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LOW ENERGY CROSS SECTION OF MESIC MOLECULES FORMATION

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Cross section of the formation of mesic molecular complex (MMC) in the reactions $(\mathrm{t}\mu)_F + (\mathrm{D}_2)_{\nu_i K_i} \to [(\mathrm{dt}\mu)_S^{Jv}\mathrm{dee}]_{\nu_f K_f}$ is calculated at low energies ($E < 100$ K). We assume the Breit-Wigner energy dependence of the cross section. In calculating the entrance widths the electron screening and various orbital moments of the incident tμ-atom were taken into account. Owing to anharmonic corrections to a purely harmonic approximation the reduced entrance width remains nonzero at $E \to 0$. As the result the conventional $1/v$ law for energy dependence of the MMC formation cross section is restored. The entrance width for the transition ($\nu_i = 0$, $K_i = 0$) → ($\nu_f = 2$, $K_f = 1$) proves to be the most important. The wave functions of D_2 and MMC were calculated in the Morse potential which proves to give the level positions with a high precision and includes anharmonic corrections to full extent. Accounting for the anharmonism modifies considerably both the energy dependence and the value of the MMC formation cross section. Comparing calculated rates with the experimental data one can estimate the positions of the resonance levels.

1. Recent experiments at Los Alamos and SIN have demonstrated high formation rates of the dtμ mesic molecules (more exactly, the mesic molecular ions–MMI) in the temperature range 20–800 K [1,2]. It implies that the energy of tμ mesic atoms (MA) necessary to form the mesic molecular complex (MMC) in the reaction

$$(\mathrm{t}\mu)_F + (\mathrm{D}_2)_{\nu_i K_i} \to \left[(\mathrm{dt}\mu)_S^{Jv}\mathrm{dee}\right]_{\nu_f K_f}$$

is very small. Here F and S are the total spins of the MA and of the MMI, respectively; $J = v = 1$ are the quantum numbers of the weakly bound state of the dtμ molecule; $\nu_i = 0$ and $\nu_f = 2$ are the vibrational quantum numbers of the D_2 molecule and of the MMC [(dtμ)dee]; K_i and K_f are their rotational quantum numbers.

The cross section of the reaction (1) is described by the Breit-Wigner formula. One must know the positions of the resonances $\varepsilon_{K_i K_f}^{(\nu)}$ as well as the entrance and exit widths in this formula. These quantities can be obtained immediately at low temperatures of the DT mixture. In this case only a few D_2 levels are excited ($K_i = 0, 1$), and only low orbital moments of the incident tμ atom are essential

($kR \sim 1$ at $T = 10^2$ K, see below). Also, only those levels which are close to the zero energy contribute to the cross section. Comparing the MMC formation rates calculated at low energies with the data we can fix the position of one MMC level. The relative energies of other levels are known from precise theoretical calculations [6]. Using this information, the cross sections and the MMC formation rates can be calculated at high temperatures as well.

2. The entrance widths have been estimated and calculated by a number of authors [3–5,7–10]. The most detailed approach has been suggested by L.I. Menshikov [9]. We shall follow ref. [9] in this section.

In the dipole approximation the transition operator for the reaction (1) has the form

$$\hat{V} = -\boldsymbol{d}\frac{\partial U}{\partial \rho}\hat{\rho}, \quad \hat{\rho} = \boldsymbol{\rho}/\rho$$

where $\boldsymbol{d}$ is the dipole moment of the MMI and U is the ${}^1\Sigma_g$ term of the H_2 molecule ($\hbar = e = m_e = 1$). Equation (2) takes into account the electron screening of the deuteron interacting with dtμ [11], ρ is the distance between dtμ center of mass and the deuteron (fig. 1). The initial tμ + D_2 wave function is

$$\Psi_i = \exp(\mathrm{i}\boldsymbol{p}\boldsymbol{X}) \cdot \phi_{1s}(\boldsymbol{r}_t - \boldsymbol{r}_\mu)\frac{\tilde{\chi}_0\left(\boldsymbol{r}_d^{(2)} - \boldsymbol{r}_d^{(1)}\right)}{\left|\boldsymbol{r}_d^{(2)} - \boldsymbol{r}_d^{(1)}\right|}Y_{K_i m_i}\left(\frac{\boldsymbol{r}_d^{(2)} - \boldsymbol{r}_d^{(1)}}{\left|\boldsymbol{r}_d^{(2)} - \boldsymbol{r}_d^{(1)}\right|}\right) \qquad (3)$$

