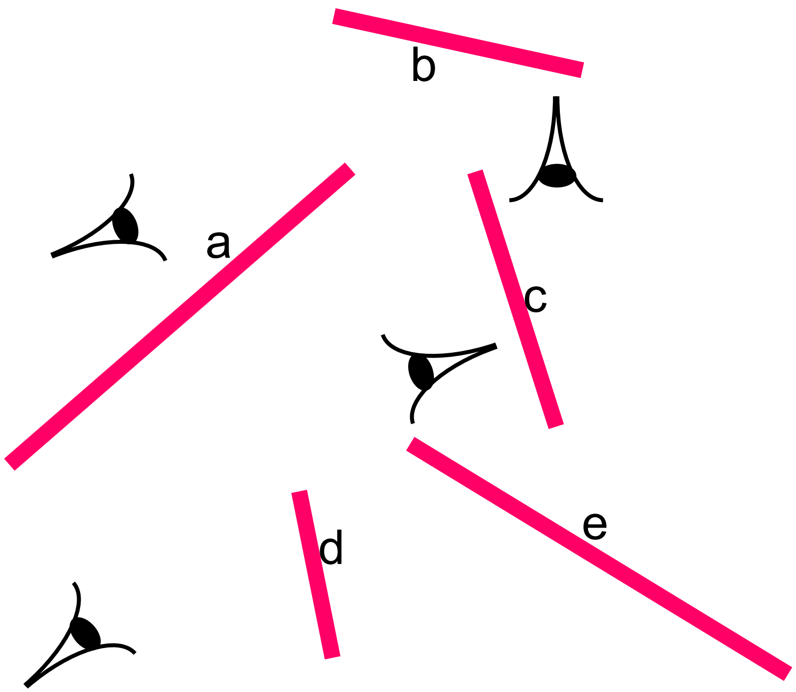
The Visibility Problem
and
Binary Space Partition

(slides by Nati Srebro)

March 8, 2005

The Visibility Problem



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Algorithms

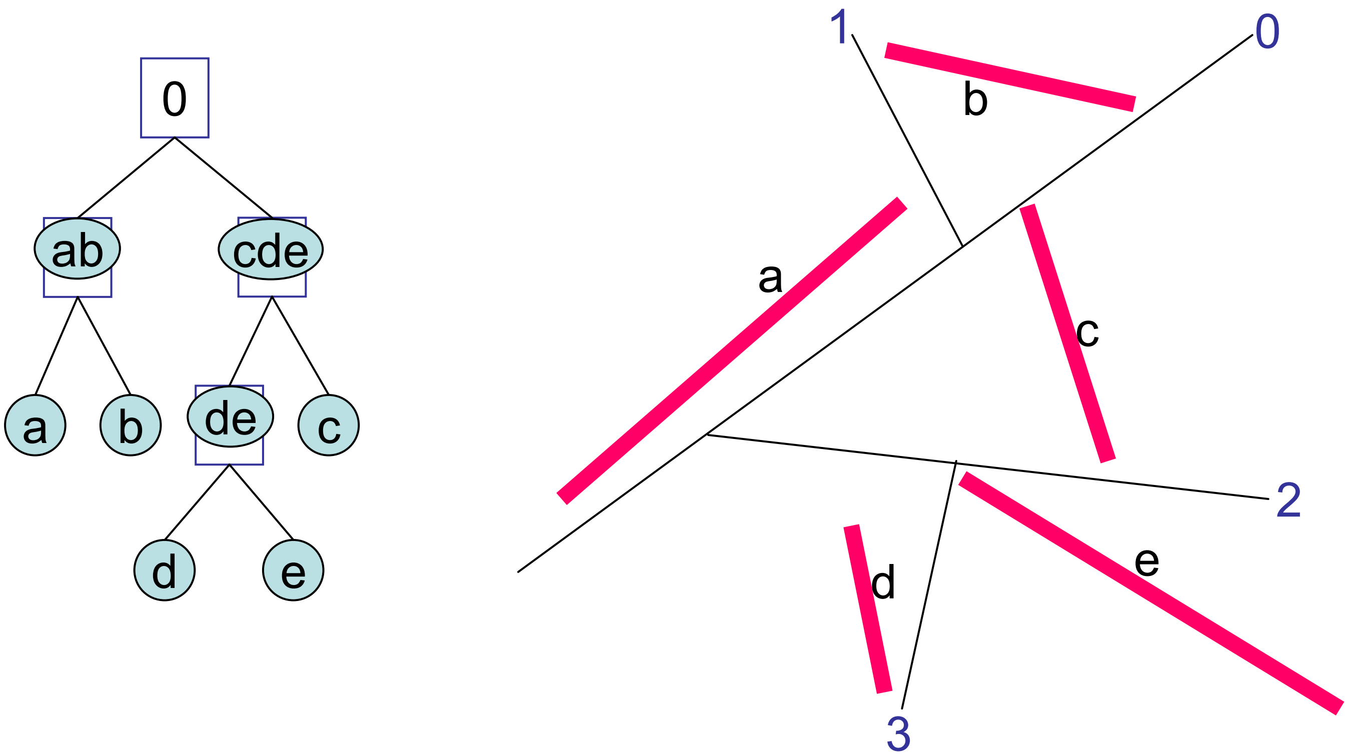
- Z-buffer:
  - Draw objects in arbitrary order
  - For each pixel, maintain distance to the eye (“z”)
  - Only draw pixel if new “z” is closer
- Painter’s algorithm:
  - draw objects in order, from back to front

