Summary

River trip is an interesting and popular activity around the world. This paper discusses river trip scheduling strategies for *Big Long River*. With a length of 225 miles, *Big Long River* has *Y* campsites along, and currently it has *X* trips during the six months of open period annually. We focus on scheduling strategies which would better utilize the campsites and minimize travelers' contact with other groups of people.

To simplify the problem, we first put forward some assumptions. Based on these assumptions, we bring forward two models to discuss the scheduling strategy for ideal situation and common one respectively.

In an ideal situation, model 1, *Maximum Capacity Model,* shows the best way to maximize number of trips and to minimize the contact of people in different trips to zero. The maximum number of trips possible annually is 2928 trips. Night of campsites occupied is 17568 annually.

In a more common situation, model 2, the *Energy Field Model,* gives a satisfactory result. In this model, boats on the river are modeled as particles each "dragged" by several forces in an energy field. These forces work together and make up an energy field which determines the action of the "boat particles" in it. We obtain a schedule for the 6 months' Big Long River trips by simulation.

In this schedule, there are 2227 trips every year. Although each boat has some contact with other boats, however, campsites on the river are utilized even more than schedule in model 1. In addition, trip mix in Energy *Field Model* is diverse, which caters to people's various preference.

We then compare the result of model 2 with the river trip schedule of *Grand Canyon River,* and show that our schedule shares certain features with the real one, suggesting that our result meets people's preference to a certain degree.

We also analyze in details how the changing of number of campsites affects the result and shows the effective range of Y to obtain a reasonable result from model 2. To improve model 2, we also provide a solution to deal with Y out of the effective range.

Memos for Managers

- We first obtain the optimal schedule which utilizes the campsite as much as possible with Model 1, Maximum Capacity Model.
- Maximum Capacity Model is based on an ideal situation and we assume Y=224 with one campsite every one mile from the start line.
- In schedule derived from Maximum Capacity Model, there are 16 6-night motor-propelled trips every day. The maximum number of trips possible in a six-month open period (183 days) is 2928 trips. Night of campsites occupied is 17568 annually.
- Model 2, *Energy Field Model,* is based on some common premises.
- In *Energy Field Model,* boats on the river are modeled as particles each "dragged" by several forces in an energy field: repulsion force from other boats on the river, force of time in a day, and force of schedule. These forces work together and make up an energy field which determines the action of the "boat particles" in it.
- With *Energy Field Model*, we obtain a schedule for the 6 months' Big Long River trips by simulation.
- In this schedule derived from *Energy Field Model*, there are 2227 trips every year, or more than 12 trips on average each day. Although each boat has approximately 2.42 hours per day during which it has contact with other boats, however, campsites on the river are utilized even more than schedule in model 1. In addition, trip mix in *Energy Field Model* is diverse, which caters to people's preference for trips of different duration.
- Evaluating the results from these aspects, we believe the schedule derived *from Energy Field Model* is better, which not only meets market demand, but also helps to utilize the campsites better and boost number of trip.
- Compared the result of model 2 with the river trip schedule of *Grand Canyon River,* our schedule shares certain features with the real one, suggesting that our result meets people's preference to a certain degree.
- Sensitivity analysis in details available on how the changing of number of campsites affects the result.
- An approach provided to improve *Energy Field Model.*

Table 1: List of Notes

Notes	Definitions
C_m	Month Capacity
d_i	The days boat i has spent on the river
d_{std}	Standard trip duration
N_i	The number of nights on which campsite $i(i \le Y, i = 1, 2, 3, ...)$ is occupied annually
f_b	Sum of forces from other "boat particles"
$f_{b-ahead}$	Forces from the boats ahead
$f_{b-behind}$	Forces from the boats behind
$f_c(i)$	The compound force of all the forces in the energy field for boat i .
f_t	Forces from time in a day
$f_s(i)$	Forces from schedule for boat i
$G_{camp}, G_{setoff}, G_{v4}, G_{v8}$	Threshold of compound force which determines the actions of "boat particles"
i, j	Index of boats on the river
l_i	distance of boat i from the start
$L = 225 miles$	Length of Big Long River
v_i	Velocity of boat i
t_c	Current time
$t_e = 6pm$	End time of sailing time in a day
$T_1, T_2, T_{31}, T_{32}, \varepsilon$	Constants
T_c	Contact Time
N_m	Night of Campsites Occupied
X	Current number of trips annually
Y	Number of campsites

Contents

1. Introduction

1.1 Interpretation of the problem

We are required to provide scheduling strategies for *Big Long River* to meet the increasing demand for river trips. *Big Long River* (225 miles long), with Y campsites distributing fairly uniformly along the river, now offers X trips annually during its six months of open period. Trips range from 6 to 18 nights. Travellers can take either oar-propelled rafts, which travel 4 mph on average, or motor- propelled boats, which travel 8 mph on average. But one campsite could allow travellers from the same trip.

We will find out ways to determine the carrying capacity, i.e. the maximum trips possible annually.

Then we should provide scheduling strategy to obtain two major goals:

- To utilize the campsites in the best way possible. Let N_i denotes the number of nights on which campsite $i (i \le Y, i = 1, 2, 3, ...)$ is occupied. This goals aims at maximizing the number of $\sum_{i=1}^{Y} N_i$
- To offer the travellers more experience with the nature, i.e. to minimize the contact of travellers with people of other trips

1.2 Basic Assumptions and Hypothesis

- All oar-propelled rafts or motor- propelled boats are of the same capacity, i.e., they can carry up to the same amount of people in one single boat
- All campsites are of the same capacity, i.e., each campsite, either small or large, would accommodate only the group of people on one boat
- People can see within 2 miles; objects 2 miles away or more would be indistinctive to people's eyes.
  In a postgraduate thesis [4], it is mentioned that people's eyes are capable to distinguish objects 4km away, which is approximately 2.48 miles. To simplify the calculation process, we assume people's distance of eye sight is 2 miles.
- In the six-months of open period, there are 183 days

1.3 Terms

- **Capacity**: The maximum number of trips available in the 6 months of open period
- **Potential customers**: People who are planning to take a river trip but are not travellers yet
- **Travellers**: People who take part in the river trips
- **A Group Of Travellers**: People in the same trip
- **Contact of A Group Of Travellers With Others:** If the distance between two groups of people is less than 3 miles (People's distance of sight defined in section 1.2), they can see each other, which therefore result in a contact.

2. Design of Models

2.1 Model 1: Maximum Capacity Model (MCM)

2.1.1 Introduction and Motivation

We first consider the optimal situation to reach the upper bound of the capacity, utilize the campsites as much as possible, and at the same time, minimize the contact of each group with others. Although the assumptions of this model are over-ideal, however, it tells the maximum trips possible on the Big Long River.

2.1.2 Development

To develop this model, we first add some more assumptions in addition to the assumptions listed in section 1.2. But the assumptions in this section are only valid in section 2.1, not in others in this paper.

Assumptions in this section include:

- **Seller's market**: There are so many potential customers that for any river trips available, people are willing to join in and become travellers until all trips are sold out.
- **Y =224** and there is **one campsite every 1 mile** from the start along the river.
- All oar-propelled rafts or motor- propelled boats set off between 9am to 1pm every day
- For the sake of safety, travellers are recommended to stay at campsites from 6pm through 9am of the next day

Based on considerable background research, we find that *Great Canyon River* is statistically similar to *Big Long River* we concern in this problem. For example, Great Canyon River is 275 miles long and its commercial river trip is available only 6 months annually. [1] On *Great Canyon River*, there is about 180 campsites [2], with one campsite every approximately 1.5 miles. To simplify the problem, we assume that there is one campsite every 1 mile on *Big Long River*. This assumption is statistically practical and reasonable.

Under this assumption, we could figure out the optimal schedule which would maximize the capacity and minimize the contact, regardless of the market.

The number of trips during the open period depends on the speed of each trip and the campsites available. For a fixed number of campsites, if each trip finishes in a relatively short time, more trips are available, and therefore more travellers join in, for each campsite allows only the group of people in one trip: The longer the time travellers stay in the river, the more campsites they require in a trip, and therefore the less the number of other trips available because these travellers in the river occupy the campsite for the potential customers. However, other requirements should be considered to minimize the contact of a group of people with others.

It is a straight and direct idea that such optimal schedule consists entirely of 6-night trips, with motorized boats. Because there is no other possible schedule that would ensure a faster speed of each trip. At the same time, as motor boats can travel longer distance in a single day, we can better arrange the campsite for travellers so that the campsites would be full or nearly full. We may also arrange the schedule so that there is no contact between people of different trips.