The plane wave in eq. (3) describes the relative motion of tμ and D_2 with the momentum $\boldsymbol{p}$ and $\boldsymbol{X}$ is the distance between their centers of mass,

$$\boldsymbol{X} = \frac{\boldsymbol{r}_t M_t + \boldsymbol{r}_\mu M_\mu}{M + M} - \frac{\boldsymbol{r}_d^{(1)} + \boldsymbol{r}_d^{(2)}}{2}$$

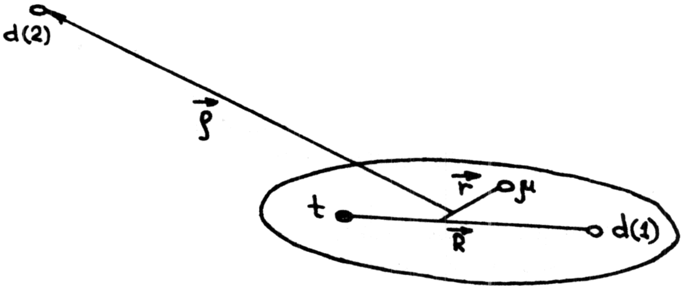

Fig. 1. The choice of coordinates in the MMC formation problem. $\boldsymbol{R} = \boldsymbol{r}_t - \boldsymbol{r}_d^{(1)}$ is the separation of nuclei in dtμ; $\boldsymbol{r}$ is the distance between the muon and t + d(1) (c.m.); ρ is the distance between d(2) and dtμ (c.m.).

ϕ_{1S} is the wave function of the MA 1s state, $\tilde{\chi}_0$ and $Y_{K_i m_i}$ are the radial and the angular parts of the D_2 nuclei wave function.

The final MMC wave function is

$$\Psi_f = \psi_0(\boldsymbol{r}, \boldsymbol{R}) \frac{\chi_\nu(\rho)}{\rho} Y_{K_f m_f}(\hat{\rho}), \quad \hat{\rho} = \rho/\rho.$$

Here $\psi_0(\boldsymbol{r}, \boldsymbol{R})$ is the wave function of the $J = v = 1$ state [4] of the MMI, χ_ν and $Y_{K_f m_f}$ are the radial and the angular parts of the MMC wave function, respectively.

The MMI state $J = v = 1$ is very weakly bound: its binding energy is $\varepsilon_{11} = -0.601$ eV. The energy ε_{11} is calculated with respect to the 1s level of a free $t\mu$ atom (taking into account the hyperfine splitting $\Delta\varepsilon_{FS} = 0.036$ eV for $F = 0$, $S = 1$ [12] and also other corrections, $\Delta\varepsilon_{11}^{\mathrm{tot}} = 0.023$ eV [13], to the nonrelativistic value $\varepsilon_{11}^{\mathrm{NR}} = 0.660$ eV) [14]. Thanks to the small value of ε_{11} the MMI appears to be so extended that it almost decays into $t\mu + d$. For this reason one can neglect their interaction with each other [9,15], and at the distances of the order of the $dt\mu$ size one can replace the exact wave function by its asymptotic form:

$$\psi_{0j}(\boldsymbol{r}, \boldsymbol{R}) = c_a \frac{1 + \kappa R}{\sqrt{\kappa} R^2} e^{-\kappa R} \sqrt{\frac{3}{4\pi}} \hat{R}_j \phi_{1s}(r)$$

$$\kappa = \sqrt{2\mu_a |\varepsilon_{11}|}, \quad \hat{R} = \boldsymbol{R}/R.$$

Equation (6) describes the free relative motion of the MA and d with the orbital moment $J = 1$. Here μ_a is the reduced MMI mass (see table 1). The value $\kappa^{-1} = 20.8$ in mesic atomic units corresponds to a large size of the MMI. The only quantity requiring the knowledge of the exact wave functions is the normalizing coefficient $c_a = 0.574$ which has been determined in ref. [9]. Owing to the fact that the μ is mostly close to the triton one has $\boldsymbol{r}_\mu = \boldsymbol{r}_t$ (hence $\boldsymbol{r} \simeq -M_d \boldsymbol{R}/(M_t + M_d)$), and the expression for the MMI dipole moment is quite simple: $\boldsymbol{d} = -\beta_2 \boldsymbol{R}$ (see β_2 in Table 1). After averaging over the initial and summing over the final spin states one has for the transition matrix element

$$V|^2 = \frac{1}{2k_i + 1} \sum_{m_i, m_f, j} |V_j|^2$$

$$V_j = \int \Psi_f \hat{V} \Psi_i \, d^3R \, d^3r \, d^3\rho$$

Table 1
Constants used in $|V|^2$ calculations

Quantity	Formula	Value, a.u.
	$M_d/2$	
	$M_d(M_d+M_t+M_\mu)/(2M_d+M_t+M_\mu)$	
	$M_d(M_t+M_\mu)/(M_d+M_t+M_\mu)$	
	$2M_d(M_t+M_\mu)/(2M_d+M_t+M_\mu)$	
	μ_i/M_d	
	μ_a/M_d	
	μ_i/μ_f	
	$\sqrt{2\mu_a\|\varepsilon_{11}\|}$	
	–	
	–	
	–	