Painter's algorithm

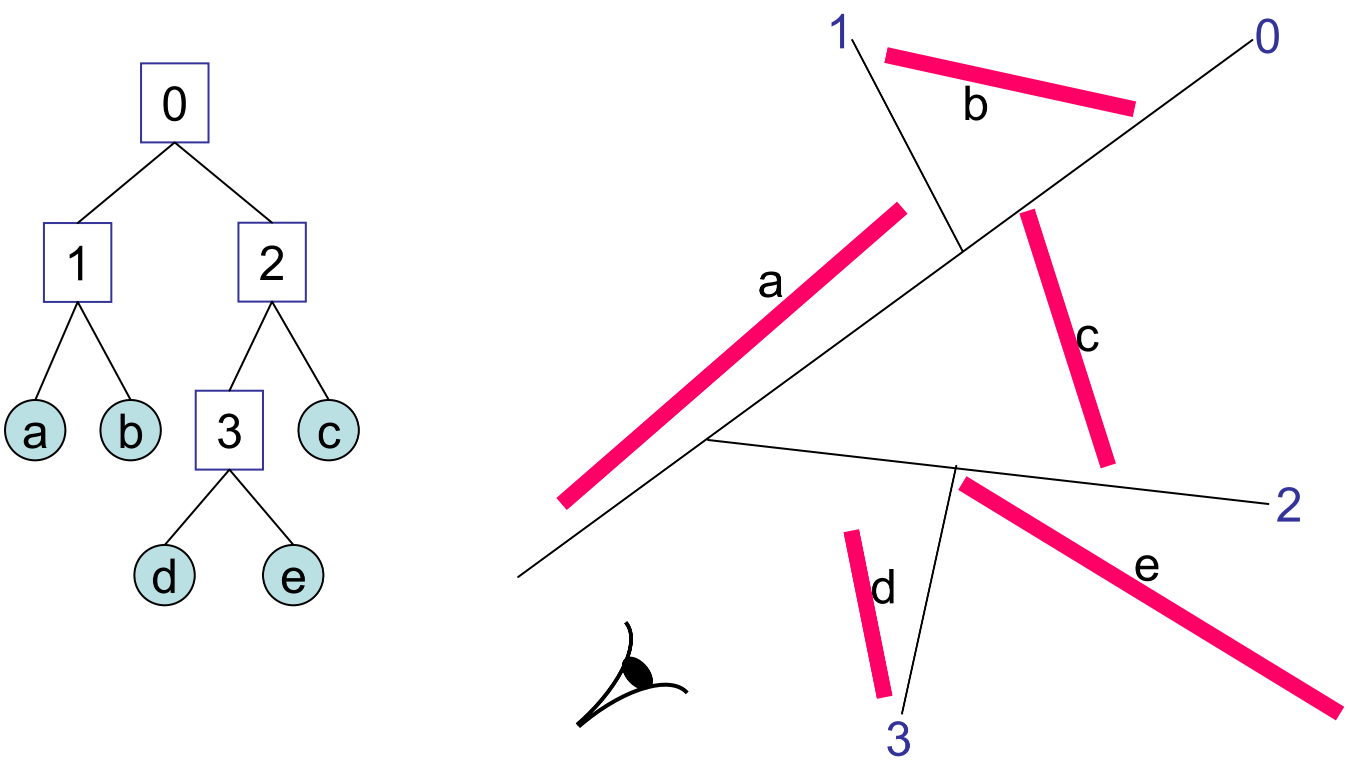
- Can one always order objects from front to back ? That is, is "A occludes B" a partial order ?

  Assuming:
  - Simple objects, e.g., segments or triangles
  - Objects disjoint
- In 2D: Yes
- In 3D: No
- We will have to **split** sometimes

