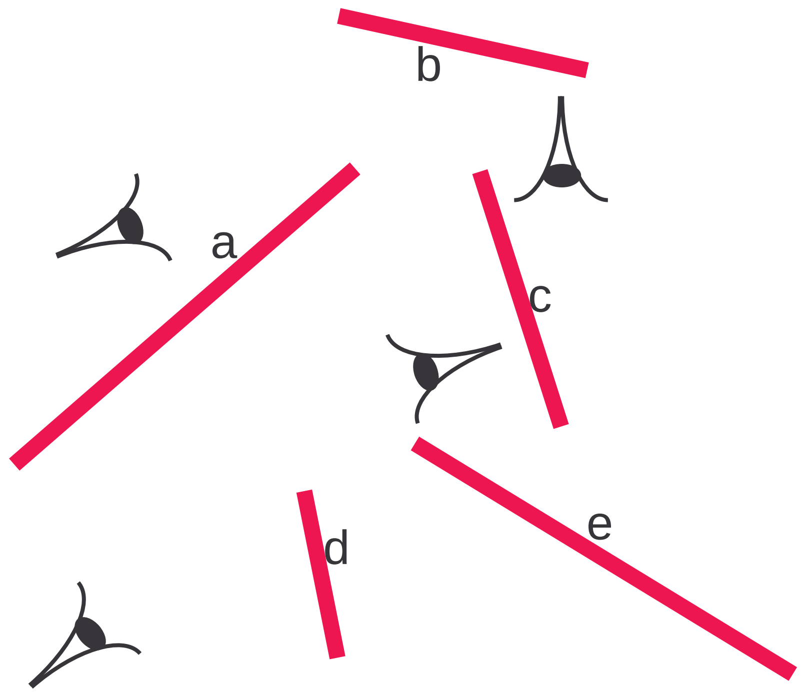
The Visibility Problem
and
Binary Space Partition

(slides by Nati Srebro)

The Visibility Problem



Z-buffering

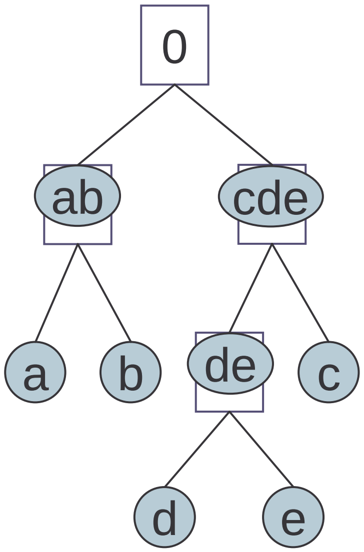
- Draw objects in arbitrary order
- For each pixel, maintain depth (“z”)
- Only draw pixel if new “z” is closer

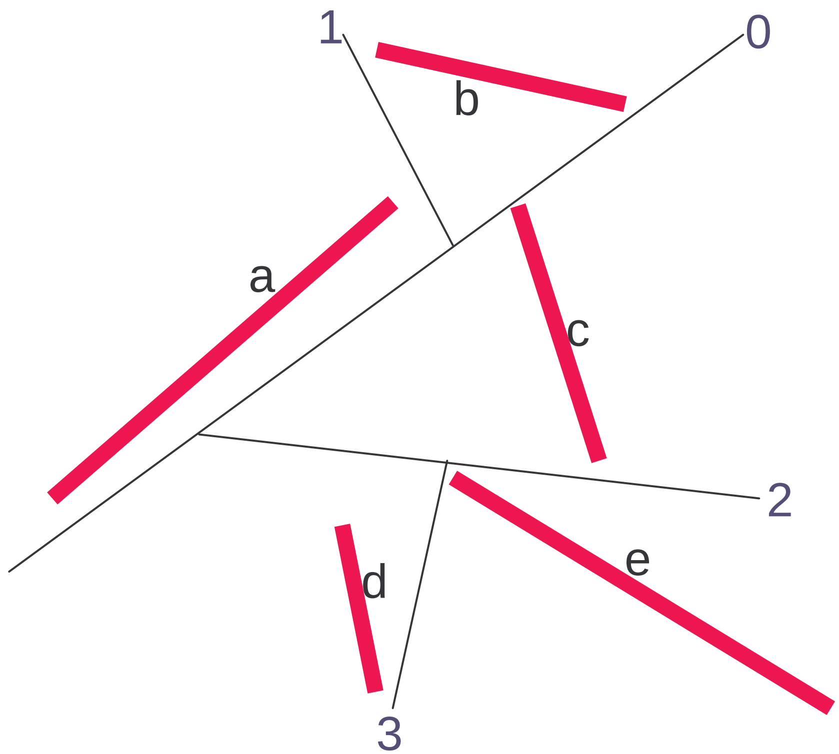
Instead:

draw objects in order, from back to front

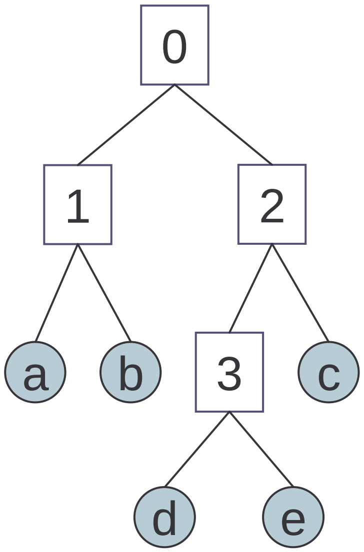
(“painter’s algorithm”)

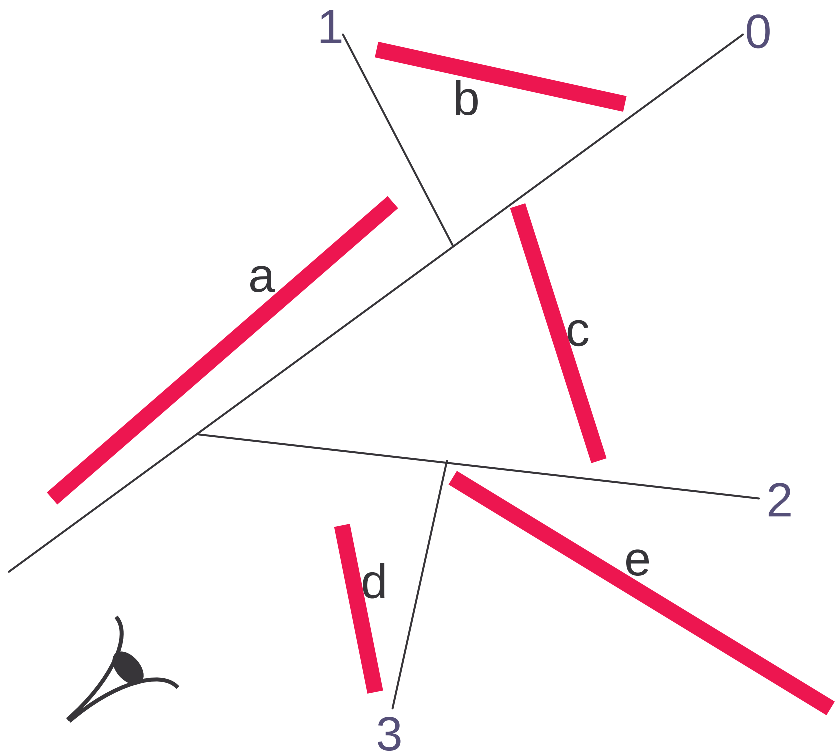
