Photon , Poisson noise

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Synonyms

- Shot noise
- Schott noise

Related Concepts

- Photon
- Sensor
- Sensor noise
- Dark current

Definition

Photon noise, also known as Poisson noise, is a basic form of uncertainty associated with the measurement of light, inherent to the quantized nature of light and the independence of photon detections. Its expected magnitude is signal-dependent and constitutes the dominant source of image noise except in low-light conditions.

Background

Image sensors effectively measure scene irradiance by counting the number of discrete *photons* incident on the sensor over a given time interval. In digital sensors, the photoelectic effect is used to convert photons into electrons, whereas film-based sensors rely on photo-sensitive chemical reactions. In both cases, the independence of random individual photon arrivals leads to *photon noise*, a signal-dependent form of uncertainty that is a property of the underlying signal itself.

In computer vision, it is widespread to describe imaging noise as signal *independent*, often using a zero-mean additive Gaussian. While this model has the advantage of simplifying analysis, and may suffice for some applications, it is physically unrealistic. In real imaging systems, photon noise and other sensor-based sources of noise contribute in varying proportions at different signal levels, leading to noise which is often highly dependent on the scene. Understanding photon noise and modeling it explicitly is especially important for low-level computer vision tasks that treat noisy images [8,2], and for the analysis of imaging systems that consider different exposure levels [5,10,1] or sensor gains [4].

Theory

Individual photon detections can be treated as independent events that follow a random temporal distribution. As a result, photon counting is a classic Poisson

process, and the number of photons N measured by a given sensor element over a time interval t is described by the discrete probability distribution

$$\Pr(N = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!} \ , \tag{1}$$

where λ is the expected number of photons per unit time interval, which is proportional to the incident scene irradiance. This is a standard Poisson distribution with a rate parameter λt that corresponds to the expected incident photon count. The uncertainty described by this distribution is known as *photon noise*.

Because the incident photon count follows a Poisson distribution, it has the property that its variance is equal to its expectation, $\mathrm{E}\left[N\right] = \mathrm{Var}\left[N\right] = \lambda t$. This shows that photon noise is signal dependent, and that its standard deviation grows with the square root of the signal.

In practice, photon noise is often modeled using a Gaussian distribution whose variance depends on the expected photon count [8,2,5,10,1,4],

$$N \ \sim \ \mathcal{N}(\lambda t, \lambda t) \ . \tag{2}$$

This approximation is typically very accurate. For small photon counts, photon noise is generally dominated by other signal-independent sources of noise, and for larger counts, the central limit theorem ensures that the Poisson distribution approaches a Gaussian.

Since photon noise is derived from the nature of the signal itself, it provides a lower bound on the uncertainty of measuring light. Even under ideal imaging conditions, free from all other sensor-based sources of noise (e.g., read noise), any measurement would still be subject to photon noise. When photon noise is the only significant source of uncertainty, as commonly occurs in bright photon-rich environments, imaging is said to be *photon-limited*.

In general, the only way of reducing the effect of photon noise is to capture more signal. Though the signal-to-noise ratio grows with the square root of the number of photons captured, capturing more photons requires longer exposure times, which may be undesirable, and the number of photons captured in a single shot is limited by the full well capacity of the sensor. Note that while squeezed coherence lasers and other forms of nonclassical light can achieve amplitude noise below the photon noise limit [11], such exotic lighting configurations are typically not relevant for computer vision applications.

In digital sensors, a related source of noise that also follows a Poisson distribution is dark current noise. Dark current refers to "phantom" photon counts due thermal energy causing the sensor to release electrons at random. While photon noise is a property of the signal itself, dark current comes from the embodiment of the sensor and depends on both temperature and exposure time.

Application

Photon noise is inherent to the measurement of light, has no parameters to be calibrated, and is independent of other noise sources. As a result, the effect of photon noise on imaging can be characterized using the radiometric response function that relates the photon count and the expected pixel intensity [6,3].

To handle the signal dependence caused by photon noise, a first step is to estimate the noise variance for each pixel. This can be approximated in a straightforward way by inverting the forward model for imaging noise [9,6,4]. For increased accuracy, several other factors can be taken into account as well: the coupling between signal and noise leads to a recursive estimation [3]; pixels near saturation have reduced variance which can lead to bias [8,2]; and on-camera processing such as demosaicking may introduce spatial correlation [8].

Image processing methods that explicitly incorporate more realistic signal-dependent models of noise, either calibrated [6,3,4] or inferred from the image [8,7], adapt naturally to pixels of different intensities. As a result, for a variety of computer vision tasks such as denoising [8,3] and edge detection [7], these methods can perform better than those handicapped by the assumption of scene-independent noise.

An alternative approach for handling signal-dependent noise is to transform the image using a *variable-stabilizing transformation* that amounts to applying per-pixel non-linearities that effectively reduce the signal dependence [9,2]. Because the transformed signal approximates one with signal-independent noise, it may be processed using methods that assume a simpler noise model.

Recommended Readings

[1] Amit Agrawal and Ramesh Raskar. Optimal single image capture for motion deblurring. In *Proc. IEEE Conference on Computer Vision and Pattern Recognition*, pages 2560–2567, 2009.

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[7] Youngbae Hwang, Jun-Sik Kim, and In-So Kweon. Sensor noise modeling using the Skellam distribution: application to the color edge detection. In *Proc. IEEE Conference on Computer Vision and Pattern Recognition*, pages 1–8, 2007.

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