercury** can ever appear in the opposite direction in the sky from the Sun, because both these planets are much closer to the Sun than the Earth, meaning that they always appear close to the Sun in our sky. The **Sun and Earth** can also never appear opposite the Sun.

Likewise, because they are closer to the Sun than they are to the Earth, **Venus and Mercury** (and obviously the **Sun and Earth** themselves) cannot possibly appear at an angle of 90° from the Sun, since the hypotenuse (line segment connecting a planet to the Sun) of any right triangle must be longer than either leg (line segments connecting Earth to the Sun and planet):



Mercury can never stray more than 23° from the Sun, while Venus never appears more than 46° from the Sun. The outer planets can appear at any angle, since they can lie on the opposite side of the Sun to Earth (0°), behind Earth (180°), or anywhere in between (like ±90°).

All of the planets except the **Earth** can appear in the same direction as the Sun from a terrestrial vantage point, since the planets between Earth and the Sun (Venus and Mercury) can appear in front of the sun, while the outer planets (Mars and Jupiter) can appear on the opposite side of the Sun, assuming that they are not too far away to see from Earth:



However, it might be difficult for the naked eye to observe a star or the Moon right beside the Sun, since strong sunlight would obscure the Moon and small stars.

Problem 3(a.) – Phases of the Moon

We experienced a full moon on April 2, and we will see no moon on April 17. Thus, assuming we can see the sky, we should see the **waning gibbous** moon (phase F) on the night of April 6, as the full moon continues waning toward new moon, and nearly a **third-quarter** moon (phase G) on April 9, since the phase will be approximately halfway between full moon and new moon. Finally, on April 24, one full week after the new moon and therefore one quarter cycle through the phase, we should see nearly a **first-quarter** moon (phase C).

Problem 3(b.) – Easter Sunday

Two predominant methods exist to determine the date of Easter. The ecclesiastical rules designate Easter as the Sunday following the first ecclesiastical full moon on or after the vernal equinox (March 21). Tables define the first ecclesiastical full moon as the moon appearing fourteen days after a new moon. Altogether, these rules mean that Easter occurs on the first post-March 21 Sunday following a full moon (fourteen days after a new moon).

The astronomical rules similarly celebrate Easter on the Sunday immediately following the first full moon after the vernal equinox; however, these rules hold that the vernal equinox occurs when the apparent ecliptic longitude is zero, so March 21 is no longer the fixed earliest possible date of Easter. Instead, the possible dates for the pre-Easter full moon begin as soon as the ecliptic longitude reaches zero, and, when that full moon occurs, Easter falls on the next Sunday.

Problem 4 – Kepler's Third Law

Moon	Orbital Period P [sec]	Semimajor Axis a [m]	P^2/a^3 [sec^2/m^3]
EARTH			**EARTH**
Earth's Moon	2360448	384000000	9.84E-14
MARS			**MARS**
Phobos	27648	9400000	9.20331E-13
Deimos	108864	23500000	9.13198E-13
JUPITER			**JUPITER**
Io	152928	422000000	3.11198E-16
Ganymeade	618624	1070000000	3.12394E-16
Ananke	54518400	21200000000	3.11945E-16
SATURN			**SATURN**
Titan	1378080	1222000000	1.04072E-15
Iapetus	6851520	3561000000	1.03958E-15

As the astronomical data reveals, the value of $\frac{P^2}{a^3}$ remains approximately constant for any planetary system. For Earth, that constant appears $\approx 9.84 \times 10^{-14}$; for Mars, the constant is $\approx 9.2 \times 10^{-13}$; for Jupiter, that value is $\approx 3.12 \times 10^{-16}$; and for Saturn, the system constant seems to be $\approx 1.04 \times 10^{-15}$.

We can trace the reason for this seeming mystery back to Newton's Second Law, which holds that any sort of acceleration requires a force; applying this to an orbiting body of mass m centripetally accelerating about a much more massive planet of mass M under the force of gravity, Newton enlightens us to the relationship:

$$F_{gravity} = m \cdot a_{centripetal}$$

$$\frac{G \cdot M \cdot m}{r^2} = m \cdot \frac{v^2}{r}$$

$$\frac{G \cdot M}{r^2} = \frac{v^2}{r}$$

$$\frac{G \cdot M}{r} = v^2$$

$$\frac{1}{r \cdot v^2} = \frac{1}{G \cdot M}$$

$$\frac{1}{r \cdot \left(\frac{2\pi r}{P}\right)^2} = \frac{1}{G \cdot M}$$

$$\frac{1}{r \cdot \left(\frac{4\pi^2 r^2}{P^2}\right)} = \frac{1}{G \cdot M}$$

$$\frac{P^2}{4\pi^2 \cdot r^3} = \frac{1}{G \cdot M}$$

$$\boxed{\frac{P^2}{r^3} = \frac{4\pi^2}{G \cdot M}}$$

Assuming that the moons' elliptical orbits are nearly circular, the semimajor axis is approximately the radius of the uniform circular orbit: $a \approx r$:

$$\boxed{\frac{P^2}{a^3} = \frac{4\pi^2}{G \cdot M}}$$

Since *G* is the universal gravitational constant, the right-hand side of the equation remains constant for all bodies orbiting the same central mass *M*; consequently, the left-hand side, too, depends only on the mass *M*. Therefore, since the quantity $\frac{P^2}{a^3}$ depends only on the mass *M* of the central planet, all moons orbiting the same planet should have matching values of $\frac{P^2}{a^3} = \frac{4\pi^2}{G \cdot M}$, where *M* is the planet's mass. Obviously, this constant will differ for planets of different masses, as we saw in data.