

Turbulent Mixed Convection from an Isothermal Plate

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Abstract

When forced flow over an isothermal plate is turbulent, its total mixed convection can be computed as an algebraic function of only the forced and natural convections and the orientation of that surface.

Presented are new correlations for turbulent mixed convection from an isothermal rectangular surface having at least one horizontal edge and flow parallel to an edge of that surface.

Also presented are series of total convection measurements at Reynolds numbers from 2500 to 25000 of the five combinations of horizontal and vertical plate orientation with turbulent horizontal and vertical flow, as well as at some intermediate angles.

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1. Scope

Convection Measurement Apparatus and Methodology [1] describes the Convection Machine apparatus constructed in 2016. Its wind-tunnel chassis was small enough ($1.3 \text{ m} \times .65 \text{ m} \times .65 \text{ m}$) to measure and characterize mixed convection at various orientations of the plate and flow.

Table 1 shows the combinations of flow types and orthogonal orientations. • marks the combinations measured by the apparatus with its 0.305 m square plates; ◦ marks those combinations requiring a smaller smooth plate; ○ marks those requiring a larger plate.¹

Natural	Forced	Down	Up	Vertical	Opposing	Aiding
laminar	laminar	◦	◦	◦	◦	◦
laminar	turbulent	•Figure 5	•Figure 7	•Figure 9	•Figure 11	•Figure 13
laminar	rough	•Figure 4	•Figure 6	•Figure 8	•Figure 10	•Figure 12
turbulent	turbulent	×	○	○	○	○
turbulent	rough	×	○	○	○	○

Table 1 Mixed convective modes

The word “rough” in the “Forced” column indicates that the plate had significant roughness on its surface, whose convection is governed by correlation (7) instead of correlation (6). This distinction makes negligible difference in the present approach, which takes forced convection as an input to the mixing function.

The finding in Section 4 that there is no transition between laminar and turbulent natural convection is evidence that the model developed here will work seamlessly with turbulent natural convection as well.

2. Overview

Section 3 reviews existing articles on the subject and organizes the form for the theory to be developed.

Section 4 reviews the natural convection correlations for upward-facing, downward-facing, and vertical plate orientations.

Section 5 presents the correlations used to model forced convection from the Convection Machine’s bi-level plate.

Section 6 recounts the process discovering the correlations combining the forced and natural components in each orthogonal orientation. Section 13 presents graphs of mixed convection in each orthogonal orientation.

Section 7 synthesizes a natural convection correlation as a function of angle θ which is continuous over $-90 \leq \theta \leq 90$. The function is in reasonable agreement with Convection Machine measurements.

Section 8 combines the function from Section 7 with the correlations developed in Section 6 to develop a general formula for mixed convection from an inclined plate. At θ values which are integer multiples of 90° , the general formula simplifies to the correlations from Section 6. The general formula is then compared with measurements of a plate at angle $\theta = 84.5^\circ$.

Section 13 presents graphs of convection measurements of both plates in the five orthogonal combinations of plate and flow orientations.

¹ In order to measure laminar-laminar mixed convection in room temperature air, the forced characteristic length would need to be reduced by a factor of at least 3. Producing turbulent natural convection in air from a 0.305 m (or smaller) square plate would require it to be heated to impractically high temperatures.

3. Prior Work

Much has been written about mixed convection, most of it about local laminar mixed convection, and most of that about aiding and opposing flows adjacent to a vertical object. The focus here is on total (not local) mixed convection where the forced component is turbulent. So most of this class of prior works is not germane to the topic at hand.

The few turbulent mixed convection experiments which have been published have been limited to local convection and a single orientation.

In *An Experimental Study of Mixed, Forced, and Free Convection Heat Transfer From a Horizontal Flat Plate to Air* [2], X. A. Wang proposes local correlations for upward natural, mixed, and forced convection from a horizontal plate, but the integral for averaging the local convection fails to converge under some conditions of practical interest.

Experimental Validation Data for CFD of Steady and Transient Mixed Convection on a Vertical Flat Plate [3] by Blake describes an apparatus for measuring local conditions throughout a vertical wind channel with one wall heated. As with *Skin-Friction and Forced Convection from an Isothermal Rough Plate* [4], convection can occur adjacent to plates under isobaric conditions, but flow in closed channels is not isobaric; skin-friction causes the pressure to drop in the channel. Measurements of a closed channel are not applicable to plates.

While the total mixed convection could be derived from a complete model for local mixed convection, no complete model for local mixed convection from an isothermal plate has been published. As is the case for forced convection in Jaffer [4] and natural convection in Jaffer [5], total convection formulas can be discovered, even when the local convection is intractable.

In *A general expression for the correlation of rates of transfer and other phenomena* [6] Churchill and Usagi write that correlations of the form:

$$\text{Nu}^n(z) = \text{Nu}_0^n(z) + \text{Nu}_\infty^n(z) \quad (1)$$

are “remarkably successful in correlating rates of transfer for processes which vary uniformly between these limiting cases.” Nu is the Nusselt number, which is proportional to the convective heat transfer.

Lin, Yu, and Chen in *Comprehensive correlations for laminar mixed convection on vertical and horizontal flat plates* [7] state the form as:

$$\text{Nu}_M^n = \text{Nu}_F^n \pm \text{Nu}_N^n$$

They report that n takes on the values of 2, 3, and 4 in prior work ($n = 3$ for uniform heat flux). They settle on $n = 4$ for local laminar correlations in horizontal and vertical orientations of uniform-wall-temperature, with aiding being the sum of terms and opposing being the difference.