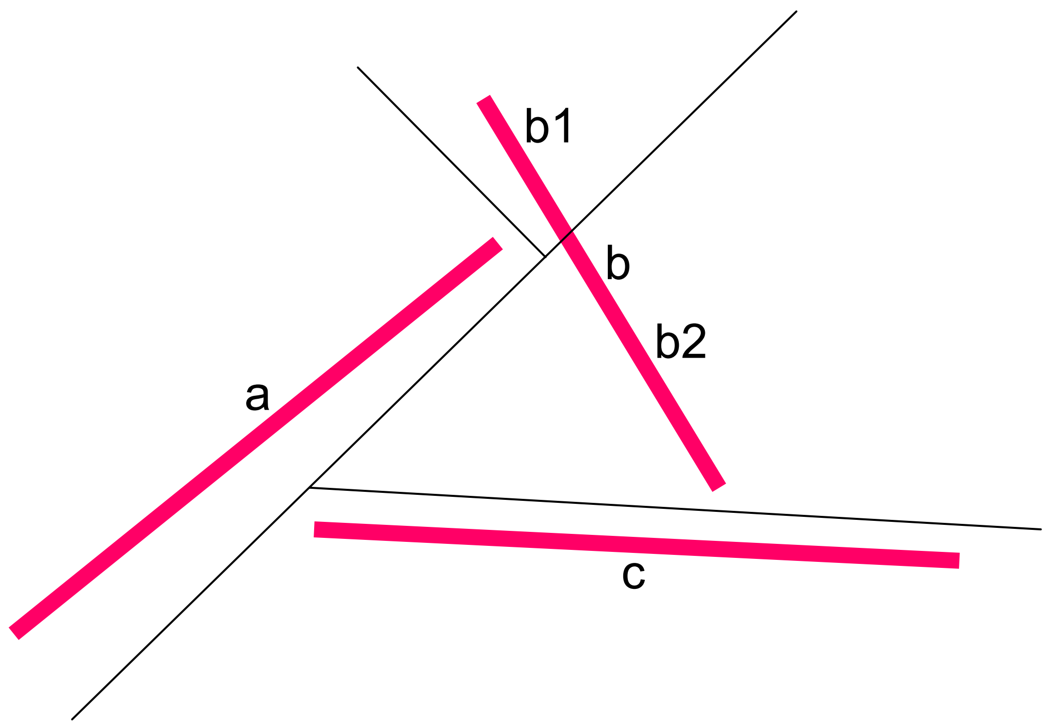
Binary Planar Partitions



Painter's Algorithm



Binary Planar Partitions



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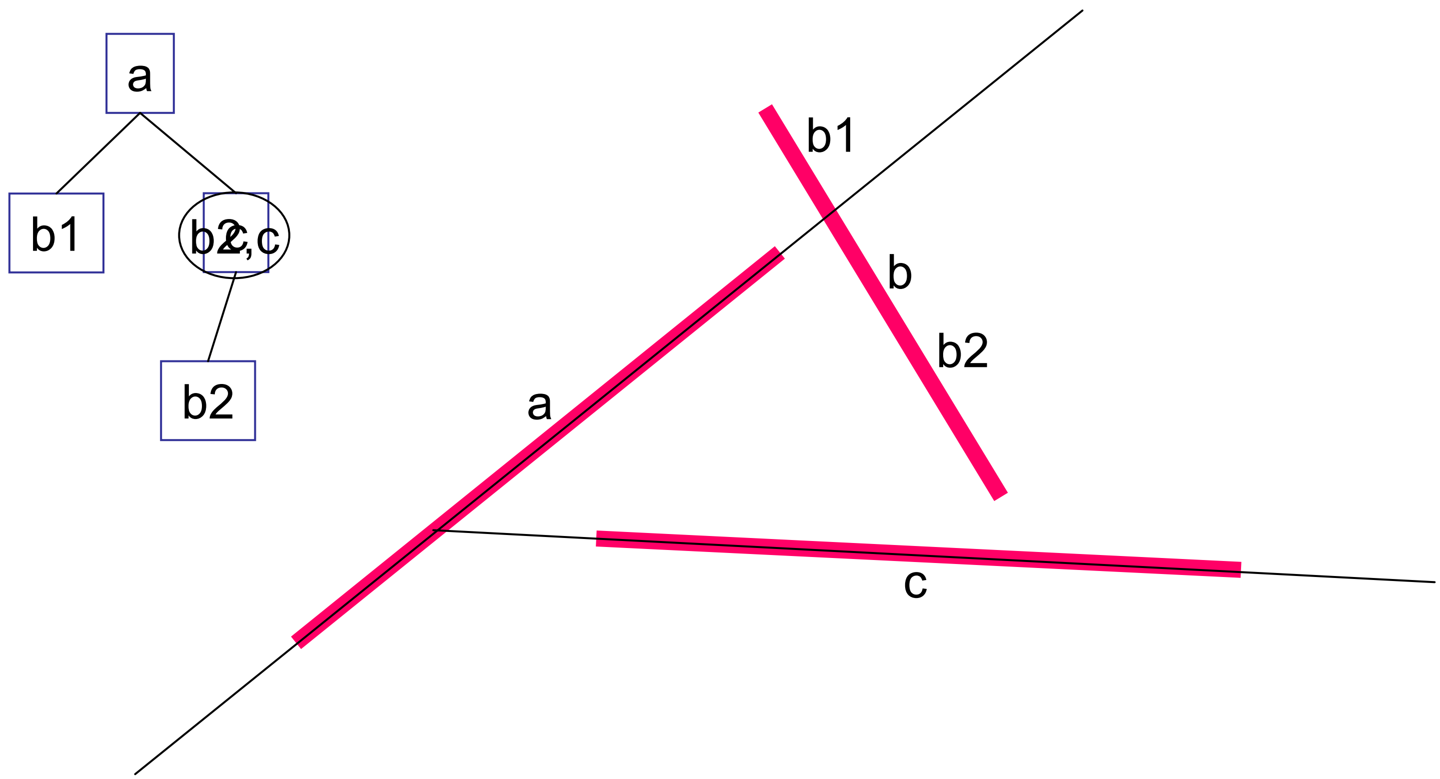
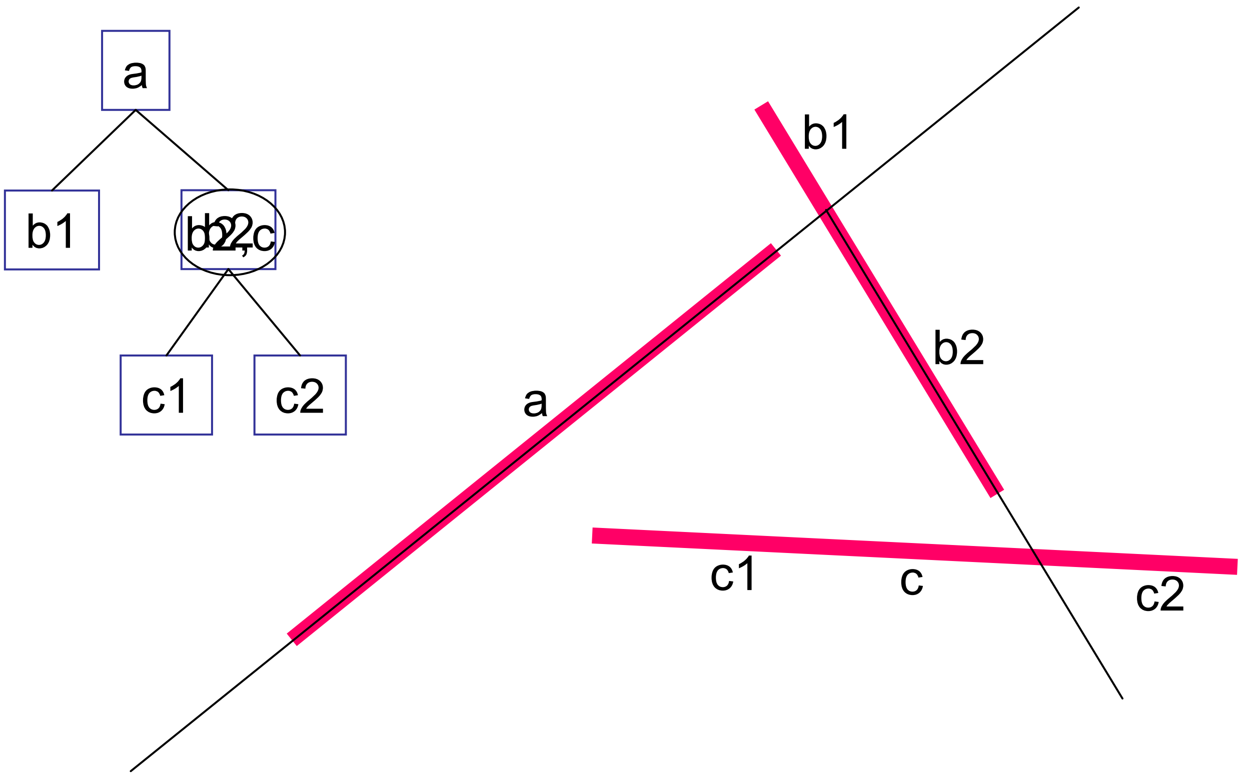
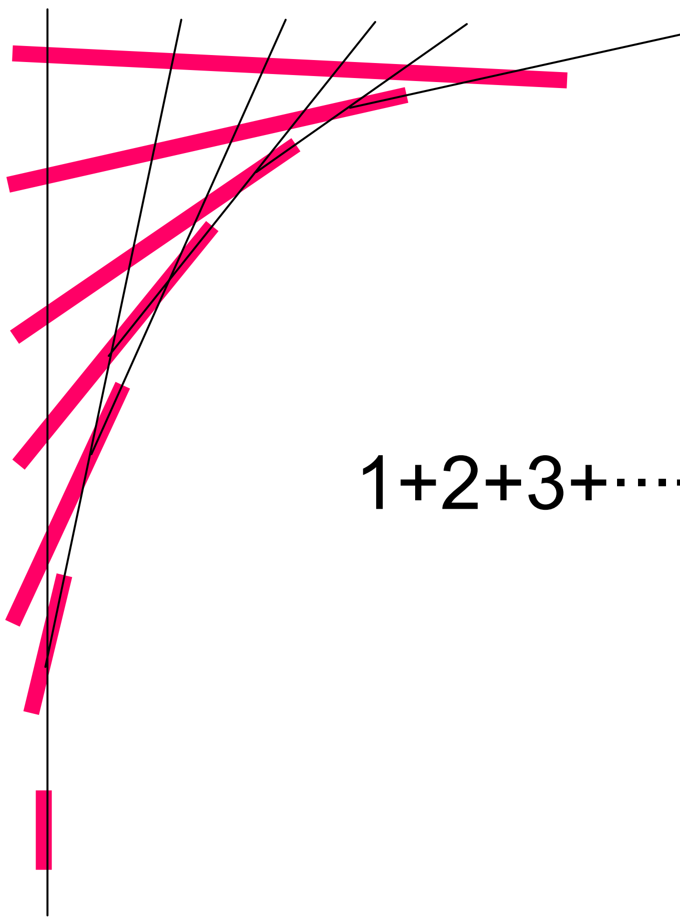
Auto-partitions



