

## **Problem Set VII**

### **Rings, Moons, Comets, and Asteroids**

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#### **Problem 11.16 – Impact Craters on the Surface of Titan**

The surface of Titan contains few impact craters, with only three impact craters – Menrva, Ksa, and Sinlap – definitively identified and named as of August 2006, and only a few more unconfirmed impact structures seen from Cassini. Certainly, no small impact craters can pepper the face of Titan because the thick atmosphere would vaporize smaller projectiles, essentially shielding the surface from small impact threats. Meanwhile, the active surface and atmospheric processes of Titan constantly rework the surface of its medium-sized craters, eroding and burying existing impact craters regularly, with some aid from viscous relaxation. All in all, as we would expect from a moon shielded by such a thick atmosphere in the presence of such active chemistry, small craters are nearly non-existent, while even medium craters are few and far between.

#### **Problem 11.17 – The Color of Particles in Rings**

One possible explanation for the brightness difference between the particles in Saturn's rings and the particles in Uranus and Neptune's rings relates to the planets' relative distances from the Sun. Farther from the Sun, more and more asteroids and comets in the outer solar system have the dark structure seen in the ring particles of Uranus and Neptune, suggesting that these other objects could have fragmented to contribute to the ring particles; furthermore, Uranus' ten inner satellites also chemically comprise black carbon and hydrocarbon compounds, suggesting that an inner moon could also have fragmented into ring particles, therefore explaining their chemical similarity. Saturn, on the other hand, is closer to the Sun, and its ring particles comprise mainly water ice rather than the black carbons hypothesized to exist in Uranus and Neptune's rings. Thus, the difference in brightness likely results from the particles' differences in chemical composition and consequently from the types of projectiles pulled or broken into the rings.

### Problem 11.22 – Revolution Period of Saturn’s Rings

Assuming that the dust and particles in Saturn’s rings are much smaller than the planetary body’s mass, we tap Kepler’s Third Law to determine the orbital period of the inner and outer edge particles in Saturn’s rings. Kepler’s Third Law holds that

$$T = \frac{2\pi\sqrt{a^3}}{\sqrt{G \cdot M_{\text{Saturn}}}}$$

For the innermost set of rings,

$$T_{\text{inner}} = \frac{2\pi\sqrt{(75,000,000 \text{ m})^3}}{\sqrt{\left(6.672 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.684 \times 10^{-26} \text{ kg})}} \approx 20,955.2955 \text{ seconds} \approx 5.8209 \text{ hours}$$

For the outermost set of rings,

$$T_{\text{outer}} = \frac{2\pi\sqrt{(137,000,000 \text{ m})^3}}{\sqrt{\left(6.672 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.684 \times 10^{-26} \text{ kg})}} \approx 51,734.7965 \text{ seconds} \approx 14.3708 \text{ hours}$$

The difference between the two periods is approximately

$$\Delta T = T_{\text{outer}} - T_{\text{inner}} \approx 30,779.5 \text{ seconds} \approx \mathbf{8.54986 \text{ hours}}$$

### Problem 12.19 – Comets in the Oort Cloud

If ten new comets approach the Sun each year, then, assuming that the solar system is approximately 4.5 billion years old, 45 billion comets have been “used up” since the beginning of the solar system, which amounts to approximately

$$\frac{45 \times 10^9 \text{ comets used up}}{10^{12} \text{ comets in Oort cloud}} = 0.045 = 4.5\%$$

therefore suggesting that approximately 4.5% of comets have vaporized since the solar system’s origin.