2.1.3 Results and Interpretation

We now offer the optimal strategy under this ideal situation:

1. There is only one kind of duration available: 6-night trip
2. There is only one kind of propulsion available: motor
3. From 9am to 12:45pm every day, a motorized boat sets off every 0.25 hour
4. The boats runs at even velocity on the river
5. For the first day of the trip, each boat stops at 2:45pm; for the next four days to come, each boat runs 33 mile per day; for the last day, each boat rushes to the Final Exit.

We can prove that with this strategy, there is no contact between different groups of people on the river and every campsite is occupied every night with Big Long River reaching its maximum capacity of trips annually.

For the first day of the trip, each boat stops at 2:45 pm. The first boast to set off (boat E on fig. 1) travels a distance of 54 miles on the first day and the last (boat F on fig. 1) 24 miles. As boats sets off every 0.25 hour and boats run at even speed of 8 mph, the distance between the boat and the one next to it is always 2 miles so that people would have no contact with other groups of people, and, at the same time, people on each boat could find their campsite as long as the group before is able to find its. As the first boat can find its campsite, so would the rest boats.

For the next four days, each boat travels a distance of 33 miles every day. Obviously, they each are able to find their campsite to rest at night. The first boat to set off (boat E on fig. 1) rests in the campsite 54miles from the start on its first night of trip, while the last boat to set off on the day before (boat D on fig. 1) rests in the campsite 24+33 =57 miles from the start. As the distance between these two closest boats is more than 2 miles, they have no contact with each other.

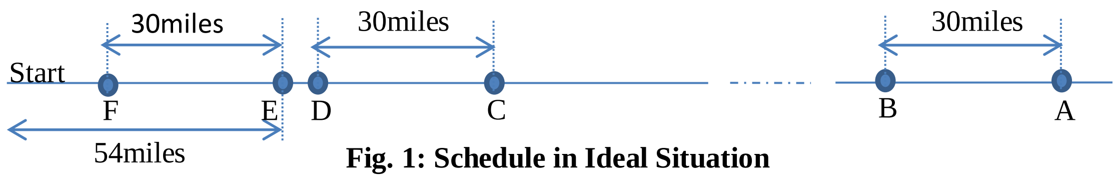


Fig. 1: Schedule in Ideal Situation

On the sixth night, the last boat to set off (boat B on fig. 1) is 24+33*5=189 miles away from the start and 36 miles away from the Final Exit. On the last day of the trip, it travels 4.5 hours to the Final Exit, which is within the time limit mentioned in section 1.2.

From the discussion above, we know that this optimal schedule in ideal situation can maximize the carrying capacity to 4/0.25 =16 trips per day and minimize the contact to zero.

2.1.4 Comments on the Model

Maximum Capacity Model is based on an ideal situation in which there is only one kind of river trip to maximize the carrying capacity and minimize the contact of groups of people of different trips. However, as mentioned before, this model is ideal. It is not necessary in the reality that there are so many potential customers to take part in so many river trips of the same kind.

Nor is it possible that boats travel at even speed to keep an exactly distance of 2 miles to boats nearby. However, this model is a good way to estimate the carrying capacity of *Big Long River*.

2.2 Model 2: Energy Field Model (EFM)

2.2.1 Introduction and Motivation

Maximum Capacity Model introduces the optimal strategy under ideal assumptions to illustrate the best situation possible. However, *Maximum Capacity Model* is to a large degree impractical. In a more common model, the travellers should be allowed more freedom on the river. For example, they should be allowed to take a rest after they set off and then resume the trip for the rest of the day. What's more, few of them may also want to take a trip at night.

In addition, we should base our schedule on the demand of the market. As seller's market does not exist, trip schedule should be diverse to meet different potential customers' preference and therefore attract more travellers so that the campsite can be utilized better.

To overcome the problem of flexibility and diversity mentioned above, we introduce model 2 , the *Energy Field Model* .

In *Energy Field Model*, travellers are allowed more freedom in the trip, and schedule includes much more diversified trip mix. In the discussion of *Energy Field Model*, we will show that although the contact of different groups of people increase, the diversity of trip mix booms with just a little decrease on the number of trips available.

Later in the analysis of *Energy Field Model*, we find that it meets the actual market demand very well. With close study of the actual schedule of *Grand Canyon River*[3], which we believe to represent the potential customers' preference for river trip, we find that both the actual schedule and the one derived from *Energy Field Model* share some common features. For example, they both have 2 peaks in the distribution of trip duration (Fig.2). *Energy Field Model* gives a reasonable and practical schedule which meets the actual need.

2.2.2 Development

To develop this model, we first add some more assumptions in addition to the assumptions listed in section 1.2. But the assumptions in this section are only valid in section 2.2, not in others in this paper.

Assumptions in this section include:

- All oar-propelled rafts or motor- propelled boats set off from the start between 9am to 1pm every day;
- Travellers are advised to stay at campsites from 6pm through 6am of the next day.
- A few permits would be issued to allow some trips to travel at night.

In this model, boats on the river are each modeled as particle which is "dragged" by several factors in an energy field: other boats on the river, time in a day, and schedule. The effects of these factors are showed below.

- **Other boats on the river:** Each "boat particle" on the river has repulsion force to each other. This is because people on different boats do not want to make contact with each other to better enjoy the nature. This kind of force aims at limiting the contact of different groups of people. Let f_b denotes the sum of forces from other "boat particles", $f_{b-ahead}$ the forces from the boats ahead and $f_{b-behind}$ the forces from the boats behind. i, j is the index of boats on the river. Let l_i be the distance of boat i from the start and v_i its velocity. T_1 is a constant. Inspired by the famous Colon's Law[4], we give the function of $f_{b-ahead}$ and $f_{b-behind}$ as below. If the distance between boat i and j is 0, we have another strategy to determine $f_{b-ahead}$ and $f_{b-behind}$.

$$f_b(i) = f_{b-ahead}(i) + f_{b-behind}(i)$$

$$f_{b-ahead}(i) = \sum_{\substack{j \neq i \\ l_j > l_i}} -\frac{T_1}{v_j\left(l_i - l_j\right)^2}$$

$$f_{b-behind}(i) = \sum_{\substack{j \neq i \\ l_j < l_i}} -\frac{T_1}{(12 - v_j)\left(l_i - l_j\right)^2}$$

  The function of $f_{b-ahead}$ differs from that of $f_{b-behind}$ because boats of different velocity has different effects when it is ahead or behind. For example, when an oar-propelled raft is ahead, its repulsion force is larger than that of a motorized boat because it is easier to be caught up with. Similarly, a motorized boat behind has a larger repulsion force as it is more likely to catch up with the boat ahead. Whenever the boat catches up with the boats ahead of it, contact happens.

  The force-distance relation goes like the Fig. 2 (Let T_1=2000, v_j =4 and calculate the repulsion force between boat i and j in case of different distance in between)

  As we can see in the Fig. 2, if the distance between boats is smaller than 20(miles), the repulsion force is extremely weak. But as the distance decreases, the repulsion increases exponentially and hence drags the two boats apart avoiding unnecessary contact.

- **Time in a day**: Each boat on the river is dragged by time to meet the requirement of daily sailing time from 6am to 6pm. let f_t denotes the forces from time in a day, t_c the current time, $t_e = 6pm$, the end line of daily sailing time. T_2, ε are constants. Then we give the function for the force of time as below.

$$f_t = -T_2 e^{(t_c - t_e + \varepsilon)}$$

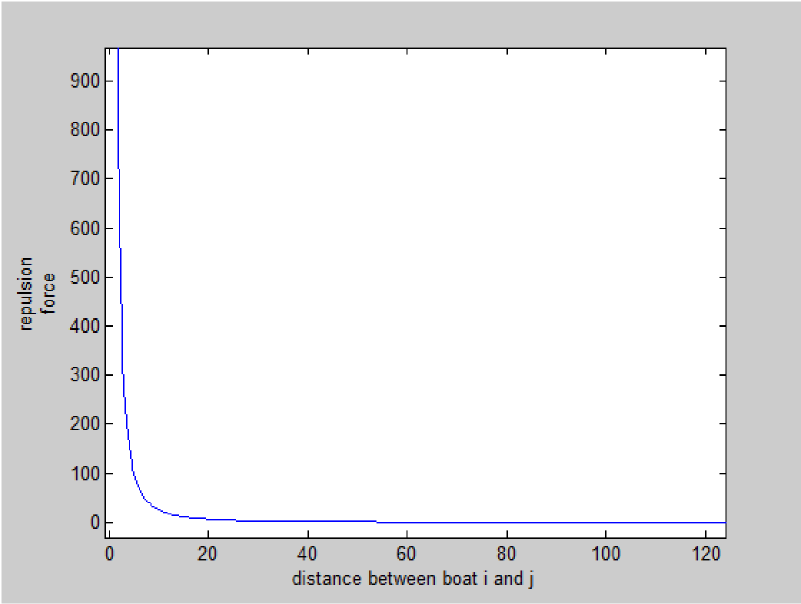


Fig. 2 Force-distance Relation

As time approaching 6pm, the force of time grows exponentially, dragging the boats to stop at a certain campsite.