Auto-partitions



What is the complexity of BSP using auto-partitions ?



$$1+2+3+\cdots+n = n(n+1)/2 = O(n^2)$$

Binary Planar Partitions

Goal:

Find binary planer partition,
with small number of fragmentations

Random Auto-Partitions

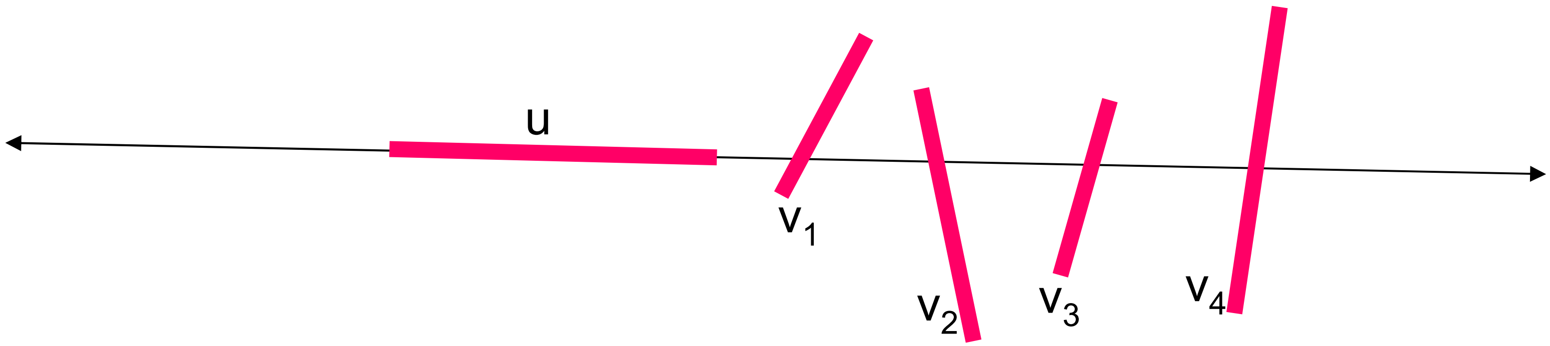
Choose random permutation of segments

$(s_1, s_2, s_3, \ldots, s_n)$

While there is a region containing more than one segment,

separate it using first s_i in the region

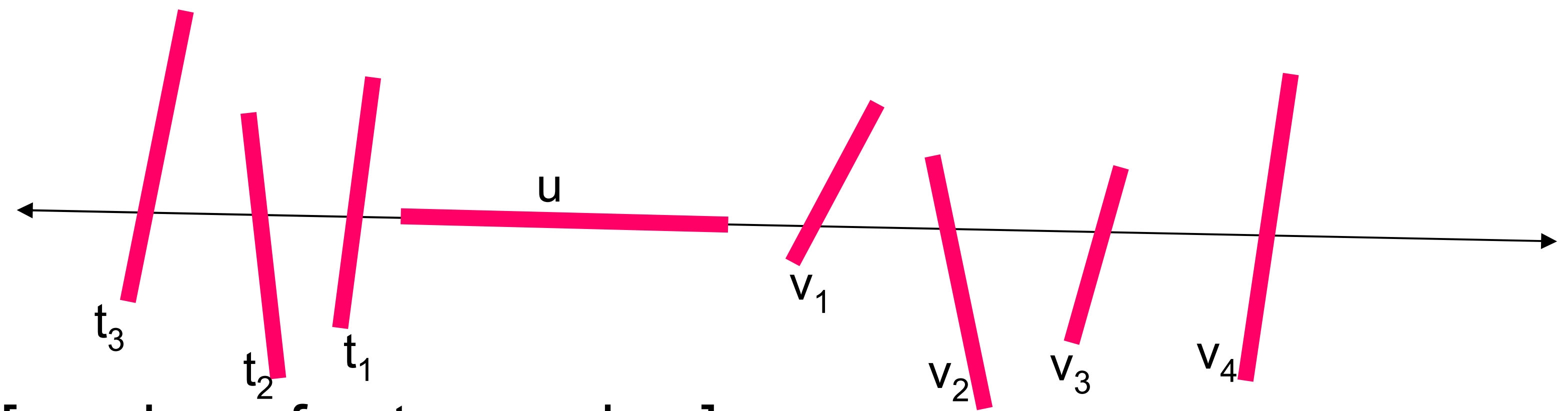
Analysis



u can cut v_4 only if *u* appears before v_1, v_2, v_3, v_4 in random permutation