The following formula can be obtained [9]

$$V_j=\int \mathrm{d}Q_{\hat{\rho}} Y^*_{K_f m_f}(\hat{\rho}) Q_j(\hat{\rho}) Y_{K_i m_i}(\hat{\rho}) \tag{8}$$

where the $\boldsymbol{Q}$ vector is given by

$$\boldsymbol{Q}=\beta_2 C_a\sqrt{\frac{3\pi}{\kappa}}\int_{-\infty}^{\infty}\mathrm{d}x\,\exp\{-\mathrm{i}\beta_1(\boldsymbol{p}\cdot\hat{\rho})\rho_0-\mathrm{i}\beta_2(\boldsymbol{p}\cdot\hat{\rho})x-\kappa|x|\}$$

$$\times I_{0,\nu}(x)\left[\ x|\hat{\rho}-\mathrm{i}\beta_3 p\frac{x}{\kappa}\hat{s}\right] \tag{9}$$

$$I_{0,\nu}(x)=\int_0^{\infty}\mathrm{d}\rho\chi_\nu(\rho)\frac{\partial U}{\partial\rho}\chi_0(\rho-\beta_2 x)\exp\{-\mathrm{i}\beta_1(\boldsymbol{p}\cdot\hat{\rho})(\rho-\rho_0)\}$$

(ρ_0 is the position of the minimum of the molecular term). Here $x=(\boldsymbol{R},\ \rho)$ is the projection of the vector $\boldsymbol{R}$ to the MMC axis (the integration over the transverse components of $\boldsymbol{R}$ and over the muon coordinates has been carried out in deriving eq. (9)), $\hat{s}$ is the unit vector perpendicular to this axis,

$$\hat{s}=[\boldsymbol{p}-\hat{\rho}(\boldsymbol{p}\cdot\hat{\rho})]/p. \tag{9a}$$

The quantities $\beta_{1,2,3}$ are defined in table 1.

3. If one neglects the variation of $I_{0,\nu}(x)$ inside the MMI size κ^{-1}, then at low energies E eq. (9) gives the expression for the matrix element $|V|^2_{0\to 1}$ of the transition $K_i = 0 \to K_f = 1$ which has been obtained earlier in refs. [16,3]

$$V|^2_{0\to 1} = |Q|^2 = \frac{4\pi}{3} I^2_{0,\nu}(0) d^2_{fi}.$$

Here d_{fi} is the matrix element of the dipole moment. Using the wave functions (6) we obtain a very simple expression for the dipole moment (until now it has been known only from numerical calculations [4]):

$$d_{fi} = \frac{3c_a\beta_2}{\kappa^{5/2}} = \frac{3c_a\beta_2}{(2\mu_a|\varepsilon_{11}|)^{5/4}}$$

The value of $I_{0,\nu}(0)$ is

$$I_{0,\nu}(0) = \left\langle 0 \left| \frac{\partial U}{\partial \rho} \right| \nu \right\rangle$$

It should be noted, however that the corrections to the approximation (10) are considerable since the corresponding parameter is of the order of

$$\beta_2\sqrt{\mu\Omega}/\kappa \simeq 0.34$$

where μ are the masses and Ω are the oscillation frequencies of MMC and D_2 molecule. Therefore, in eq. (10) $I_{0,\nu}(0)$ should be replaced by $I_{0,\nu}(x)$ averaged with the MMI wave functions,

$$\bar{I}_{0,\nu} = \sqrt{3/4\pi}\,|Q|/d_{fi}.$$

In the harmonic oscillator approximation considered in ref. [9] $U_{\mathrm{harm}} = -Kz^2/2$ where $z = \rho - \rho_0$ $(-\infty < z + \infty)$ and $\rho_0 = 1.40$ a.u. is the position of the minimum of the molecular term. The frequencies and the energy levels in the initial and final states are:

$$\Omega_{if} = \sqrt{K/M_{if}},\ E_{\nu_i\nu_f} = \Omega_{if}(\nu_{i,f} + 1/2)$$

where $M_{i,f}$ are the reduced masses of the D_2 and the MMC, respectively (see table 1). In this approximation the function I_{0,ν_f} is odd in x at even ν_f (since $\partial U_H/\partial z = -Kz$ is odd in z), and in eq. (9) the integral over x is zero at $E = 0$

($I_{0,\nu_f} = 0$). Thus the reduced entrance widths for a harmonic oscillator vanish at $E \to 0$ [9].

