Penalty Functions for Automatic Rigging and Animation of 3D Characters

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1 Summary

In the interest of reproducibility, we describe here in detail the penalty functions we use in [Baran and Popović 2007] for discrete optimization and for embedding refinement. We plan to release the source code to Pinocchio in the future.

2 Skeleton Joint Attributes

The skeleton joints can be supplied with the following attributes to improve quality and performance (without sacrificing too much generality):

- A joint may be marked symmetric with respect to another joint. This results in symmetry penalties if the distance between the joint and its parent differs from the distance between the symmetric joint and its parent.
- A joint may be marked as a foot. This results in a penalty if the joint is not in the bottommost position.
- A joint may be marked as "fat." This restricts the possible placement of the joint to the center of the σ largest spheres. We use $\sigma = 50$. In our biped skeleton, hips, shoulders and the head are marked as "fat."

3 Discrete Penalty Basis Functions

The discrete penalty function measures the quality of a reduced skeleton embedding into the discretized character volume. It is a linear combination of basis penalty functions (Pinocchio uses nine). The weights were determined automatically by the maximum-margin method described in the paper and are (0.27, 0.23, 0.07, 0.46, 0.14, 0.12, 0.72, 0.05, 0.33) in the order that the penalties are presented.

The specific basis penalty functions were constructed in an ad hoc manner. They are summed over all embedded bones (or joints), as applicable. Slight changes in the specific constants used should not have a significant effect on results. We use the notation $s_{a\to b}^{c\to d}(x)$ to denote the bounded linear interpolation function that is equal to b if $x < a$, d if $x > c$, and $b + (d - b)(x - a)/(c - a)$ otherwise.

Pinocchio uses the following discrete penalty functions:

1. It penalizes short bones: suppose a reduced skeleton bone is embedded at vertices v_1 and v_2, whose spheres have radii r_1 and r_2, respectively. Let d be the shortest distance between v_1 and v_2 in the graph, and let d' be the distance between the joints in the unreduced skeleton. If $d + 0.7(r_1 + r_2) < 0.5d'$, the penalty is infinite. Otherwise, the penalty is

$$\left(S_{0.5\to 0}^{2\to 3}\left(\frac{d'}{d + 0.7(r_1 + r_2)}\right)\right)^3$$

2. It penalizes embeddings in which directions between embedded joints differ from those in the given skeleton. More precisely, for every pair of joints that are either adjacent or share a common parent in the reduced skeleton, we compute c, the cosine of the angle between the vectors $v_2 - v_1$ and $s_2 - s_1$ where v_1 and v_2 are the joint positions in the embedding and s_1 and s_2 are the joint positions in the given skeleton. The penalty is then infinite if $c < \alpha_1$, and is $0.5 \max(0, \alpha_2 \cdot (1 - c)^2 - \alpha_3)$ otherwise. If the joints are adjacent in the reduced skeleton, we use $(\alpha_1, \alpha_2, \alpha_3 = (0, 16, 0.1))$ and if they share the parent, we use $(\alpha_1, \alpha_2, \alpha_3 = (-0.5, 4, 0.5))$, a weaker penalty.

3. It penalizes differences in length between bones that are marked as symmetric on the skeleton. Suppose that two bones have been marked symmetric and have been embedded into v_1–v_2 and v_3–v_4 with these vertices having sphere radii r_1, r_2, r_3, and r_4, respectively. Suppose that the distance along the graph edges between v_1 and v_2 is d_1 and the distance between v_3 and v_4 is d_2. Let

$$q = 0.2 \max\left(\frac{d_1}{d_2}, \frac{d_2}{d_1}\right) + \\ + 0.8 \max\left(\frac{d_1}{d_2 + 0.7(r_3 + r_4)}, \frac{d_2}{d_1 + 0.7(r_1 + r_2)}\right).$$

Then the penalty for this pair of bones is $\max(0, q^3 - 1.2)$.

4. It penalizes two bone chains sharing vertices. If two or more bone chain embedding share a vertex whose distance to the surface is smaller than 0.02, the penalty is infinite. If a bone chain is embedded into a path $v_1, \ldots, v_k$ such that v_1 is the child joint and v_k is the parent joint, and if S is the subset of these joints occupied by a previously embedded bone chain, the penalty is $0.5 + \sum_{v_i \in S} \frac{1}{2i^2}$ if S is not empty.

5. It penalizes joints that are marked as feet if they are not in the bottommost possible position. For each such joint, the penalty is the y coordinate difference between the graph vertex with the minimum y and the vertex into which the joint is embedded.

6. It penalizes bone chains of zero length. This penalty is equal to 1 if a joint and its parent are embedded into the same vertex.