P(*u* cuts v_4) $\leq$ 1/5

Random Auto-Partitions



E[number of cuts *u* makes]
= E[num cuts on right] + E[num cuts on left]
= $E[C_{v1}+C_{v2}+\cdots] + E[C_{t1}+C_{t2}+\cdots]$
= $E[C_{v1}]+ E[C_{v2}]+\cdots + E[C_{t1}]+ E[C_{t2}]+\cdots$
$\leq 1/2+1/3+1/4+\cdots+1/n + 1/2+1/3+1/4+\cdots+1/n$
= O(log n)

C_{v1}=1 if u cuts v_1, 0 otherwise

E[total number of fragments] = n + E[total number of cuts]
= n + Σ_uE[num cuts *u* makes] = n+nO(log n) = O(n log n)

Random Auto-Partitions

Choose random permutation of segments

$(s_1, s_2, s_3, \ldots, s_n)$

While there is a region containing more than one segment,

separate it using first s_i in the region

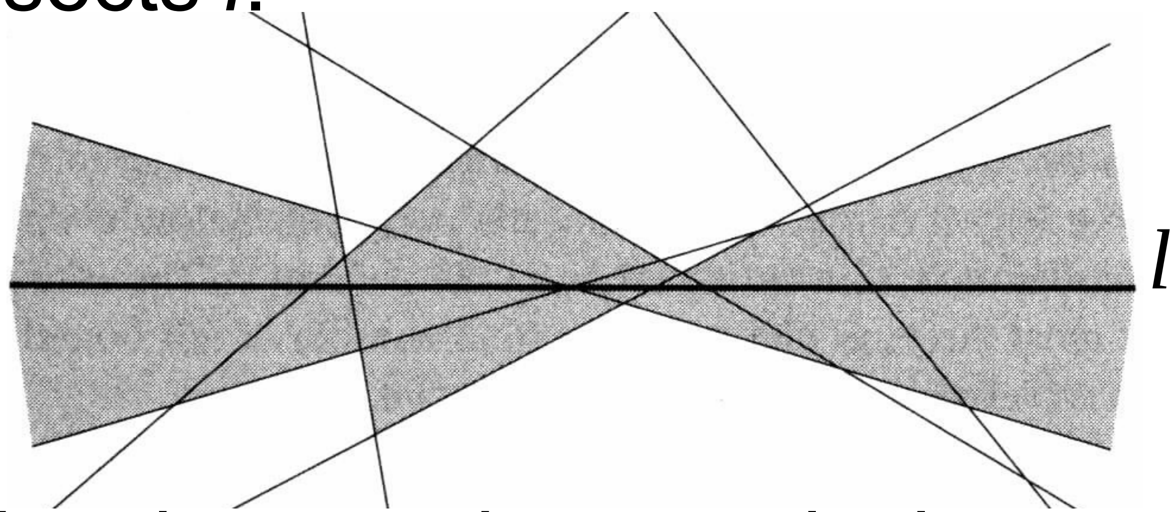
O(n log n) fragments in expectation

What about 3D ?

- Assume arbitrary order of triangles
- What is the complexity of the BSP ?
  - Each triangle can be split using n-1 planes
  - From the perspective of the triangle, it is split using n-1 lines
  - Complexity of arrangement: $O(n^2)$
  - Total complexity: $O(n^3)$
- Also, at least $\Omega(n^2)$

Zones

- The *zone* of a line *l* in an arrangement *A*(*L*) is the set of faces of *A*(*L*) whose closure intersects *l*.



- Note how this relates to the complexity of inserting a line into a DCEL…

Zone Complexity

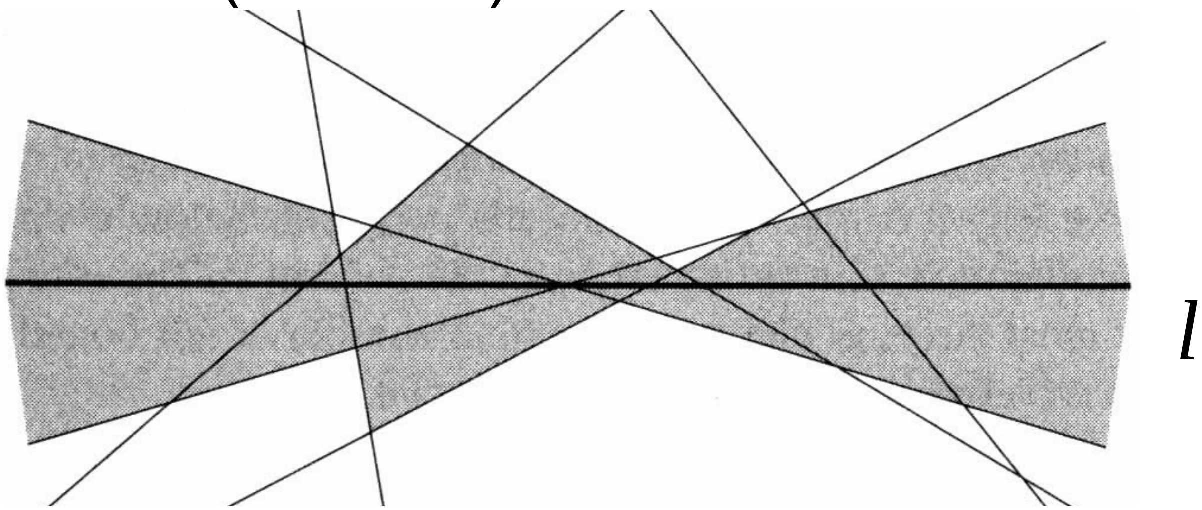
- The complexity of a zone is defined as the total complexity of all the faces it consists of, i.e. the sum of the number of edges and vertices of those faces.
- The time it takes to insert line l_i into a DCEL is linear in the complexity of the zone of l_i in $A(\{l_1,\ldots,l_{i-1}\})$.