Again, the force-time relation goes like the figure 3, regardless of the sign (Let ε =3, T_2 =20 and calculate f_t of different time t_c).

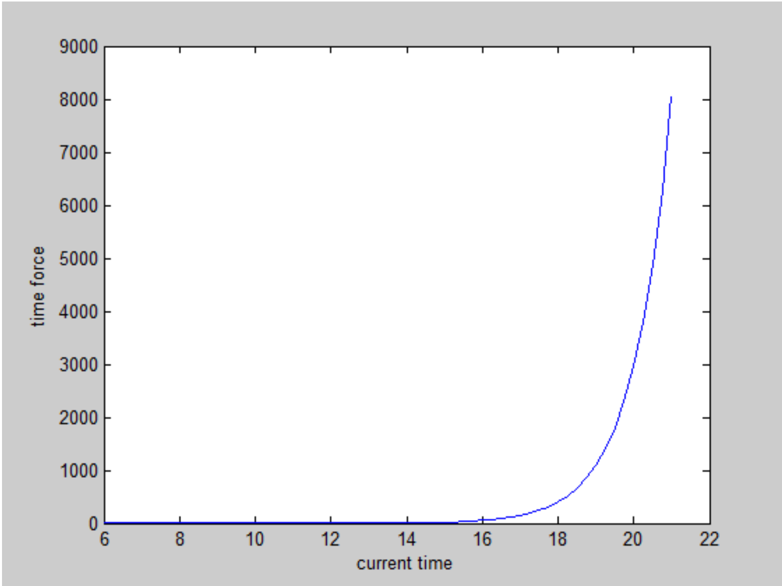


Fig. 3 Time-distance Relation

As shown in the figure above, the time force is quite weak from 6:00 to 16:00. And as time goes on and approaches 18:00, the time force increases a lot faster and even exponentially, which urges the boat to stop at a certain campsite.

- **Schedule**: Each boat on the river is dragged by schedule to meet the requirement of trip duration, which is supposed to range from 6 to 18 nights. Let $f_s(i)$ denotes the forces from schedule for boat i, d_i the days boat i has spent on the river, l_i be the distance of boat i from the start. L is a constant representing the total length of *Big Long River*. d_{std} denotes the standard trip duration, which can be modified to change the distribution of trip duration. T_{31}, T_{32} are constants. We first calculate a variable called $offset$, and then determine $f_s(i)$ according to the sign of $offset$.

$$offset = \frac{d_i}{d_{std}} - \frac{l_i}{L}$$

$$\Rightarrow f_s(i) = \begin{cases} T_{31} \times e^{T_{32} \times offset} - T_{31}, offset > 0 \\ 0, offset = 0 \\ -T_{31} \times e^{-T_{32} \times offset} + T_{32}, offset < 0 \end{cases}$$

In the standard schedule with a duration of d_{std}, we assume that the distance boat i has traveled is in proportion to the days that has elapsed. As boat i travels ahead of the schedule of standard duration, the force of schedule slows it down by dragging it back and similarly, if it travels behind the schedule, this force speed it up by dragging it forward.

The force-offset relation goes like Fig. 4 (Let T_{31}=50, T_{32}=7 and calculate the schedule-force for each schedule-offset).

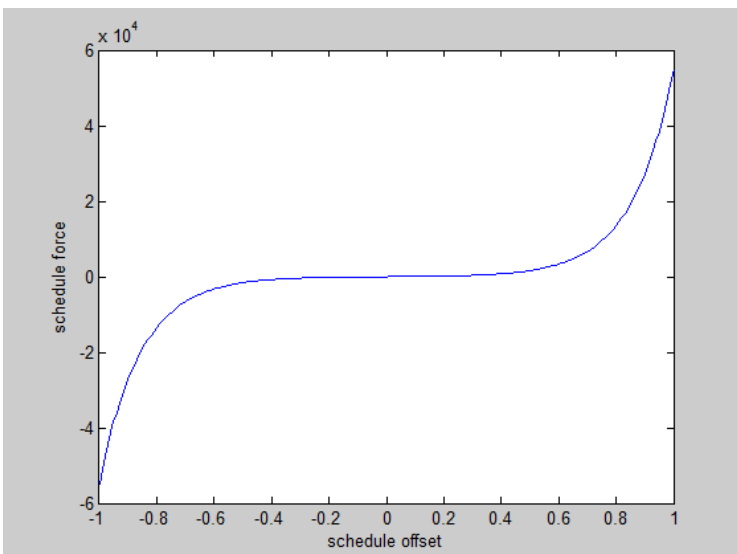


Fig. 4 Schedule-distance Relation

We can see from Fig. 4 that when the current schedule match the standard schedule completely, such as the case in which the standard duration d_{std} equals 12(days) and on the 6th day, boat i has covered 6/12 of the total river, the schedule-force equals to 0. And if the boat is behind or before the standard schedule the schedule-force will increase exponentially.

To maximize the diversity of the trip durations, we choose not to fix the standard duration (measured in days, not nights, but convertible) d_{std}. Instead we adopt a randomized strategy. In simulation, we update, together with other data, the standard duration every 15 minutes (1/4 hour). The random number will be generated between 7 days (6 nights) and 19 days (18 nights). The result shows that this strategy greatly improved the diversity of the trip durations.

These factors work as different kinds of force to drag the "boat particle" and determine how these "boat particles" act. We create an energy field consist of several major forces. The "trip particles" are like objects in Energy Field, whose reaction is determined by the effect of the whole energy field. Let $f_c(i)$ denotes the compound force of all the forces in the energy field for boat i. So we have

$$f_c(i) = f_b(i) + f_t(i) + f_s(i)$$

The rules by which "boat particles" react to f_c are discussed below.

- "Trip particle" **stops at a certain campsite**, i.e. a group of people rest at the campsite: if $f_c(i) < G_{camp}$ and boat i is moving ahead on the river, it stops and camps. For $f_c(i)$ which is small or even negative, the repulsion force drags it back, but as boats could not move backward, so the boat stops to camp.
- " Trip particle" **sets off from the campsite**, i.e. a group of people resumes their trip after a rest in a campsite: if $f_c(i) > G_{setoff}$ and boat i is resting in campsite, it resumes its trip. For large $f_c(i)$, the repulsion force pushes it forward, so the boat goes on and moves ahead. G_{setoff} denotes the threshold for boats to set off.
- **A new trip launches**, i.e. new "trip particle" is added into the river: Let f_c denote the compound force of a boat at the start. If $f_c > G_{v8}$, a motorized boats launches; if $G_{v4} < f_c \le G_{v8}$, a raft launches; if $f_c \le G_{v4}$, no trips will launch. G_{v8} and G_{v4} represent the thresholds for launching a motorized boat or a raft respectively.
  Here is why we make up this rule. At the start, both f_t and f_s falls to zero, so f_c changes with the impact from the boats ahead on the river. If the repulsion force from boats on the river is too large, we consider it improper to launch a trip as the river is probably so crowded that a new trip would result in unbearable contact. Similarly, if the repulsion force is little, probably there are a few boats on the river but it would be ok to launch a new trip, so a raft launches, which is less likely to travel faster, catch up with the boats ahead and result in contact, leaving more room for the boats ahead. In case that few

boats are on the river or most of them are close to the final exit, f_c falls to a relatively small value, we assume that there is either few contact possible or plenty of room for boats to travel, so a motorized boat can set off, which has a better travelling capability.

To better illustrate the mechanism of the *Energy Field Model*, we have a simple example here. This simple example shows how trips launch in 2 days.

