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EE 106 – Planetary Exploration

Problem Set III
Radiation and Spectra

Problem 3 – Blackbody Radiation in Planetary Exploration and Astronomy

Blackbody radiation is integral to planetary exploration and astronomy because radiation spectra serve as primary sources of information concerning both the temperature and composition – chemical and material – of bodies which we are interested in studying.

For one, Wien's Displacement Law relates the peak wavelength in an object's radiation spectrum to the blackbody temperature, allowing us to approximate the temperature of planetary surfaces, like the Sun, for example, by mere examination of the emitted spectrum.

Secondly, radiation and absorption spectra reveal which wavelength bands contain maximal (or minimal) content, allowing us to infer how planetary bodies interact with incident electromagnetic waves – either due to surface material and dielectric constant or atmospheric processes. For example, we can attribute rising temperatures to certain chemical compounds (like carbon dioxide) that partially obfuscate Earth's transmission spectrum in key wavelength bands. Likewise, we can begin to characterize atmospheric content around large stars and planets by observing the amount of reflection, scattering, or absorption of blackbody radiation from the planet, as each element has a distinct spectral signature.

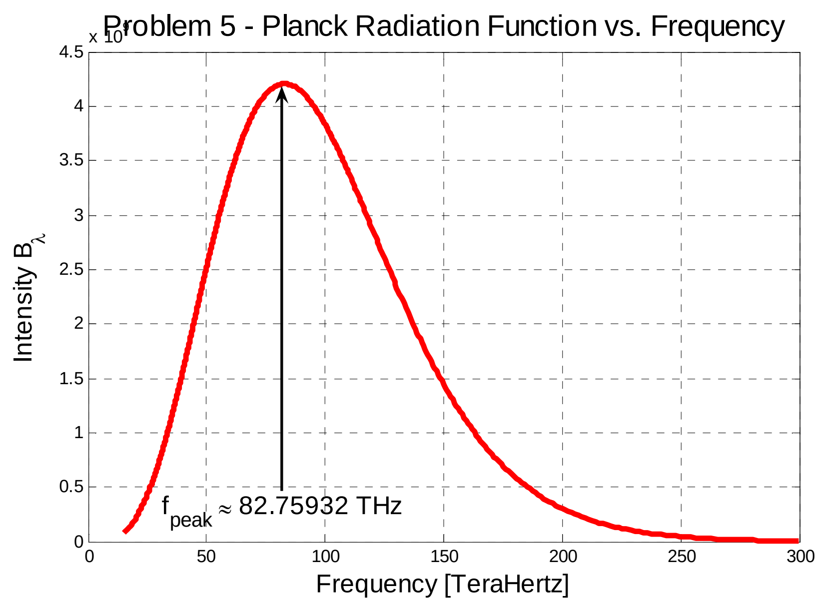
Finally, quantifying blackbody radiation from moving bodies is vital to deriving velocity information from the Doppler equation, which relates perceived wavelength shift to the motion of the emitting body.

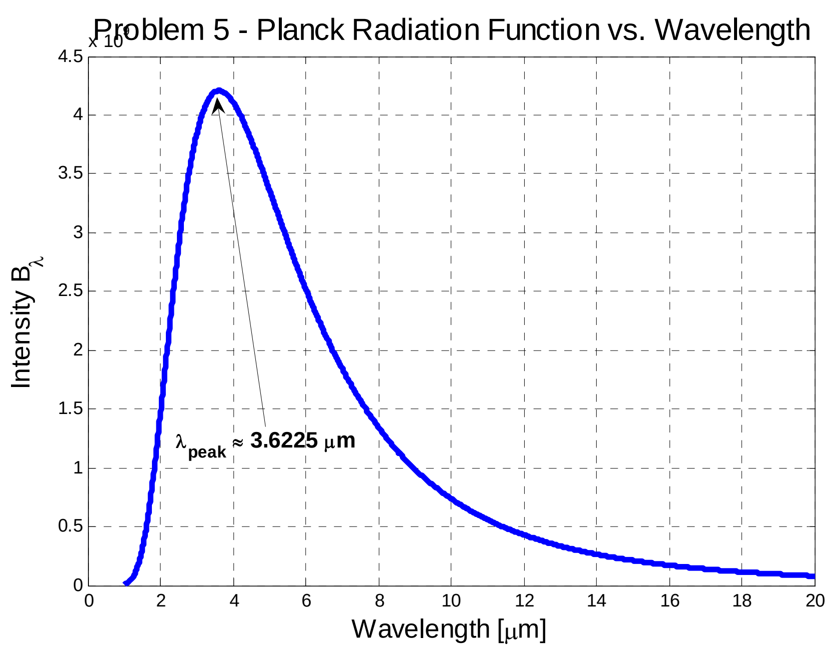
In brief, treating bright objects and planets as blackbodies makes perfect sense, even if they do not emit like ideal radiators or absorb all incident light; in fact, the closest realizable blackbody is a small hole entrance to a large and dark, soot-coated cavity. Nevertheless, bright objects and planets emit and absorb electromagnetic radiation in close accordance with Planck's Radiation Law.

Problem 4 – Radar Echoes from the Sun

The detection of radar echoes from the Sun surprised many scientists because the Sun, which emits electromagnetic radiation like a blackbody, should absorb all incident radiation; the fact that it possibly scattered or returned some radar echoes raised questions about the Sun's status as an ideal blackbody absorbing all incident radiation falling upon it.

Problem 5 – Planck's Blackbody Radiation Spectrum





Problem 4.14 – Red and Blue = Hot and Cold?

Water faucets are often labeled with a red dot for hot water and a blue dot for cold water. From a physical standpoint, the color red connotes fire, which is generally quite hot, while the color blue connotes ice, which is generally quite cold. However, by Wien's Law $\left(\lambda_{peak} = \frac{2.898\times10^{-3}\text{ m K}}{\text{T}}\right)$, blackbodies at higher (hotter) temperature actually radiate more at *shorter* wavelengths, closer to the *blue* end of the visible spectrum, whereas blackbodies at lower temperature actually radiate more at *longer* wavelengths, closer to the *red* end of the spectrum. Thus, Wien's Law contradicts layman sense because it holds that hotter objects radiate most in the *blue* while colder objects radiate in the *red*.

Problem 4.17 – Orbital Speed from Spectral Snapshots

Assuming that the star lies in the ecliptic plane, we can measure Earth's orbital speed by photographing the spectrum of the star at various times throughout the year and obtaining the variation of the star's radial velocity (toward the Sun) from the Doppler shift. In other words, the radial component of the star's velocity relates to the wavelength shift through $v_r = c \cdot \frac{\Delta\lambda}{\lambda}$. By measuring the wavelength change $\Delta\lambda$ through redshifts or blueshifts in the spectrographs and removing the effect of Earth's rotation, we can ascertain Earth's orbital velocity relative to the Sun.

Problem 4.24 – Star Temperature and Brightness

Two stars with identical diameters are equidistant from Earth. Star A has a temperature of 5800 K, whereas Star B has a temperature of 2900 K. The Stefan-Boltzmann Law $(R = \sigma T^4)$ holds that radiated energy flux depend on the fourth power of absolute temperature. If we assume that the energy flux (or emissive power) of the radiating surface directly affects the brightness we perceive, then the brightness of a radiating object varies with the fourth power of the object's temperature. Hence, because Star A is twice as hot as Star B, **Star A is 2^4 = 16 times brighter than Star B**.