Zone Theorem

- The complexity of the zone of a line in an arrangement of *m* lines on the plane is $O(m)$
- Therefore:
  - We can insert a line into an arrangement in linear time
  - We can compute the arrangement in $O(n^2)$ time

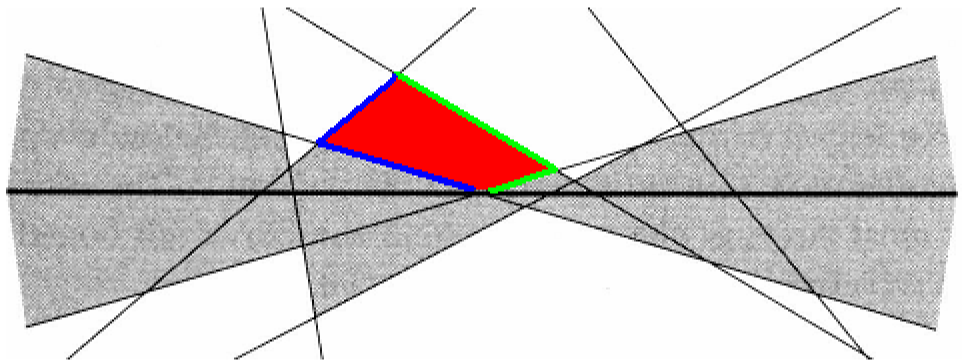
Proof of Zone Theorem

- Given an arrangement of m lines, $A(L)$, and a line l.
- Change coordinate system so l is the x-axis.
- Assume (for now) no horizontal lines



Proof of Zone Theorem

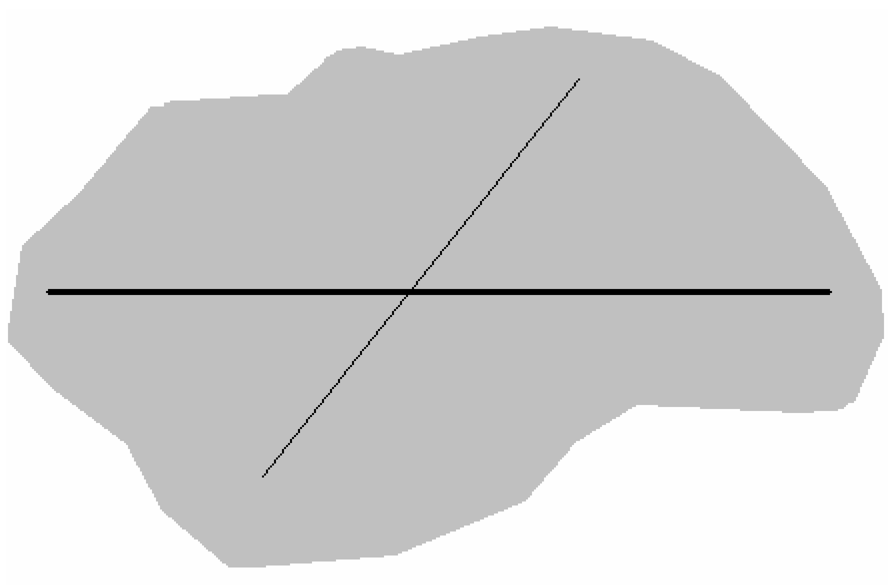
- Each edge in the zone of *l* is a *left bounding edge* and a *right bounding edge*.



- Claim: number of left bounding edges ≤ $5m$
- Same for number of right bounding edges
  → Total complexity of *zone*(*l*) is linear

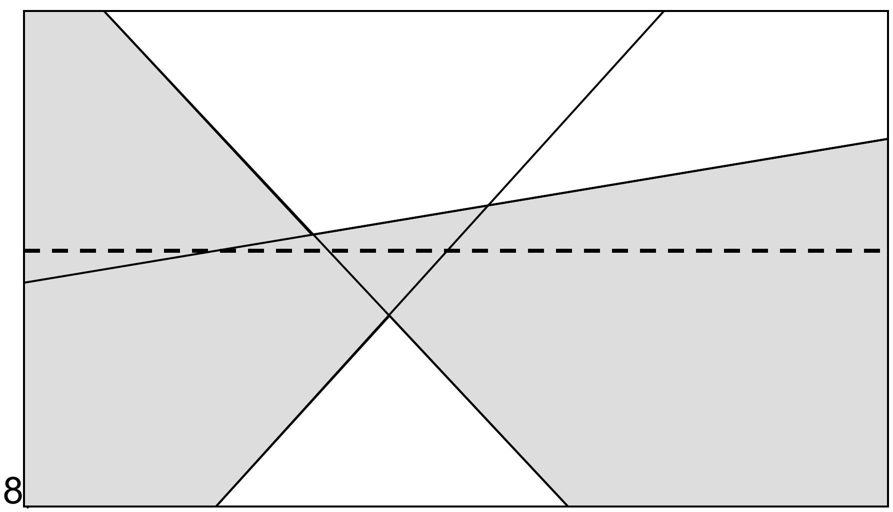
Proof of Zone Theorem
-Base Case-

- When $m=1$, this is trivially true.
  (1 left bounding edge ≤ 5)



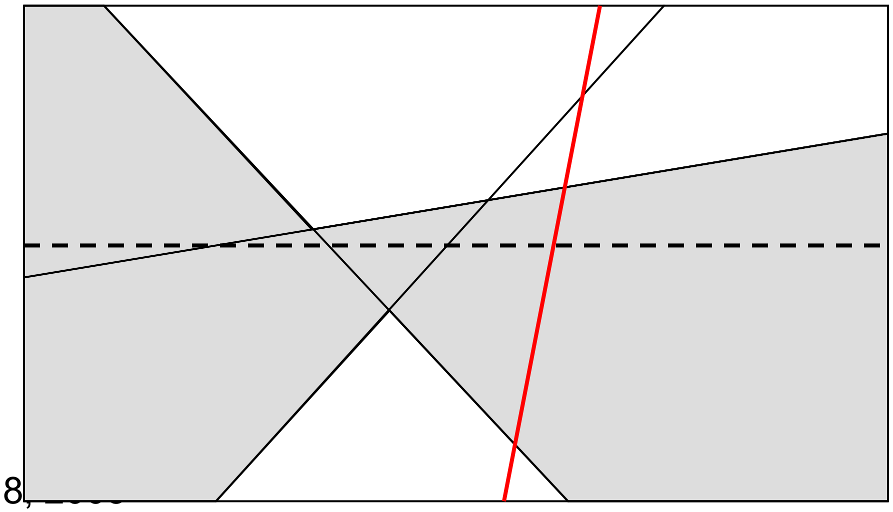
Proof of Zone Theorem -Inductive Case-

- Assume true for all but the rightmost line l_r: i.e. Zone of l in $A(L\text{-}\{l_r\})$ has at most $5(m\text{-}1)$ left bounding edges
- Assuming no other line intersects l at the same point as l_r , add l_r