Binary Planar Partitions



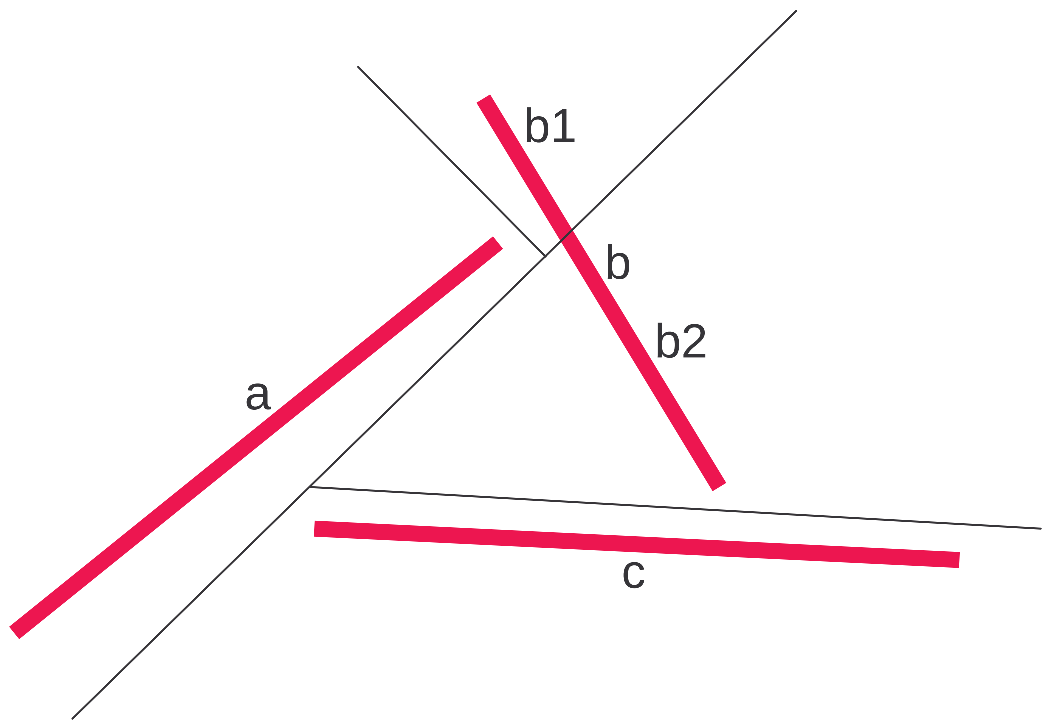


Painter's Algorithm

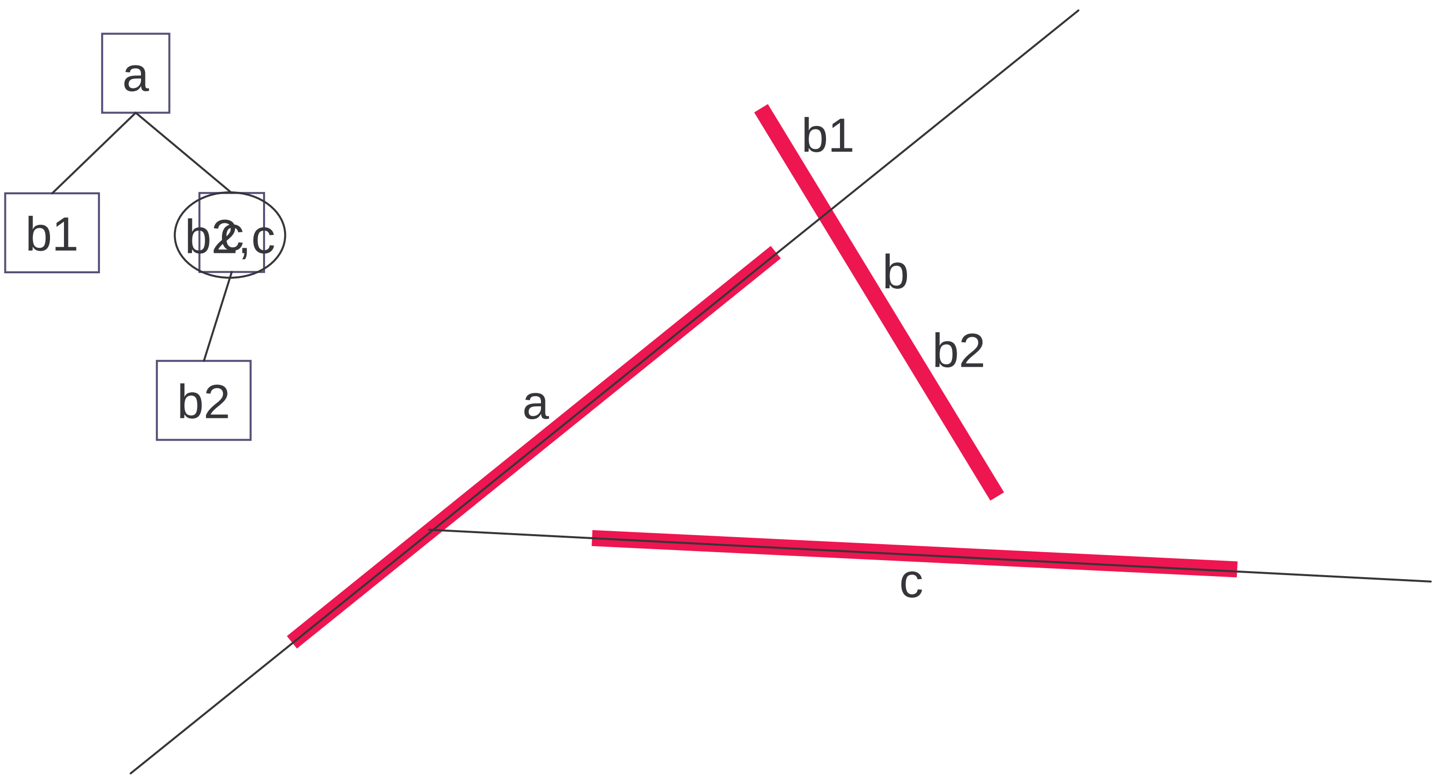