In such a case it is necessary to take into account the next anharmonic term in the potential U

$$\frac{\partial U}{\partial z} = \frac{\partial U_{\mathrm{H}}}{\partial z} + \frac{\partial U_{\mathrm{Anh}}}{\partial z} = -Kz\left(1 - \tfrac{3}{2}\alpha z\right)$$

where α can be extracted e.q. from the Morse potential for the H_2 molecule [17]: $\alpha_{\mathrm{M}} = 1.05$ a.u. Considering the anharmonism U_{Anh} as a perturbation we obtain the first order correction at $E \to 0$ ($K = \mu_f \Omega_f^2 = \mu_i \Omega_i^2$) in the form

$$\bar{I}_{02} = \tfrac{3}{4}\alpha_{\mathrm{M}}\Omega_f \left(\frac{\mu_i \mu_f \Omega_i \Omega_f}{2^6 \pi^2} \right)^{1/4} \int_{-\infty}^{\infty} \mathrm{d}t \, |t| \, \mathrm{e}^{-|t|}$$

$$\int_{-\infty}^{\infty} \mathrm{d}y \cdot y \cdot H_2(y) \exp\left\{ -\frac{1}{2}\sqrt{\frac{\mu_i}{\mu_f}} \left(y - \frac{\sqrt{\mu_f \Omega_f}}{\kappa} \beta_2 t \right)^2 - \frac{y^2}{2} \right\}$$

$$H_2(y) = 4y^2 - 2. \tag{13}$$

Here natural variables $t = \kappa x$ and $y = \sqrt{\mu_f \Omega_f z}$ are introduced and the oscillator wave functions with $\nu = 0$ and $\nu = 2$ are written down explicitly. Anharmonic corrections to these wave functions contribute to I_{02} only in the second order in U_{Anh}. It follows from eq. (12) that the anharmonic correction is $3\alpha_{\mathrm{M}}/2\sqrt{\mu_f \Omega_f} = 0.3$. The calculated values of I_{02} and $\bar{I}_{02}$ are listed in table 2. The value $I_{02}(0)$ has been also calculated numerically in ref. [11] with the wave functions for a potential close to the exact one [18]. It exceeds our result by 11%. In order to get a better accuracy we have performed exact calculations with the full Morse potential

$$U(z) = D\left(\mathrm{e}^{-2\alpha_{\mathrm{M}} z} - 2\,\mathrm{e}^{-\alpha_{\mathrm{M}} z}\right), \quad z = \rho - \rho_0,$$

$$D = 0.1745 \text{ a.u.}, \quad \rho_0 = 1.401 \text{ a.u.}, \quad \alpha_{\mathrm{M}} = 1.05 \text{ a.u.}$$

Table 2
Matrix elements for the vibrational transition $\nu_i = 0 \to \nu_f = 2$ (in atomic units)

Potential	$I_{02}(0)$	$\bar{I}_{02}$
Harmonic oscillator	0	0
Anharmonic oscillator	$1.56 \cdot 10^{-2}$	$2.09 \cdot 10^{-2}$
Morse	$1.64 \cdot 10^{-2}$	$1.68 \cdot 10^{-2}$
Kolos-Wolniewicz [18]	[11] $1.73 \cdot 10^{-2}$	$1.77 \cdot 10^{-2}$

Table 3
The values of constants A, B, C (a.u.) in eq. (16)

	A	B	C
1-st order anharmonic correction	$1.44\cdot10^{-4}$	$0.83\cdot10^{-4}$	$0.192\cdot10^{-4}$
Morse potential	$1.16\cdot10^{-4}$	$0.80\cdot10^{-4}$	$0.194\cdot10^{-4}$
Kolos-Wolniewicz potential	$1.22\cdot10^{-4}$	$0.78\cdot10^{-4}$	$0.187\cdot10^{-4}$

These parameters give the energy levels of MMC with an accuracy better than 1%. The values of I_{02} calculated from the potential (14) are only 5% less than the exact ones.