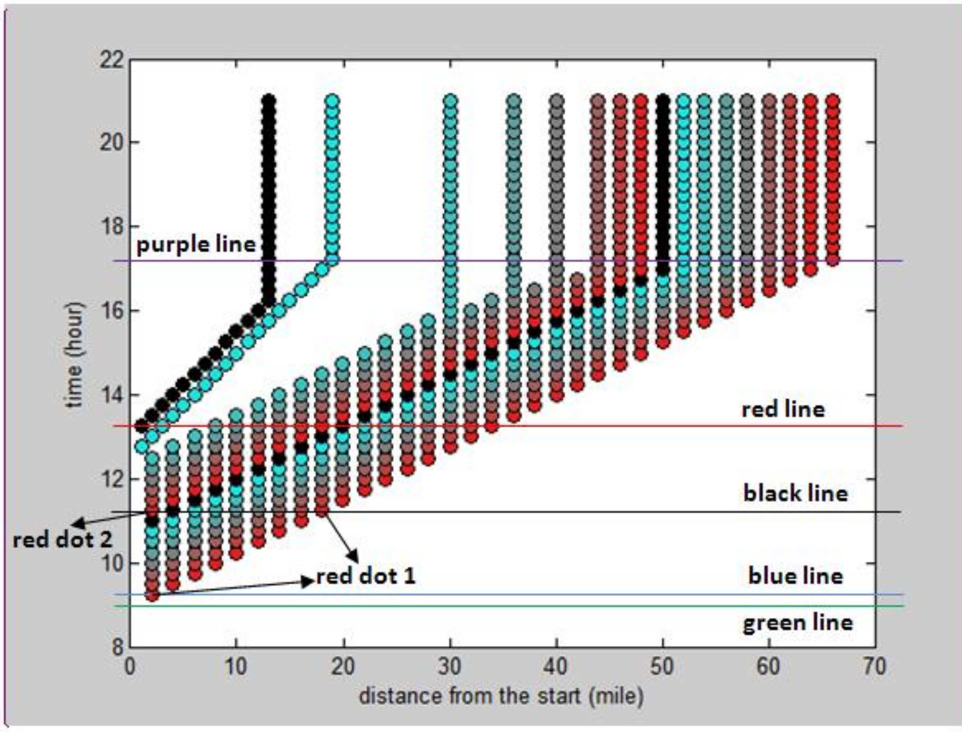


Fig. 5: Travelling Detail of the First Day

In the Fig. 5, we use different colors to distinguish different boats nearby. Each value on the time-axis corresponds to a distribution of boats on the river. For example, at 9:15 of the first day, indicated by the blue line, we have only 1 boat on the river; and at 11:15 we have 9 boats on the river. The boat mentioned above has travelled 18 miles as indicated by the red dot. One thing to is that the 2 red dots on the black line represents 2 different boats instead of the same one.

Time Force: As we can see in the Fig. 5, before 12:30, one 8-mph boat launches every 1/4 hour. As the boats are launching continuously, the repulsion force imposed on the starting spot keeps increasing. Then at 12:30 $f_c(i)$, which is negative, gets so large that $f_c(i)$ is less than G_{v8}, due to the repulsion force from the boats on the river, so it is improper to launch a 8-mph boat, but as long as $G_{v4} < f_c(i) < G_{v8}$, a 4-mph boat can still be launched. Sadly after the two 4-mph boats are launched, $f_c(i)$ gets even larger. Therefore no more boats can be launched this day.

Repulsion Force: Apparently, on Fig. 5, more and more boats stop at the campsite as time goes closer to 18:00, due to the force $f_c(i)$ from time. By 17:15, all travelers are already at the campsite resting. At 17:15, indicated by the purple line, not all boats camp next to each other. This is because under the effect of repulsion force f_b, boats tend to camp far from each other.

Schedule Force: Now let's see the situation of the second day, which starts from time=30. Fig.6 shows how boats travel on the second day.

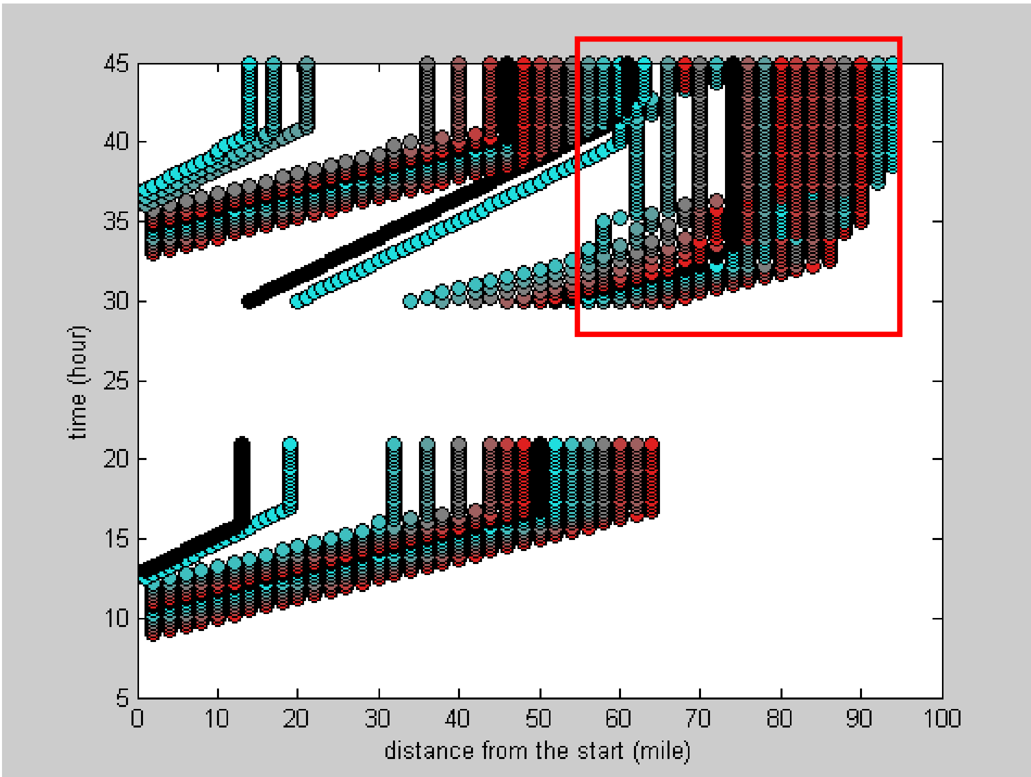


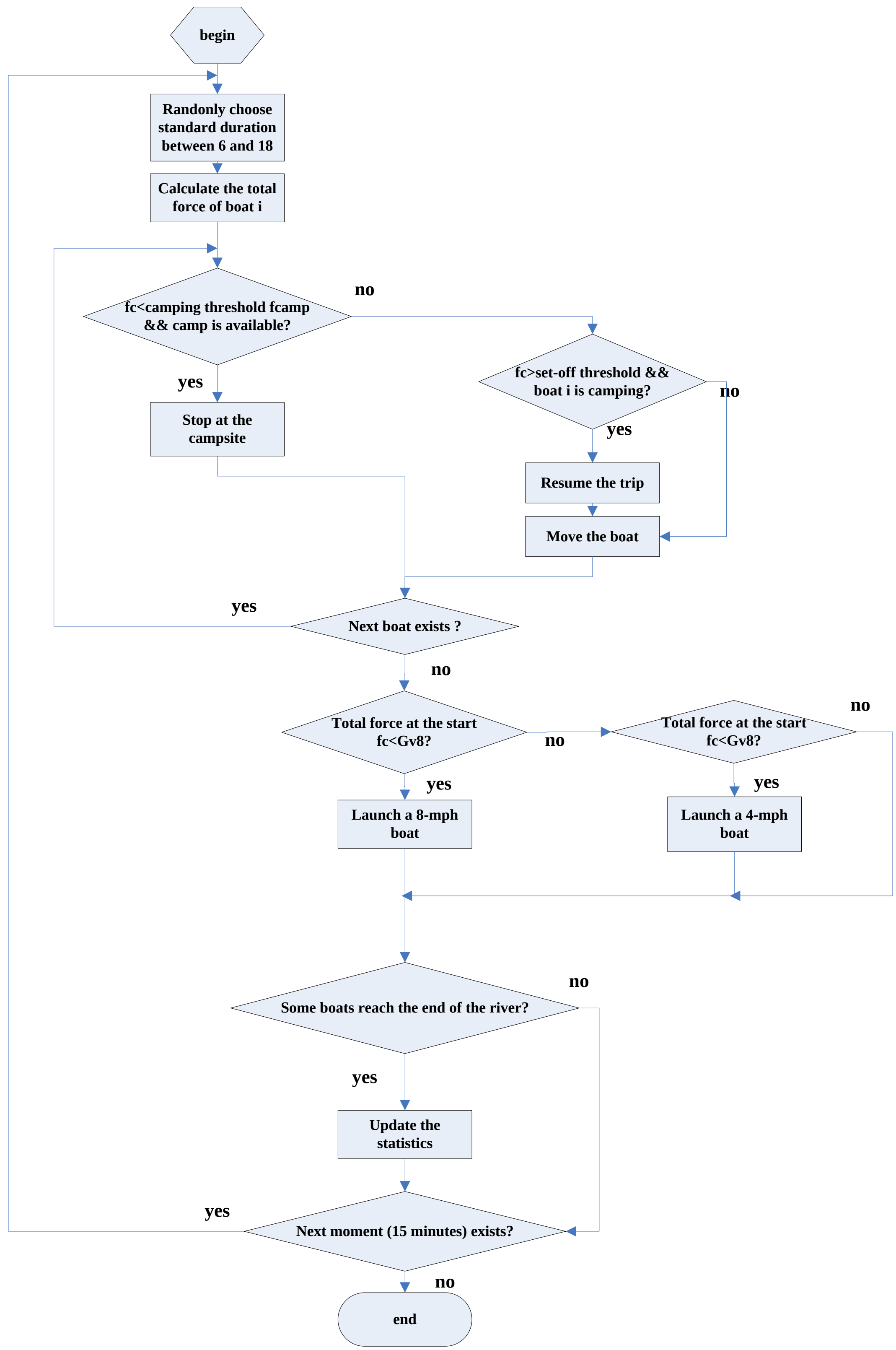
Fig. 6: Travelling Detail of the First Two Days