7. It penalizes bone segments that are improperly oriented relative to the given bones. This penalty is calculated for the unreduced skeleton, so we first extract the unreduced embedding, as we do before embedding refinement: we reinsert degree-two joints by splitting the shortest paths in the graph in proportion to the given skeleton. The penalty is then the sum of penalties over each unreduced bone. Let $\vec{u}$ be the vector corresponding to the embedded bone and let $\vec{u}'$ be the vector of the bone in the given skeleton. The penalty per unreduced bone is

$$50\|\vec{u}'\|^2 \left((1 - c)s_{-0.5\to 6}^{0\to 1}(c)\right)^2$$

where $c = \frac{\vec{u}\cdot\vec{u}'}{\|\vec{u}\|\|\vec{u}'\|}$.

8. It penalizes degree-one joints that could be embedded farther from their parents and are not. Suppose a degree-one joint is embedded into v_2 and its parent is embedded into v_1 (different from v_2). This penalty is equal to 1 if there is a vertex v_3 adjacent to v_2 in the extracted graph whose sphere is at least $1/2$ the radius of the sphere at v_2 and the following two conditions hold:

$$\frac{(v_2 - v_1) \cdot (v_3 - v_1)}{\|v_2 - v_1\|\|v_3 - v_1\|} \geq 0.95$$

and

$$\frac{(v_2 - v_1) \cdot (v_3 - v_2)}{\|v_2 - v_1\| \|v_3 - v_2\|} \geq 0.8.$$

Moreover, to improve optimization performance, we never try embedding a degree-one joint into a vertex v_1 if for every adjacent vertex v_2 there is a vertex v_3 adjacent to v_1 such that the sphere around v_3 is at least $1/2$ the radius of the sphere around v_1 and:

$$\frac{(v_3 - v_1) \cdot (v_1 - v_2)}{\|v_3 - v_1\| \|v_1 - v_2\|} \geq 0.8.$$

9. It penalizes joints that are embedded close to each other in the graph, yet are far along bone paths. More precisely, for every pair of joints v_1 and v_2 (that are not adjacent in the reduced skeleton), this penalty is 1 if

$$2d(v_1, v_2) + r_1 + r_2 < d(v_1, v_L) + d(v_2, v_L)$$

where d is the distance along graph edges, r_1 and r_2 are the radii of spheres into whose centers v_1 and v_2 are embedded, and v_L is the embedding of the least common ancestor (in the oriented reduced skeleton) of the two embedded joints.

4 Embedding Refinement Penalty Function

This penalty function is used to refine the discrete embedding. It was also constructed ad hoc. It is the weighted sum of the following four penalty functions over all bones. The weights we use are $(\alpha_S, \alpha_L, \alpha_O, \alpha_A) = (15000, 0.25, 2, 1)$ for the respective penalties.

1. Pinocchio penalizes bones that are not near the center of the object. The penalty is the average of

$$r(0.003, \min(m(q_i), 0.001 + \max(0, 0.05 + s(q_i))))$$

over 10 samples q_i on the bone, where $r(a, x)$ is 0 if $x < a$ and is x^2 otherwise, $m(p)$ is the distance from p to the nearest sampled medial surface point, and $s(p)$ is the signed distance from p to the object surface (positive when p is outside).

2. It penalizes bones that are too short when projected onto their counterparts in the given skeleton. Suppose a bone has endpoints q_1 and q_2 in the embedding and endpoints s_1 and s_2 in the given skeleton. The penalty is:

$$\max\left(0.5, \frac{\|s_2 - s_1\|^2}{((q_2 - q_1) \cdot (s_2 - s_1))^2 / \|s_2 - s_1\|^2}\right).$$

3. It penalizes improperly oriented bones. Suppose a bone has endpoints q_1 and q_2 in the embedding and endpoints s_1 and s_2 in the given skeleton. Let θ be the angle between the vectors $q_2 - q_1$ and $s_2 - s_1$. The penalty is $(0.3 + 0.5\theta)^3$ if θ is positive and $10 \cdot (0.3 + 0.5\theta)^3$ if θ is negative.

4. It penalizes asymmetry in bones that are marked symmetric. If a bone has endpoints q_1 and q_2 and its symmetric bone has endpoints q_3 and q_4 then the penalty is:

$$\max\left(1.05, \frac{\|q_1 - q_2\|^2}{\|q_3 - q_4\|^2}, \frac{\|q_3 - q_4\|^2}{\|q_1 - q_2\|^2}\right).$$

This penalty appears in the sum once for every pair of symmetric bones.

References

BARAN, I., AND POPOVIĆ, J. 2007. Automatic rigging and animation of 3d characters. *ACM Transactions on Graphics 26*, 3. to appear.