If the energy is nonzero, but still low we can write down eq. (9) for Q in the form

$$Q = -\left[(A + \mathrm{i}Bp\hat{\rho})\hat{\rho} + \mathrm{i}\beta_3 Cp\hat{s}\right] \mathrm{e}^{-\mathrm{i}\beta_1 \mathbf{p}\hat{\rho}\rho_0}$$

The values of A, B, C are approximately constant: in the energy range $E < 100$ K they vary by several percent. Substituting eq. (15) into eqs. (7), (8) we obtain

$$\begin{aligned} |V|^2_{K_i \to K_f} = (2K_f + 1) \sum_{L=0}^{\infty} (2L+1) \\ &\times \left\{ \left[Aj_L(\xi) - (B - \beta_3 C) pj'_L(\xi) \right]^2 \right. \\ &\times (T_+ + T_-) + 2\beta_3 Cp \left[j_{L+1}(\xi) T_+ - j_{L-1}(\xi) T_- \right] \\ &\times \left. \left[Aj_L(\xi) - (B - \beta_3 C) pj'_L(\xi) \right] + \beta_3^2 C^2 p^2 j_L^2(\xi) T_0 \right\} \end{aligned}$$

where $T_\pm$, T_0 are expressed through the $3j$ symbols:

$$T_\pm = [2(L \pm 1) + 1] \begin{pmatrix} K_i & L \pm 1 & K_f \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} L & 1 & L \pm 1 \\ 0 & 0 & 0 \end{pmatrix}^2$$

$$T_0 = \begin{pmatrix} K_f & L & K_i \\ 0 & 0 & 0 \end{pmatrix}^2 .$$

The quantity $\xi = \beta_1 \rho_0 p = 10.75\sqrt{E\ (\mathrm{eV})}$ plays here the role of the kR parameter; $j_L(\xi)$ is the spherical Bessel function and $j'_L(\xi)$ is its derivative. At $B = C = 0$, eq. (16) comes to the formula by M. Leon [8] who has taken into account the moments $L \neq 0$ of the incident MA but has neglected the electron screening of

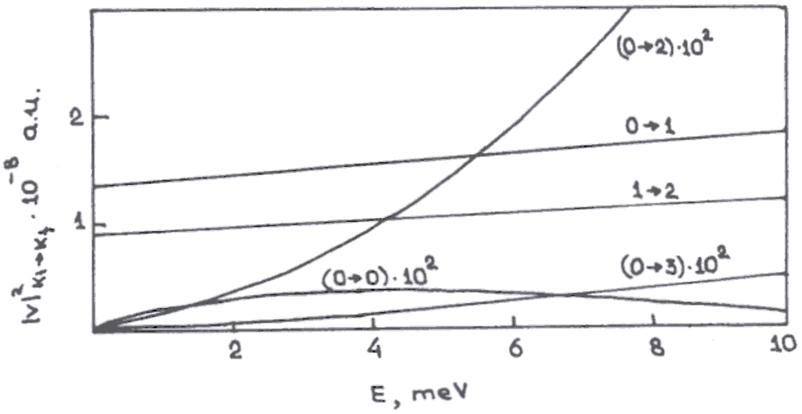

Fig. 2. Dependence of matrix elements $|V|^2_{K_i \to K_f}$ on energy E (c.m.) for the reaction (1).

the MMC. Let us note that in ref. [8] in comparing the theory with the data the value $d_{\text{fi}} = 380$ m.a.u. $= 6.18 \cdot 10^{-4}$ a.u. has been used, while eq. (11) gives at $\kappa = 9.933$ a.u. $d_{\text{fi}} = 2072$ m.a.u. $= 3.369 \cdot 10^{-3}$ a.u. This difference explains the order of magnitude discrepancy between the theoretical and experimental rates pointed out by M. Leon [8,11].

4. The energy dependence of $|V|^2_{K_i \to K_f}$ for the transitions $K_i = 0 \to K_f = 0, 1, 2, 3$ and $K_i = 1 \to K_f = 2$ to be used below for fitting the level position, are shown in fig. 2. Let us note that the matrix elements of the $K_i = 0 \to K_f = 1$ and $K_i = 1 \to K_f = 0, 2$ transitions remain nonzero at $E \to 0$ and the conventional $1/v$ dependence of the inelastic cross sections at low energy is restored. It can be seen from fig. 2 that at low energy the transition $K_i = 0 \to K_f = 1$ is dominant (see also [10]).