In the rectangle at the right-top corner, we can see that a lot of boats stop and rest at the campsite even if the repulsion force from behind is relatively strong. This is because these boats are extremely before the standard schedule, which means the schedule force is strong enough to push the boats back and make them rest at the campsite.

This simple example shows how trips launch during these 2 days. It helps us to understand the forces and rules by which the boats act better.

Overall, the algorithm can be vividly described in Flow Chart 1.

Generally speaking, in *Energy Field Model*, boats on the river are modeled as particles each "dragged" by several forces in an energy field: repulsion force from other boats on the river, force of time in a day, and force of schedule. These forces work together and make up an energy field which determines the action of the "boat particles" in it.



Flow Chart 1: Algorithm of *Energy Field Model*

Then we simulate to obtain the schedule by rules listed in this section.

2.2.3 Results and Interpretation

We simulate with Matlab to obtain the schedule. In simulation, we update the forces and locations of boats every 15 minutes. To test the model, we first run a simulation of 1 month (30 days) and obtain one month's schedule listed in Table 3. The parameters used in the simulation and their value are listed in Table 2.

Table 2: Parameters and Values

Parameters	ε	T_1	T_2	T_{31}	T_{32}	G_{v4}	G_{v8}	G_{setoff}	G_{camp}	Y **[Note]**
value	3	100	2000	50	7	200	100	-200	-155	224

[Note]: We assume there is one campsite every 1 mile from the start.

Now we will evaluate the schedule in terms of three: first, how it utilizes the campsite, or the monthly capacity; second, people's contact; third, the trip mix. To evaluate the schedule on a quantified basis, we define 3 evaluation terms below and then give a numeric picture of the schedule.

Monthly carrying capacity C_m : It evaluates the number of boats launched in a month, i.e. the number of trips launched monthly.

Night of Campsites Occupied N_m : It evaluates the number of nights on which campsites are occupied. Let n_i be the number of nights boat i spends on the river and B the total number of boats launched in the month. Then $N_m = \sum_i^B n_i$.

Contact time T_c : It evaluates the time travellers make contacts with those of another boat. Let t_i be the period of time during which boat i make contacts with other boats, i.e. its distance with another boat is less than 2 miles, and B the total number of boats launched in the month. Then $T_c = \sum_i^B t_i$. In simulation, we update the data every 15 minutes. In an update, if the distance of a certain boat with another is less than 2 miles, we add 15 minutes to t_i . So the final contact time we provide is an approximation of the real contact time.

Trip duration distribution: We list the number of trips of different duration. We will later compare this data with that of the *Grand Canyon River* trip schedule.

Table 3: Schedule Table for a month

DATE	1	2	3	4	5	6	7
TRIP	M-7, M-7 M-7,M-7 M-7,M-7 M-7,M-9 M-9,M-9 M-9,M-9 O-16,O-17 O-17	M-8,M-8 M-8,M-8 M-8,M-8 M-8,M-8 M-8,M-8 O-10,O-17 O-17,O-18	M-9,M-10 M-10, M-10 M-10, M-10 M-10, M-10 M-10, M-10 M-10 O-17,O-17 O-17	M-9,M-9 M-9,M-9 M-9,M-9 M-10,M-11 M-11,M-11 O-16,O-16 O-16,O-16	M-10,M-10 M-10,M-10 M-10,M-10 M-10,M-11 M-11,M-11 O-15,O-15 O-15,O-15	M-9,M-9 M-10,M-10 M-10,M-10 M-11,M-11 M-12 O-15,O-15 O-15,O-16	M-8,M-10 M-10,M-10 M-11,M-11 M-11,M-11 M-11 O-15,O-16 O-16,O-17
DATE	8	9	10	11	12	13	14
TRIP	M-8,M-9 M-9,M-9 M-9,M-10 M-11,M-12 M-15 O-17,O-17 O-17,O-17	M-8,M-8 M-9,M-11 M-11,M-12 M-12,M-12 M-12 O-16,O-16 O-16,O-16	M-8,M-8 M-9,M-9 M-11,M-11 M-11,M-13 M-13 O-15,O-15 O-15,O-15	M-7,M-8 M-11,M-11 M-11,M-12 M-12,M-13 M-13 O-14,O-15 O-15,O-16	M-9,M-11 M-11,M-12 M-12,M-12 M-12,M-12 O-15,O-15 O-15,O-15 O-15	M-9,M-10 M-10,M-11 M-11,M-12 M-12,M-13 O-15,O-15 O-16,O-16 O-16	M-8,M-10 M-12,M-12 M-12,M-12 M-13,M-13 O-15,O-15 O-15,O-15 O-15
DATE	15	16	17	18	19	20	21
TRIP	M-7,M-9 M-11,M-12 M-12,M-13 M-13,M-13 O-14,O-14 O-14,O-14 O-15	M-9,M-9 M-10,M-10 M-11,M-12 M-13 O-16,O-16 O-16,O-17 O-17	M-9,M-9 M-9,M-12 M-13,M-13 M-13,M-14 O-16,O-16 O-16,O-16 O-17	M-11,M-11 M-12,M-12 M-12,M-13 M-14 O-16,O-16 O-16,O-16 O-16	M-9,M-9 M-9,M-9 M-10,M-11 M-12 O-15,O-16 O-16,O-16 O-16	M-9,M-9 M-9,M-9 M-10,M-11 M-12,M-14 O-15,O-15 O-15,O-15 O-15	M-8,M-11 M-11,M-11 M-12,M-13 M-13 O-14,O-16 O-16,O-16 O-16
DATE	22	23	24	25	26	27	28
TRIP	M-7,M-10 M-10,M-10 M-10,M-11 M-11,M-12 O-15,O-15 O-15,O-15 O-16	M-10,M-11 M-11,M-11 M-12,M-12 M-12 O-15,O-15 O-15,O-15 O-15	M-10,M-11 M-11,M-11 M-11,M-12 M-12 O-14,O-16 O-16,O-16 O-16	M-10,M-10 M-10,M-10 M-10,M-10 M-11,M-12 O-15,O-15 O-15,O-16 O-16	M-9,M-10 M-11,M-11 M-11,M-11 M-14 O-15,O-15 O-15,O-15 O-15	M-10,M-10 M-11,M-11 M-11,M-13 M-13,M-14 O-14,O-14 O-14,O-14 O-15	M-10,M-10 M-12,M-12 M-12,M-13 M-13 O-14,O-15 O-16,O-18 O-19

Table 3: Schedule Table for a month(Continued)							
DATE	29	30					
TRIP	M-8,M-8 M-12,M-12 M-12,M-13 M-13,M-13 O-15,O-15 O-15,O-15 O-15	M-11,M-12 M-12,M-12 M-12,M-13 M-14 O-14,O-14 O-14,O-15 O-17					

The trip calendar in table 3 lists the launch schedules in a month of 30 days. Each river trip is labeled on the calendar according to its planned schedule. The first abbreviation represents the propulsion type of trip, with 'M' standing for "Motor" and 'O' for "Oar". The second number represents the duration of the trip. For example, trip labeled "O-14" is a 14-night oar-propelled one.