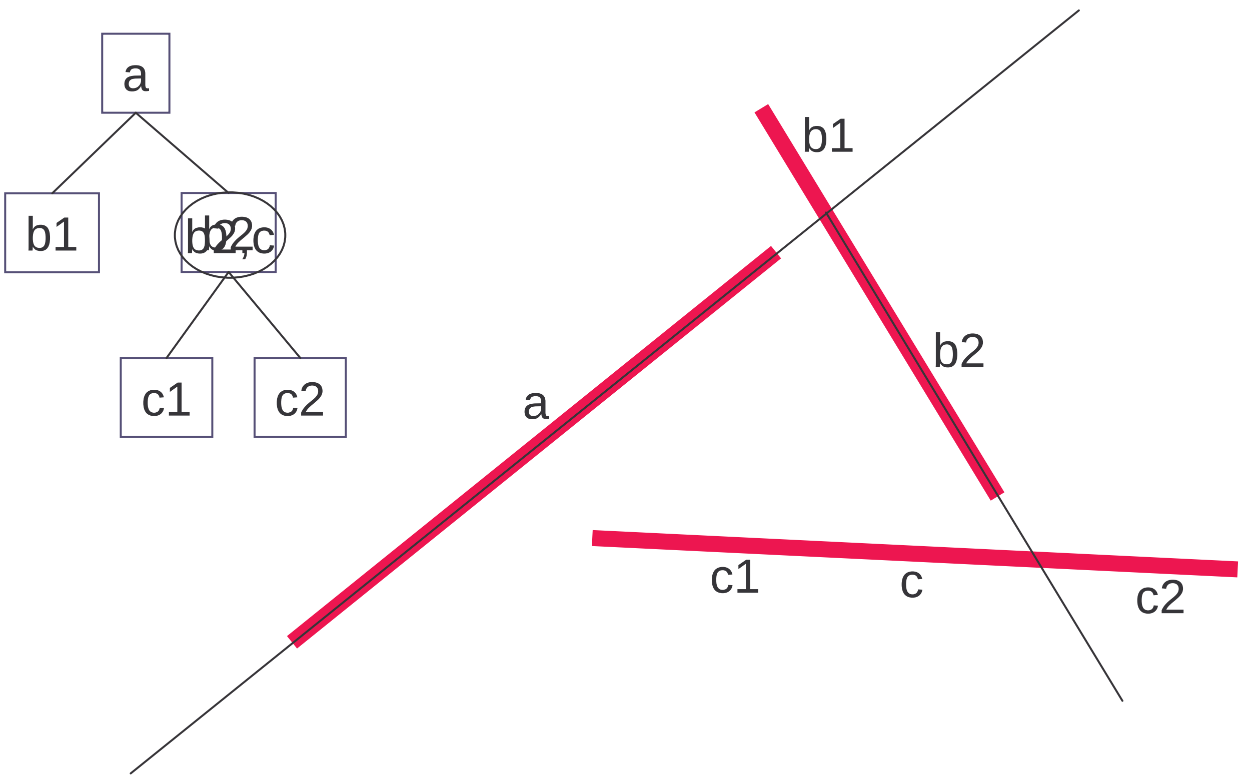
Binary Planar Partitions



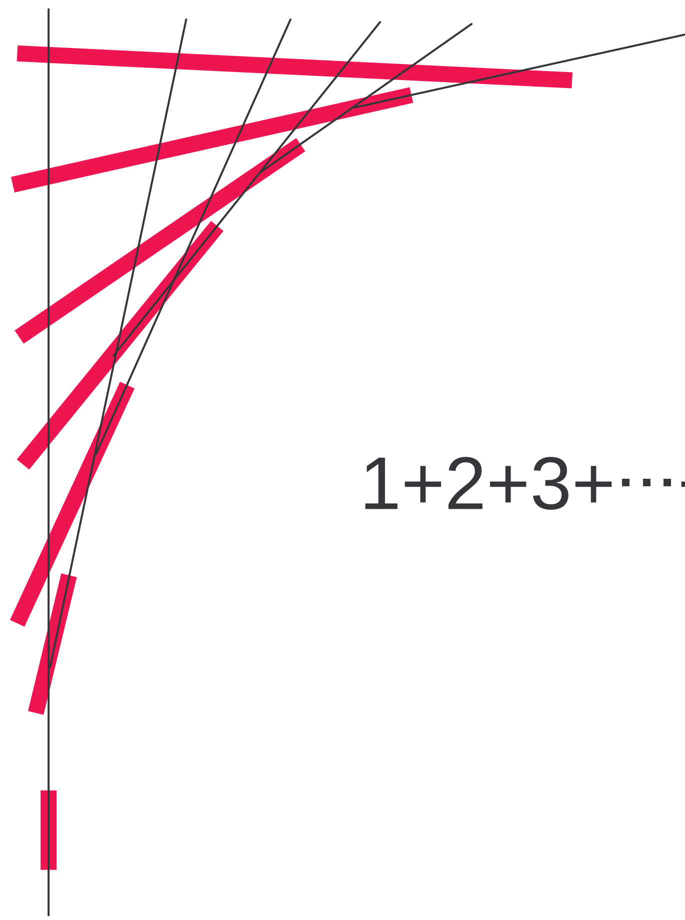
Auto-partitions



Auto-partitions



Auto-partitions



$$1+2+3+\cdots+n = n(n+1)/2 = O(n^2)$$

Binary Planar Partitions

Goal:

Find binary planer partition,

with small number of fragmentations

Random Auto-Partitions

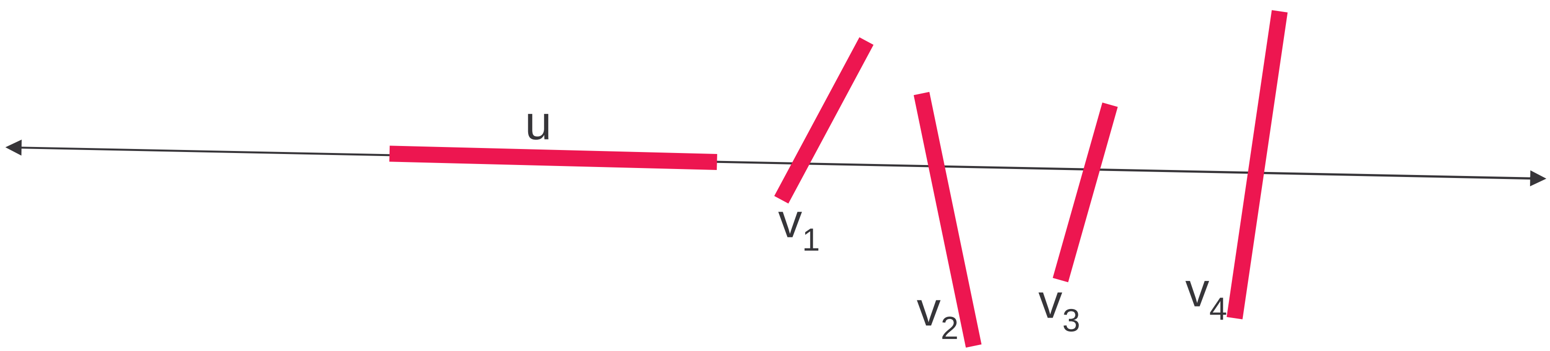
Choose random permutation of segments

$(s_1, s_2, s_3, \ldots, s_n)$

While there is a region containing more than one segment,

separate it using first s_i in the region

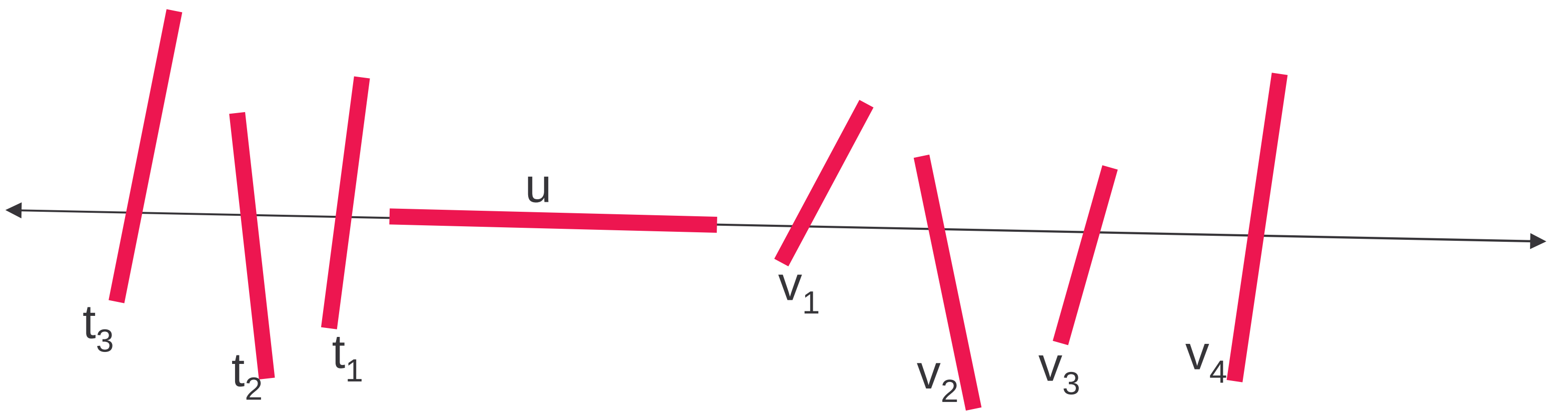
Random Auto-Partitions



u can cut v_4 only if u appears before v_1, v_2, v_3, v_4 in random permutation

$$P(u \text{ cuts } v_4) \leq 1/5$$

Random Auto-Partitions



E[number of cuts *u* makes]

= E[num cuts on right] + E[num cuts on left]