The MMC formation rate $\lambda_{\text{dt}\mu}$ at the density $N_0 = 4.25 \cdot 10^{22}\ \text{cm}^{-3}$ is

$$\lambda_{\text{dt}\mu} = N_0 \int \sigma_{\text{dt}\mu}(E)\, v_{\text{c.m.}}\, f(E)\, \mathrm{d}E$$

$$\sigma_{\text{dt}\mu} \cdot v_{\text{c.m.}} = \sum_{K_i K_f} w(K_i)\, |V|^2_{K_i \to K_f} \frac{\Gamma_{\text{c}}}{\left(E - \varepsilon^{(2)}_{K_i K_f}\right)^2 + \frac{1}{4}\Gamma^2} \tag{18}$$

Here $\sigma_{\text{dt}\mu}$ is the MMC formation cross section, $v_{\text{c.m.}}$ is the incident MA velocity, $f(E)$ is their energy spectrum, $w(K_i)$ are the populations of the D_2 levels with given K_i; $\varepsilon^{(2)}_{K_i K_f}$ are the resonance energies. The tμ capture width Γ_{c} is a sum [5] of (i) the Auger transition width $\Gamma_{\text{EM}} = 0.84$ meV [19], (ii) the Γ_{coll} corresponding to the collisions with the neighbour molecules, which transfer MMC to states with other values of K and ν. The value of Γ_{coll,K_f} is proportional to the density. According to the measurements [20,2], $\Gamma_{\text{coll},K_f} \simeq 2\Gamma_{\text{EM}}$, at the relative density $\phi = N/N_0 = 1$. The total width Γ is the sum $\Gamma = \Gamma_{\text{EM}} + \Gamma_{\text{coll},K_f} + \Gamma_s$ where Γ_s is the back decay width

$$\Gamma_s^{K_f \to K_i} = \frac{\mu p}{\pi} \eta(K_i) \,|V|^2_{K_f \to K_i} = 4.271 \cdot 10^5 \text{ eV} \cdot \eta(K_i) \,|V|^2_{K_f \to K_i} \sqrt{E \text{ (eV)}}$$

$$V|^2_{K_f \to K_i} = \frac{2K_i + 1}{(2K_f + 1)(2J + 1)} |V|^2_{K_i \to K_f},$$

$$\eta = \begin{cases} 4/3, & K_i = 0, 2, \\ 2/3, & K_i = 1, 3, \end{cases}$$

Here $\eta(K_i)$ is the statistical factor which accounts for the identical particles in the final state (orto- and parahydrogen); μ is the reduced mass of the $t\mu + D_2$ system (see table 1).

Table 4
Energies $\varepsilon^{(2)}_{K_i K_f}$ and contributions of the transitions $K_i \to K_f$ into $\lambda dt\mu$ at $T = 2$ meV ($w(K_i = 0) = 0.964$, $w(K_i = 1) = 0.036$)

$K_i \to K_f$	$\varepsilon^{(2)}_{K_i K_f}$, meV	$\lambda^{K_i \to K_f}_{dt\mu} 10^8\ \text{s}^{-1}$	$\varepsilon^{(2)}_{K_i K_f}$, meV	$\lambda^{K_i \to K_f}_{dt\mu} 10^8\ \text{s}^{-}$
		$1.7 \cdot 10^{-3}$	-30.8	$1.2 \cdot 10^{-3}$
		1.56	-26.0	1.06
		0.018	-16.3	0.010
		0.47	-1.7	0.015
		–	17.6	–
		–		–
		–		–
		1.28		0.84
		–		
		–		

$$\lambda_{dt\mu} = \sum_{K_i, K_f} w(K_i) \lambda^{K_i \to K_f}_{dt\mu}$$

The comparison of the calculated MMC formation rate with the data is most simple at low density ($\Gamma_{\text{coll},K_f} \ll \Gamma_{\text{EM}}$) *. Low temperature of the DT mixture $T = 23$ K ($T = 2$ meV) is also a convenient choice. At this temperature only 3.6% of the D_2 levels with $K_i = 1$ are populated. Extrapolating the data of the Jones

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* The time of the $t\mu$ mesoatom slowing down and thermalization $\tau \sim 10^{-8} \div 10^{-9}$ sec$/\phi$ (where ϕ is the density) should be still much less than the muon lifetime $\tau_0 \sim 2 \cdot 10^{-6}$ sec.

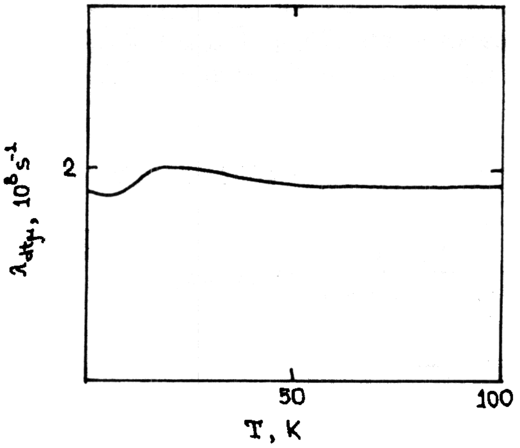

Fig. 3. Dependence of the MMC formation rate $\lambda_{dt\mu}$ on temperature $T(\varepsilon^{(2)}_{0,1} = -21$ meV).