For the one month's schedule list in table 3, we have the quantified evaluation terms as below.

Table 4: Quantified Evaluation On one month's schedule

Monthly Capacity/trips		389		**Night of Campsites Occupied/nights**				2589		**Contact Time/hour**		8920	
Trip Duration	6	7	8	9	10	11	12	13	14	15	16	17	18
Number of trips	6	13	54	44	45	17	2	1	19	23	10	9	2

Compared with 480 of Model 1, the monthly capacity of *Energy Field Model* is 389. Monthly capacity decreases by 100, which means that fewer trips are available in a month. In addition, Contact time of *Energy Field Model* is 8920 hours a month for all boats, which means each boat has approximately 2.17 hours per day during which its distance with other boats is less than 2 mile, or we can say it has contact with other boats. But for a day of trip, 2.17 hours is relatively short, which we believe is bearable for the travellers.

Night of Campsites occupied in *Energy Field Model* is 2589. Compared with the optimal 2880 in model 1, it drops slightly, which means campsites on the river are utilized very well. We also find out that trip mix in Energy *Field Model* is diverse, offering a lot of different choices to travellers.

The final schedule for the 6 months of annual open period is listed in attachment named *Six-Months-Schedule*, with some of the quantified evaluation terms listed in Table 5.

Table 5: Quantified Evaluation on Six Months' Schedule

Annual Capacity/trips		2227		**Night of Campsites Occupied/nights**				28557		**Contact Time/hour**		69200	
Trip Duration	6	7	8	9	10	11	12	13	14	15	16	17	18
Number of trips	24	59	167	261	358	247	139	196	306	284	145	27	14

Compared with 2928 of Model 1, there are 2227 trips annually in the 6-month schedule derived from *Energy Field Model*, which means that more than 12 trips on average launch each day. We can say there enough trips for travellers despite of a light drop from the optimal number. Contact time of *Energy Field Model* is 69200 hours annually for all boats, which means each boat has approximately 2.42 hours per day during which its distance with other boats is less than 2 mile, or we can say it has contact with other boats. But for a day of trip, 2.42 hours is relatively short, which we believe is bearable for the travellers.

Night of Campsites occupied in *Energy Field Model* is 28557. Compared with the optimal 17568 in model 1, it expands largely, which means campsites on the river are utilized even more. There are more campsites occupied than that of Model 1 because we allow contact in *Energy Field Model*. In Model 1, boats camp at least 2 miles away from each other, which means the campsite in between are empty. However, in *Energy Field Model*, boats may camp next to each other. This is why campsites are utilized more in *Energy Field Model*. In light of campsite utilization, *Energy Field Model* is even better than Model 1.

We also find out that trip mix in *Energy Field Model* is diverse. Among all the trips, we offer more 10-day trips and 14 –day trips. This schedule caters to people's preference for trips of different duration.

2.2.4 Error Analysis and Sensitivity Analysis

We believe that a good schedule should not only optimized the evaluation parameters such as monthly capacity and nights of campsite occupied, but should also meet market demand to offer service to travellers of different preference. We assume that actual river trip schedule in real life, after a long time of interaction between customers and service providers, represents the market demand very well. As discussed in section 2.1.2, we find *Big Long River* shares a lot in common with *Grand Canyon River*. We think the trip schedule of *Grand Canyon River* is valuable to evaluate our schedule. So we compare our one month's schedule with that of *Grand Canyon River* in July, 2011.[3]

In July, 2011, 270 trips are available on *Grand Canyon River*, compared with 389 in our *Energy Field Model*. This is probably because many other factors are considered to make up a real schedule. For example, we should also consider environmental capacity while we set off a new trip.

We mainly focus on the analysis on the trip duration distribution. Fig. 7 shows the number of trips of different duration. Remember the total number of trips per month is normalized to 1 before we draw the graph.

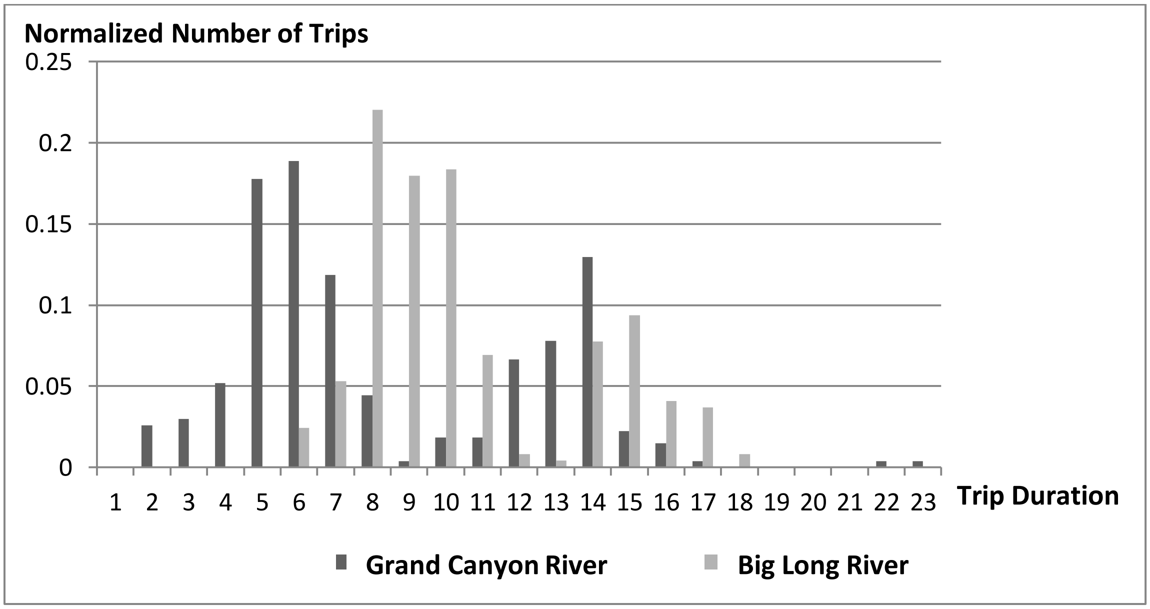


Fig. 7: Comparing the Trip Duration Distribution of Two Schedules

We find out in Fig.4 that there are 2 peaks in each of the trip duration distribution. One peak indicates trips of shorter duration, and another trips of longer time. We believe that this is because people's preference is alike: Some may want to take a short trip which costs less time and other prefer to spend more time to enjoy the nature; Or Some people, such as office workers, have less time to go for a trip, while others enjoys more leisure time so that they can take on a trip of longer duration. We think our schedule caters to such kind of people's preference.

Here is the sensitivity analysis of *Energy Field Model*, concerning how the result changes with the varying number of campsites along the river.

Y is unknown according to the problem given. Therefore, the value of Y in the section 2.2.3 comes from our close study on the background information, the *Grand Canyon River* [2]. However, we find that the change of Y has some effects on model 2, and in order to improve model 2 work, Y must be bounded within a certain range. And here are our findings about the range of Y.

1. The range of Y where model 2 is effective : within 10%
2. The range of Y where model 2 is disable: more than 10%

We will discuss these two cases below.

1. The range of Y where model 2 is effective : within 10%

If we delete the camps at the distance of N*100 (N is integer) mile to the starting point, we will get a new result (the code is attached in the appendix).

This result includes the changes in the number of camps, the contact between people, the carrying capacity, trips whose duration is out of range, and the distribution of the all kinds of trips.