$= E[C_{v1}+C_{v2}+\cdots] + E[C_{t1}+C_{t2}+\cdots]$

$= E[C_{v1}]+ E[C_{v2}]+\cdots + E[C_{t1}]+ E[C_{t2}]+\cdots$

$\le 1/2$ … $+1/n + 1/2+1/3+1/4+\cdots+1/n$

= O(l…

C_{v1}=1 if u cuts v_1,
0 otherwise

E[total number of fragments] = n + E[total number of cuts]

$= n + \sum_u$ E[num cuts *u* makes] = n+nO(log n) = O(n log n)

Random Auto-Partitions

Choose random permutation of segments

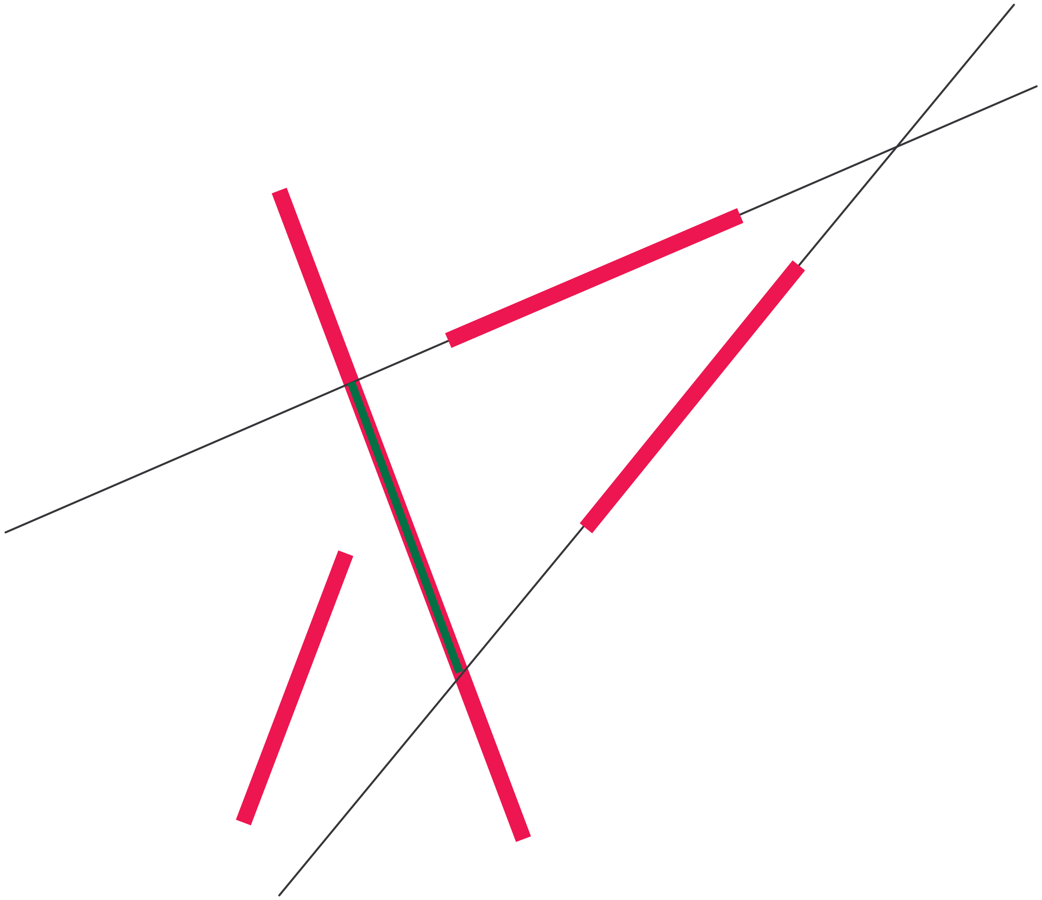
$(s_1, s_2, s_3, \ldots, s_n)$

While there is a region containing more than one segment,

separate it using first s_i in the region

O(n log n) fragments in expectation

Free cuts



Use internal fragments immediately as “free” cuts

Binary Space Partitions

- Without free cuts: $O(n^3)$
- With free cuts: $O(n^2)$