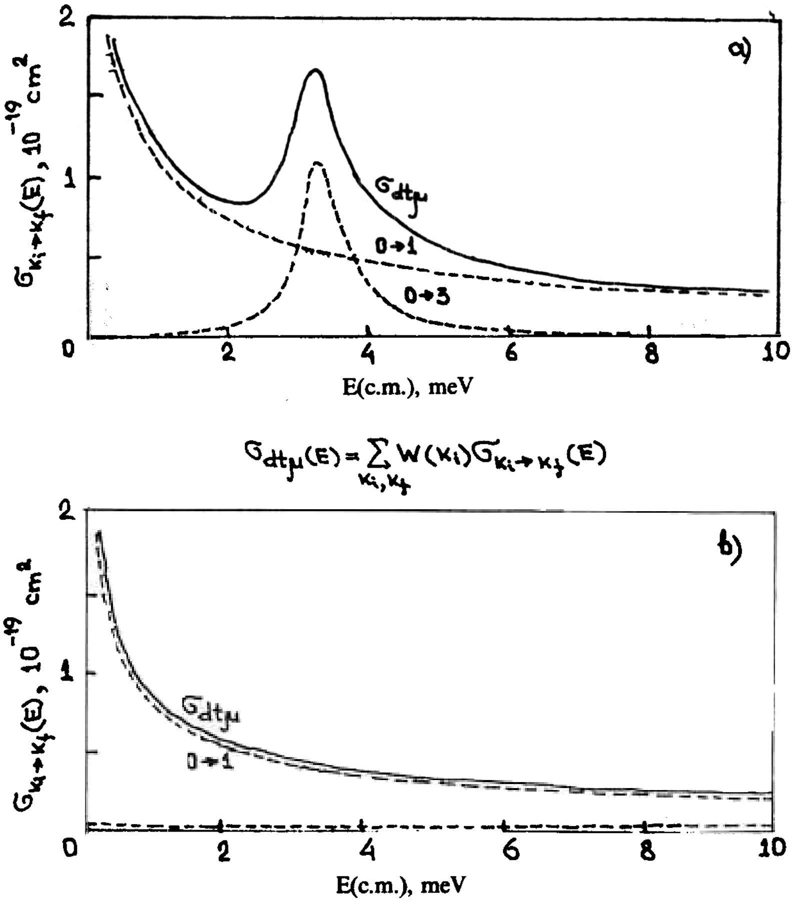

Fig. 4. Cross section of the MMC formation $\sigma_{dt\mu}(E)$ at low energy a) $\varepsilon^{(2)}_{0,1} = -21$ meV, b) $\varepsilon^{(2)}_{0,1} = -26$ meV.

group [1] to 23 K we obtain at low density the value $\lambda_{\mathrm{dt}\mu}^{\exp} = 2 \cdot 10^8\ \mathrm{s}^{-1}$. If one assumes the Maxwell MA spectrum then a crude fit gives the position of $\varepsilon_{0,1}^{(2)} = -21$ meV. The positions of other levels and their contributions into $\lambda_{\mathrm{dt}\mu}$ are listed in table 4. This fit gives for the MMI level energy $\varepsilon_{11}^{\exp} = -0.605$ meV which agrees rather well with the theoretical value $\varepsilon_{11} = -0.601$ meV from ref. [14]. The dependence of $\lambda_{\mathrm{dt}\mu}(T)$ on the temperature in the range $T < 100$ K is shown in fig. 3.

5. The calculated levels positions are not very precise. First, the experimental data at low temperatures vary considerably: the SIN group [2] gives the value of $\lambda_{\mathrm{dt}\mu}$ about two times less than that of the Los Alamos group. If one assumes the value of ref. [2] the level $K_{\mathrm{i}} = 0 \to K_{\mathrm{f}} = 1$ should be shifted further from zero by 5 meV, resulting in $\varepsilon_{0,1}^{(2)} = -26$ meV (see table 4). Second, in order to increase the precision it would be useful to calculate $|V|^2_{K_{\mathrm{i}} \to K_{\mathrm{f}}}$ with the exact dtμ wave function [21]. Finally, the use of the Maxwell spectrum $f_{\mathrm{M}}(E)$ seems doubtful. Indeed, let us compare the MMI formation cross section $\sigma_{\mathrm{dt}\mu}(E)$ shown in fig. 4 with the tμ scattering cross section (at $T = 2$ meV). It can be seen from fig. 4 that at $E \simeq 1$ meV $\sigma_{\mathrm{dt}\mu} \simeq 2 \cdot 10^{-19}\ \mathrm{cm}^2$ ($2 \cdot 10^5$b) i.e. it is of the same order as the scattering cross section $\sigma_{\mathrm{s}} \simeq 2 \cdot 10^{-19}\,\mathrm{cm}^2$ [22]. This fact indicates that there might be a considerable hardening of the spectrum $f(E)$ due to the absorption of slow tμ atoms. Such effect is well-known in neutron physics (see e.g. [23]).