Similarly, we generate the results in case that we delete the campsites m*50 miles, m*30 miles, m*15 miles and m*9 miles from the start, where m = 1, 2, 3…

- "L" is the parameter we use in deleting the N*L mile camps
- "Out of choice" indicates the trips whose duration is out of range
- To test the model with higher efficiency, we generate the schedule of just one month.
- Deviation of contact is the current value compared with the initial value in section 2.2.3

Table 6: Sensitivity Analysis of Y when Model 2 is effective (Part I)

L	100	50	30	15	9
Deviation of Y	-0.90%	-1.79%	-3.11%	-6.25%	-10.71%
Y	222	220	217	210	200
Contact	8.86E+03	9.48E+03	8.90E+03	1.02E+04	1.03E+04
Deviation of contact	-0.64%	6.26%	-0.19%	14.20%	16.00%
Number of Trips monthly/ monthly capacity	392	394	397	399	400
Deviation of total number	0.77%	1.29%	2.06%	2.57%	2.83%
Out of Choice	0	0	1	3	10
Distribution [Note]	4 32 49 42 33 16 5 10 13 17 20 4 0	13 32 44 48 33 19 5 10 11 22 13 4 0	9 23 59 53 36 7 1 8 16 19 15 1 0	26 44 58 39 13 13 3 11 23 21 6 0 0	36 41 52 41 25 2 3 12 23 23 4 0 0

[**Note**]: The number in the box indicates the number of trips of different duration. The number on top-left corner is the number of 6-night trips, the one on the top-right corner 7-night trip, with the number in the bottom indicates the number of 18-night trip.

According to the table 6, we draw the curve of Y-contact (Fig. 8), Y- monthly capacity (Fig. 9), Y- out-of-choice (Fig. 10) and Y-distribution(Fig. 11).

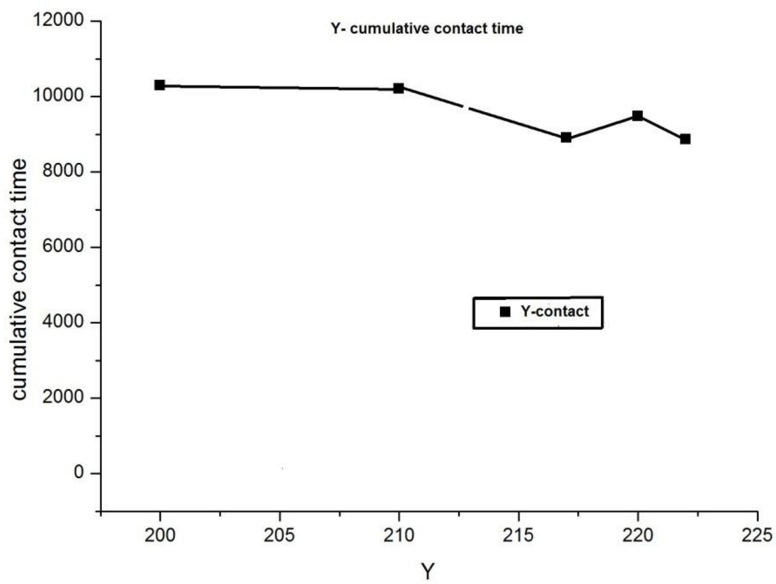


Fig. 8: Total Contact time (hour/month) Changes with Y

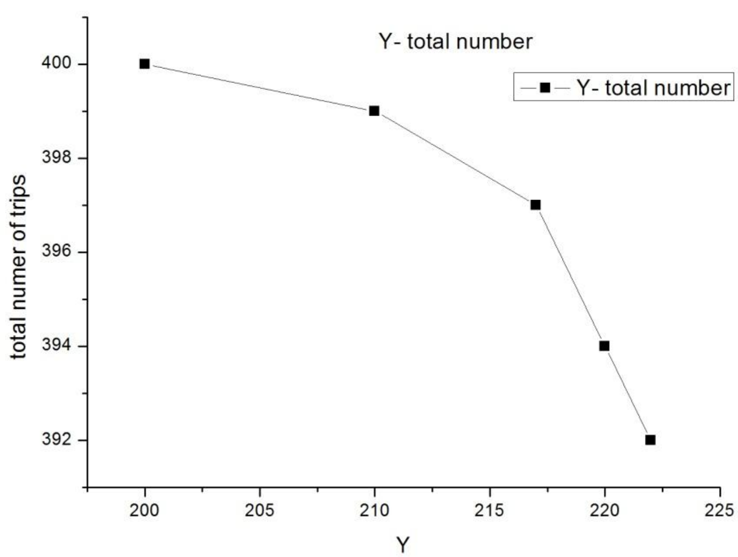


Fig. 9: Monthly Capacity (Trips/Month) Changes with Y

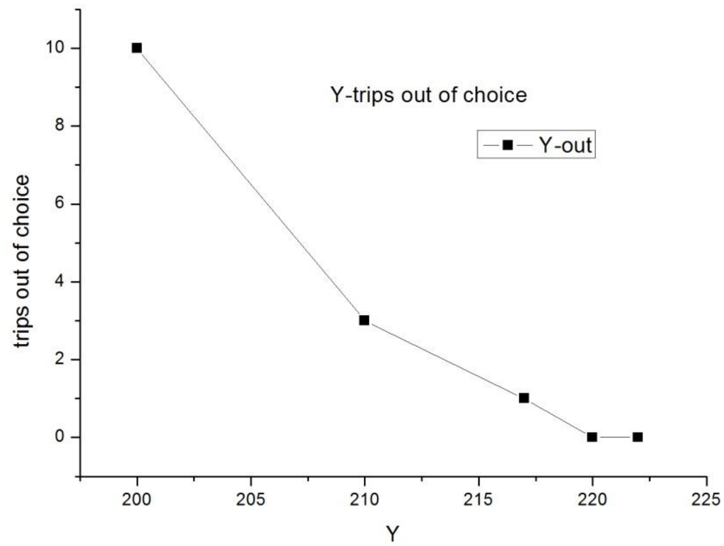


Fig. 10: Number of Trips Out of Choice Changes with Y

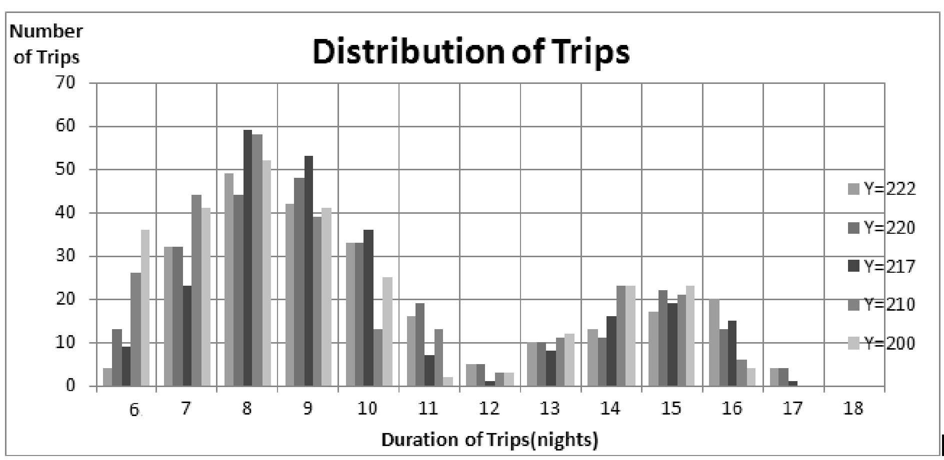


Fig. 11: Distribution of trip duration Changes with Y

In the curve of Y- contact (Fig. 8), though the contact time shown in the curve seems quite stable as Y decreases, the actual contact time should have been less than the data in Fig. 8, because contact caused by the trips whose duration is out of range is present in the graph. Therefore, the contact should have decreased slightly with the decrease of Y.

In the curve of Y-total number (Fig. 9), the total number seems quite stable. And the out-of-choice number rise slightly with the decrease of Y, which means the whole capacity, is stable.

In the curve of Y-distribution (Fig. 10), we find that the two-peak situation become more and more obvious, which means the peak becomes sharper and higher.

As you can see, within the range of 10% of Y=224, model 2 generates a satisfactory schedule, though as Y decreases, there'll be less trips around 13-night and 19-night but more around 9-night and 15-night.

2.the range of Y where model 2 is disable----more than 10%

Though model 2 works well with Y decreasing within 10%, yet if Y decreases more than 10% of 224, we find that model 2 is not quite suitable.

Some experiments are done as below.

Table 7: Sensitivity Analysis of Y when Model 2 is effective (Part II)

L	8	7	6	5	4	3
Deviation of Y	12.50%	14.80%	17.19%	20.32%	25%	33%
Y	196	192	187	180	168	150
Contact	14.80%	17.19%	20.32%	25%	33.93%	50%
Deviation of contact	9.39E+03	9.98E+03	9359	1.05E+04	8.03E+03	1.08E+04
Number of Trips monthly/ monthly capacity	5.29%	11.85%	4.92%	17.88%	-9.96%	21.07%
Deviation of total number	402	400	403	402	406	406
Out of Choice	31	19	71	33	158	81
Distribution [Note]	65 64 46 14 1 0 0 2 8 28 16 0 0	52 51 58 21 11 4 2 12 27 16 1 0 0	73 61 27 0 0 0 0 9 15 20 9 2 0	53 57 55 20 5 7 10 22 25 4 0 0 0	72 18 2 0 0 0 0 4 16 28 7 1 0	89 56 9 3 0 12 18 18 8 2 2 0 0

[**Note**]: The number in the box indicates the number of trips of different duration. The number on top-left corner is the number of 6-night trips, the one on the top-right corner 7-night trip, with the number in the bottom indicates the number of 18-night trip.