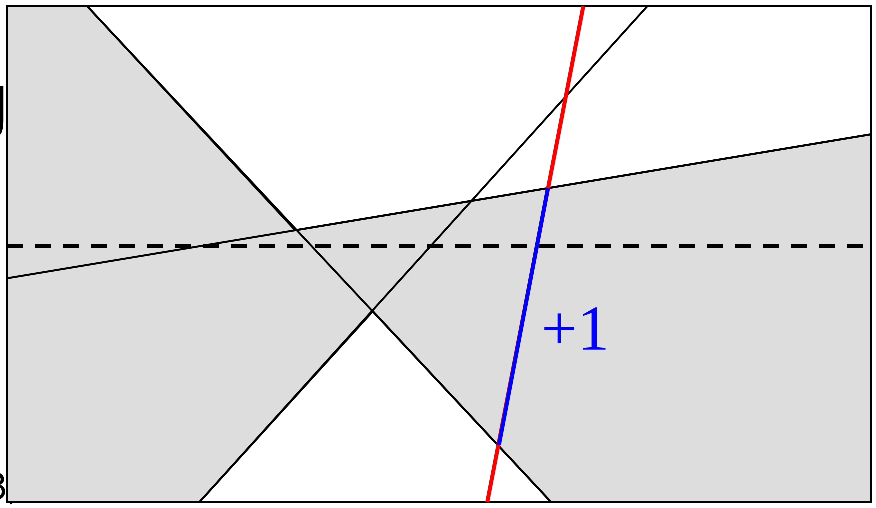
Proof of Zone Theorem -Inductive Case-

- Assume true for all but the rightmost line l_r: i.e. Zone of l in $A(L-\{l_r\})$ has at most $5(m-1)$ left bounding edges
- Assuming no other line intersects l at the same point as l_r , add l_r



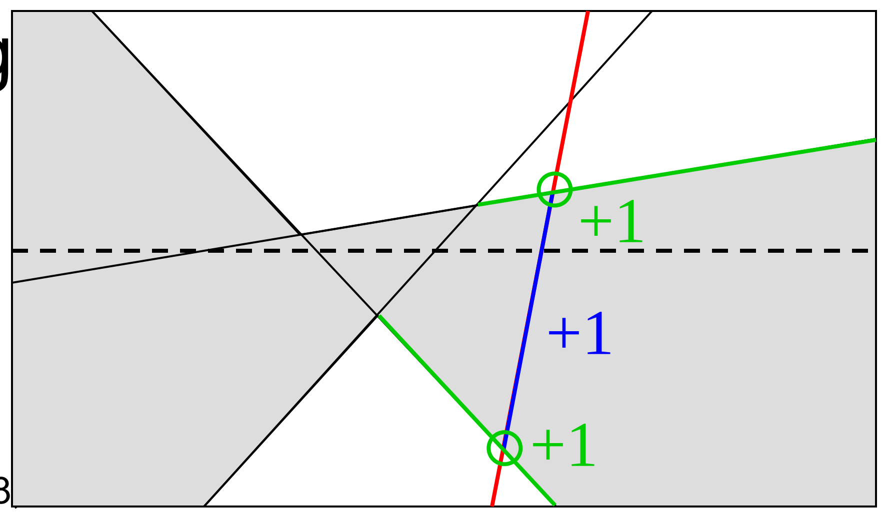
Proof of Zone Theorem -Inductive Case-

- Assume true for all but the rightmost line l_r: i.e. Zone of l in $A(L\text{-}\{l_r\})$ has at most $5(m\text{-}1)$ left bounding edges
- Assuming no other line intersects l at the same point as l_r, add l_r
  - l_r has one left bounding edge with l (+1)



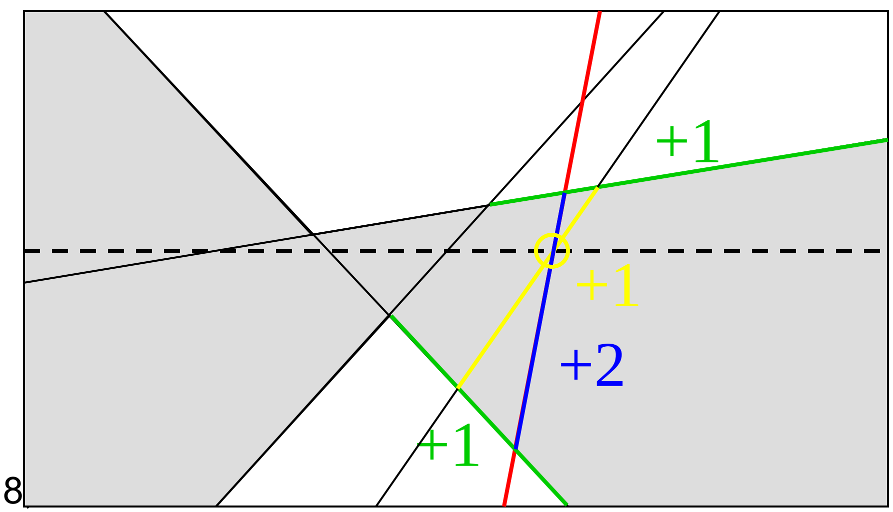
Proof of Zone Theorem -Inductive Case-

- Assume true for all but the rightmost line l_r: i.e. Zone of l in $A(L\text{-}\{l_r\})$ has at most $5(m\text{-}1)$ left bounding edges
- Assuming no other line intersects l at the same point as l_r , add l_r
  - l_r has one left bounding edge with l (+1)
  - l_r splits at most two left bounding edges (+2)