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Note added in proof: Recently we have repeated the calculations with a realistic potential [18]. The results are included in tables 2, 3. The rates of MMC formation $\lambda_{\mathrm{dt}\mu}$ are slightly different but the level position ε_{11} remains the same.

References

[1] S.E. Jones, A.N. Anderson, A.J. Gaffrey, C. De Van Siclen, K.D. Watts, J.N. Brandbery, J.S. Cohen, P.A.M. Gram, M. Leon, H.R. Maltrud and M.A. Paciotti, Phys. Rev. Lett. 56 (1986) 588.

[2] W.H. Breunlich, M. Cargnelli, P. Kammel, J. Marton, N. Maegele, P. Pawlek, A. Serinzi, J. Werner, J. Zmeskal, J. Bistirlich, K.M. Crowe, M. Justice, J. Kurck, C. Petitjean, R.H. Sherman, H. Bossy, F.J. Hartmann, W. Neumann and G. Schmidt, Preprint Lawrence Berkeley Labor., LBL-21366, 1986.

[3] S.S. Gershtein and L.I. Ponomarev, Phys. Lett. 72B (1977) 80.

[4] S.I. Vinitsky, L.I. Ponomarev, I.V. Puzynin, T.P. Puzynina, L.N. Somov and M.P. Faifman, Zh. ETF 74 (1978) 849.

[5] Yu.V. Petrov, Phys. Lett. 163B (1985) 28; Muon Catalyzed Fusion 1 (1987) 219.

[6] M.P. Faifman, L.I. Men'shikov, L.I. Ponomarev, I.V. Puzinin, T.P. Puzynina and T.A. Strizh, Z. Phys. D. 2 (1986) 79.
[7] A.M. Lane, Phys. Lett. 98A (1983) 337.
[8] M. Leon, Phys. Rev. Lett. 52 (1984) 605.
[9] L.I. Men'shikov, Yad. Fiz. 42 (1985) 1184.
[10] M. Leon, Muon Catalyzed Fusion 1 (1987) 163.
[11] J.S. Cohen and R.L. Martin, Phys. Rev. Lett. 53 (1984) 738.
[12] D.D. Bakalov, S.I. Vinitsky, V.S. Melezhik and L.I. Men'shikov, Preprint JINR P4-84-693, 1984.
[13] L.I. Ponomarev and G. Fiorentini, Muon Catalyzed Fusion 1 (1987) 3.
[14] S.I. Vinitsky, V.I. Korobov and I.V. Puzynin, Zh. ETF 91 (1986) 705.
[15] L.I. Men'shikov, Yad. Fiz. 42 (1985) 1449.
[16] E.A. Vesman, Zh. ETF (Pis'ma) 5 (1967) 113 (JETP Lett. 5 (1967) 91).
[17] A. Sharp, At. Data 2 (1971) 119.
[18] W. Kolos and L. Wolniewicz, J. Chem. Phys. 43 (1965) 2429.
[19] L.N. Bogdanova, V.E. Markushin, V.S. Melezhik and L.I. Ponomarev, Zh. ETF 83 (1982) 1615.
[20] S.E. Jones, A.N. Anderson, A.J. Caffrey, J.B. Walter, K.D. Watts, J.N. Bradbery, P.A.M. Gram, M. Leon, H.R. Maltrud and M.A. Paciotti, Phys. Rev. Lett. 51 (1983) 1767.
[21] S.I. Vinitsky and L.I. Ponomarev, Physics of Elementary Particles and Atomic Nuclei, Sov. J. Part. Nuclei 13 (1982) 557.
[22] A. Adamczak and V.S. Melezhik, Phys. Lett. A118 (1986) 181.
[23] A.M. Weinberg and E.P. Wigner, *The Physical Theory of Neutron Chain Reactors* (The University of Chicago Press, 1959).
[24] L.I. Men'shikov and L.I. Ponomarev, Zh. ETF (Pis'ma) 45 (1987) 471; see also M.P. Faifman, L.I. Men'shikov and L.I. Ponomarev, Muon Catalyzed Fusion 2 (1988) this volume, Ch. 5.