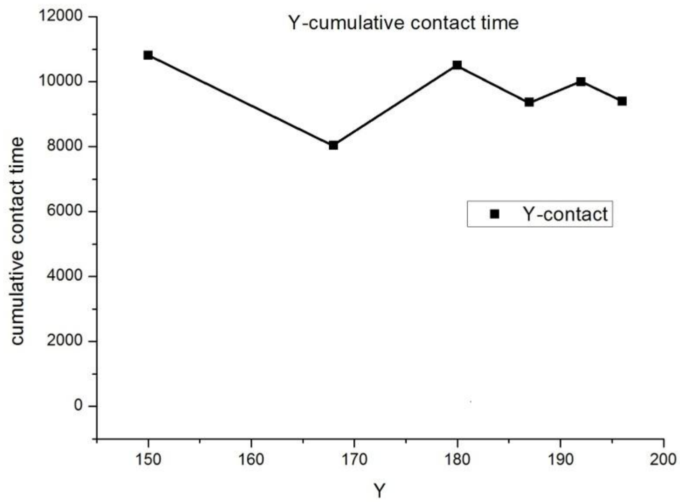


Fig. 12: Total Contact time (hour/month) Changes with Y

As is shown in table 7, the trips out of choice increase sharply with the decrease of Y. Therefore, though the curve of Y-contact seems stable, lots of contact should be deleted as the contact caused by the trips out of choice should not be considered. That is to say, the Y-contact curve has lost its meaning.

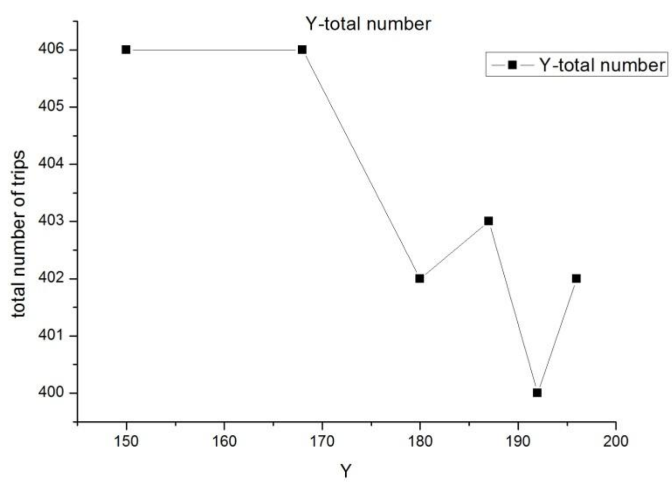


Fig. 13: Monthly Capacity (Trips/Month) Changes with Y

And also the actual number of trips should be calculated by cutting the trips out of choice from the total number of trips, so if there're too many trips out of choice, the curve of Y-total number will be meaningless as well.

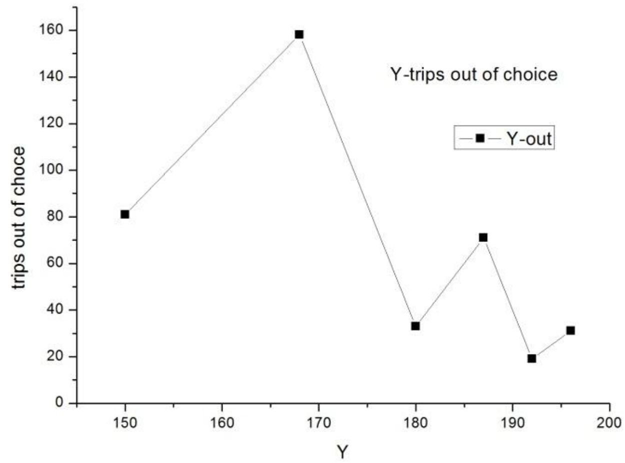


Fig. 14: Number of Trips Out of Choice Changes with Y

Of course we can just delete contact of the trips out of choice to make the model 2 suit the situation with low Y. Similarly, delete the trips out of choice in the total number to get the actual trips. However, we'll still find the two peak situation become too sharp that there are some kinds of choice lost. For example, as is showed in the table, when Y=150, 11, 18 and 19 night trips don't exist.

Therefore, model 2 is not quite suitable if Y decreases by more than 10% from its initial value 224.

For the deviation of Y is larger than 10 %, we now offer a solution to obtain a reasonable result.

Study closely the previous part of the sensitivity analysis and we can find that the problem is mainly caused by the lack of campsite. In our *Energy Field Model*, we haven't considered the force caused by the campsite, which means the change of available campsites doesn't affect the launches of boats. In this way even if the campsite are already fully occupied, boats may still be launching. Such poor boats might not have the chance to rest at the campsite and have to keep going, which will lead to very short duration (probably shorter than) and even serious disturbance.

The key to solving the problem is to control the launches of boats when the campsite available is not enough. More specifically, in the context of our Energy Field Model, we can add a campsite force to control the launches.

For example, we can track the availability rate of campsite R_a to determine the **campsite force** f_{camp}. As R_a decrease, f_{camp} will increase to control the launches of boats.

In a word, by adding the **campsite force**, the problem of the previous part can hopefully be solved.

2.2.5 Comments on the Model

With *Energy Field Model*, we obtain a satisfactory and particle schedule which meets people's preference. In this model, travellers are allowed more freedom on the river. For example, they may rest in the campsite and resumes their trip after a break. Although travellers sometimes have to make contact with other groups of people in the schedule, however, our schedule manages to utilize the campsite very well. Compared with the optimal situation, night of campsite occupied drops just a little.

In addition, this schedule offers a diverse mix of trips with varying duration and propulsion, which better meets different potential customers' demand. Comparing the actual schedule of *Grand Canyon River* trips with our schedule, we find out that both of them share a feature in common. They each have two peaks in the trip duration distribution; one indicates trips of shorter period of time, approximately a week; and the other of longer duration, about 2 weeks. Such schedules meet people's preference for trips of short and long duration.

However, as *Energy Field Model* is a model developed from physical background, we cannot prove that the schedule derived from this model is the best schedule possible. But from so much analysis we have made, we believe the schedule derived is an satisfactory and practical one.

3. Conclusion

With model 1, *Maximum Capacity Model,* we estimate the optimal situation under ideal assumptions. The maximum number of trips possible in a six-month open period (183 days) is 2928 trips, or 16 trips per day. All trips are 6-night oar-propelled trips. Night of campsites occupied is 17568 annually.

In model 2, *Energy Field Model*, boats on the river are modeled as particles each "dragged" by several forces in an energy field. These forces work together and make up an energy field which determines the action of the "boat particles" in it. With this model, we obtain a schedule of 6 months (183 days).

In the schedule derived from *Energy Field Model*, there are 2227 trips annually, or more than 12 trips on average launch each day. Contact time is 69200 hours annually for all boats, which means each boat has approximately 2.42 hours per day during which it has contact with other boats. *Night of Campsites occupied* in *Energy Field Model* is 28557. Compared with the optimal 17568 in model 1, it expands largely; indicating campsites on the river are utilized even more. This is because we allow contact in *Energy Field Model.* We also find out that trip mix in Energy *Field Model* is diverse, which caters to people's preference for trips of different duration. Evaluating the results from these aspects, we believe the schedule derived from *Energy Field Model* is better, which not only meets market demand, but also helps to utilize the campsites better and boost number of trip.

Generally speaking, *Energy Field Model* is an effective model to schedule river trips. The schedule derived from this model is practical and satisfactory.

References

[1] B. Cipra. "Mathematicians Offer Answers to Everyday Conundrums: Shooting the Virtual Rapids." Science 283:925, 1999

[2] Joanna A. Bieri and Catherine A. Roberts, Using the Grand Canyon River Trip Simulator to Test New Launch Schedules on the Colorado River

[3] Launch Calendar Jul11, Grand Canyon National Park River Permits Office

[4] T. Zhang, A research on the Private Landscape's Space in South China,

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