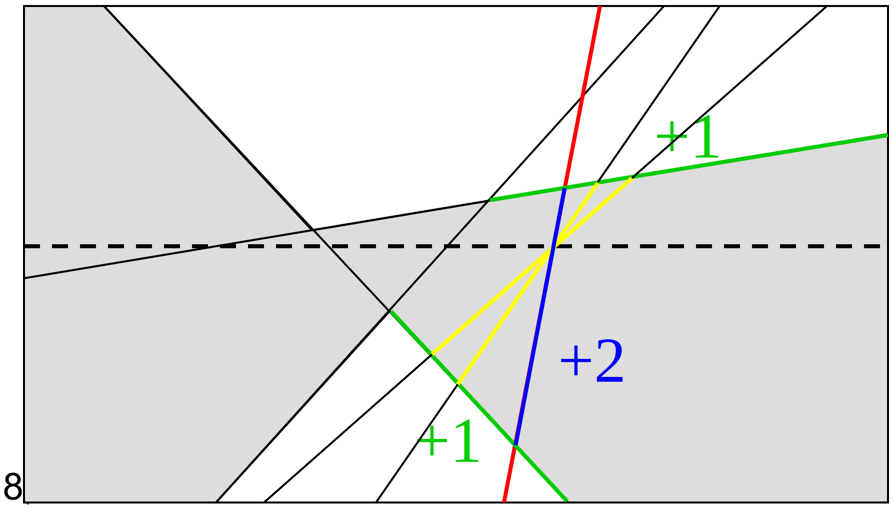
Proof of Zone Theorem
Loosening Assumptions

- What if l_r intersects l at the same point as another line, l_i does?
  - l_r has two left bounding edges (+2)
  - l_i is split into two left bounding edges (+1)
  - As in simpler case, l_r splits two other left bounding edges (+2)



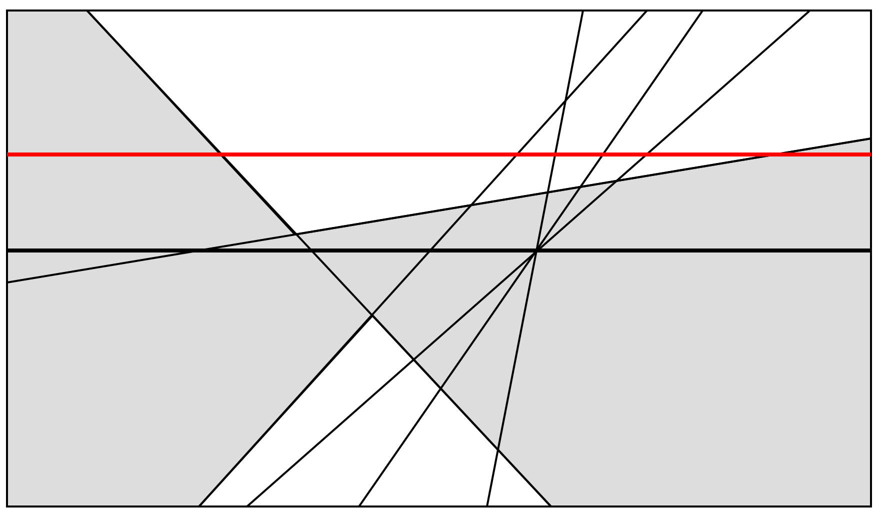
Proof of Zone Theorem
Loosening Assumptions

- What if l_r intersects l at the same point as another line, l_i does? (+5)
- What if >2 lines (l_i, l_j, …) intersect l at the same point?
  - Like above, but l_i, l_j, … are already split in two (+4)

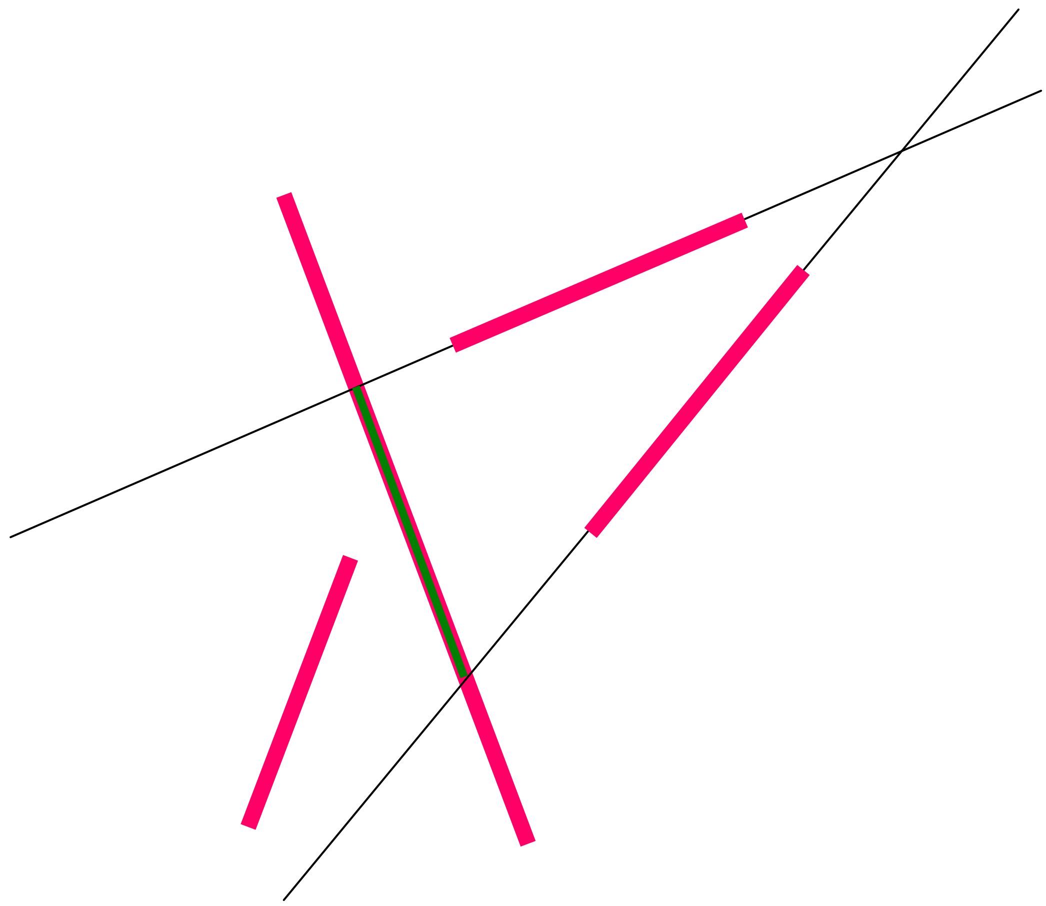


Proof of Zone Theorem -Loosening Assumptions-

- What if there are horizontal lines in *L*?
- A horizontal line introduces *not more* complexity into A(L) than a non-horizontal line.



Free cuts



Use internal fragments immediately as “free” cuts