The Visibility Problem
and
Binary Space Partition
(slides by Nati Srebro)
The Visibility Problem
Z-buffering

• Draw objects in arbitrary order
• For each pixel, maintain depth ("z")
• Only draw pixel if new "z" is closer

Instead:

draw objects in order, from back to front
("painter’s algorithm")
Binary Planar Partitions
Painter’s Algorithm
Binary Planar Partitions
Auto-partitions
Auto-partitions
Auto-partitions

\[1+2+3+\cdots+n = \frac{n(n+1)}{2} = O(n^2)\]
Binary Planar Partitions

Goal:
Find binary planer partition, with small number of fragmentations
Random Auto-Partitions

Choose random permutation of segments

\[(s_1, s_2, s_3, \ldots, s_n)\]

While there is a region containing more than one segment,
separate it using first \(s_i\) in the region
Random Auto-Partitions

$u$ can cut $v_4$ only if $u$ appears before $v_1, v_2, v_3, v_4$ in random permutation

$P(u \text{ cuts } v_4) \leq 1/5$
Random Auto-Partitions

\[ E[\text{number of cuts } u \text{ makes}] = E[\text{num cuts on right}] + E[\text{num cuts on left}] \]
\[ = E[C_{v_1} + C_{v_2} + \ldots] + E[C_{t_1} + C_{t_2} + \ldots] \]
\[ = E[C_{v_1}] + E[C_{v_2}] + \ldots + E[C_{t_1}] + E[C_{t_2}] + \ldots \]
\[ \leq 1/n + 1/n + 1/2 + 1/3 + 1/4 + \ldots + 1/n \]
\[ = O(1) \quad \text{if } u \text{ cuts } v_1, \]
\[ = 0 \text{ otherwise} \]

\[ E[\text{total number of fragments}] = n + E[\text{total number of cuts}] \]
\[ = n + \sum E[\text{num cuts } u \text{ makes}] = n + nO(\log n) = O(n \log n) \]
Random Auto-Partitions

Choose random permutation of segments 
\((s_1, s_2, s_3, \ldots, s_n)\)

While there is a region containing more than one segment, separate it using first \(s_i\) in the region

\(O(n \log n)\) fragments in expectation
Free cuts

Use internal fragments immediately as "free" cuts
Binary Space Partitions

- Without free cuts: $O(n^3)$
- With free cuts: $O(n^2)$