The present work did not find any cases where the total (not local) natural convective component was subtracted, and casts the summing form equivalently in terms of the (vector-space) ℓ^p -norm function of the natural and forced convection components:

$$\|\text{Nu}_F, \text{Nu}_N\|_p = (|\text{Nu}_F|^p + |\text{Nu}_N|^p)^{1/p}$$

The ℓ^p -norm scales linearly:

$$\|C \text{Nu}_F, C \text{Nu}_N\|_p = C \|\text{Nu}_F, \text{Nu}_N\|_p$$

Nusselt numbers should be directly combined only if they have the same characteristic-length. The convective surface conductance $h = k \text{Nu}/L$ (also $U = Ah$) is normalized relative to characteristic-length and can be combined in the ℓ^p -norm.

4. Natural Convection

For natural convection Jaffer [5] derives three formulas.

For upward-facing surfaces ($\Delta T > 0$ and $\theta < 0$ or $\Delta T < 0$ and $\theta > 0$) he derives:

$$\overline{\text{Nu}}_* = \left(0.671 + 0.370 \text{Ra}_*^{1/6}\right)^2 \quad (2)$$

For vertical surfaces ($\theta = 0$) he gives a formula close to that of Churchill and Chu [8]:

$$\overline{\text{Nu}}_V = 0.682 \left(1 + 0.5 \left[\frac{\text{Ra}_V}{\Xi_V(\text{Pr})}\right]^{1/6}\right)^2 \quad \Xi_V(\text{Pr}) = \left\|1, \frac{0.5}{\text{Pr}}\right\|_{\sqrt{1/3}} \quad (3)$$

For downward-facing surfaces ($\Delta T > 0$ and $\theta > 0$ or $\Delta T < 0$ and $\theta < 0$) he derives a formula similar to that of Schulenberg [9]:

$$\overline{\text{Nu}}_R = 0.682 \left(1 + 0.806 \left[\frac{\text{Ra}_R}{\Xi_R(\text{Pr})}\right]^{1/5}\right) \quad \Xi_R(\text{Pr}) = \left\|1, \frac{0.5}{\text{Pr}}\right\|_{\sqrt{1/3}} \quad (4)$$

In *Correlating equations for laminar and turbulent free convection from a vertical plate* [8] Churchill and Chu note that their correlation “provides a good representation for the mean heat transfer for free convection from an isothermal vertical plate over a complete range of Ra and Pr from 0 to ∞ even though it fails to indicate a discrete transition from laminar to turbulent flow.”

Jaffer [5] similarly finds no laminar-turbulent transition in the natural convection correlation (2) for upward-facing surfaces.

Schulenberg’s [9] downward-facing natural convection correlation makes no provision for turbulent flow.

As for natural convection from rough plates, the working hypothesis is that under identical conditions, the total natural convection from a rough ($0 < \varepsilon \ll L$) plate is the same as the total natural convection from a smooth plate.

The agreement of rough plate measurements with theory over the 180° range in Figure 1 means that if natural convection from rough plates is different from smooth plates, then it is a scale factor independent of angle.

5. Forced Convection

The correlations for laminar and turbulent convection from a smooth flat plate are:

$$\text{Nu}_F = 0.664 \text{Re}_F^{1/2} \text{Pr}^{1/3} \quad (5)$$

$$\text{Nu}_F = 0.037 \text{Re}_F^{4/5} \text{Pr}^{0.43} \quad (6)$$

Jaffer [4] gives a correlation (7) for a plate with root-mean-squared height-of-roughness ε :

$$\text{Nu}_F = \frac{\text{Re}_F \text{Pr}^{1/3}}{6 \ln^2(L_F/\varepsilon)} \quad 0 < \varepsilon \ll L_F \quad (7)$$

The Convection Machine plates have roughness composed of equal areas at two elevations. Being a roughness which is not self-similar, equation (7) does not apply at all Reynolds numbers. Through dimensional analysis Jaffer [1] finds the boundary between fully rough turbulence of equation (7) and smooth turbulence of equation (6):

$$\text{Re}_\ell = \frac{L/8}{0.036 L_T} \left(\frac{2\varepsilon}{0.036 L_P}\right)^{9/4} \quad (8)$$

With $\text{Re}_T = \min(\text{Re}, \text{Re}_\ell)$, Jaffer further derives a correlation for this bi-level roughness which matches measurements of both plates (1 mm and 3 mm roughness) within their measurement uncertainties:

$$\text{Nu} = \frac{\text{Re}_T \text{Pr}^{1/3}}{6 \ln^2(L/\varepsilon)} + \left[\frac{L_P - L_T}{L_P} + \left(\frac{L_T^2}{L_P^2} + \sqrt{\frac{L_T \varepsilon_{pv}}{2 L_P^2}}\right) \left(\frac{L}{L_T}\right)^{1/5}\right] [\text{Nu}_\sigma(\text{Re}) - \text{Nu}_\sigma(\text{Re}_T)] \quad (9)$$

The rough convection of correlation (7) is in effect in the mixed convection region for the 3 mm roughness plate, while a combination of smooth turbulent and rough convection are active for the 1 mm roughness plate.

6. Turbulent Mixed Convection

Jaffer [1] details the mixed convection interactions between the sides and rough face of the plate assuming a mixing function $U_M = A h_M$ (where A is the plate area). This section discovers that mixing function for horizontal and vertical plates and flows.

Consider first the case of forced flow perpendicular to the natural flow, specifically horizontal forced flow by upward-facing, downward-facing, and vertical plates. The hypothesized mixture model from Section 3 is a ℓ^p -norm function of the natural and forced convection components:

$$\|h_F, h_N\|_p = (|h_F|^p + |h_N|^p)^{1/p}$$

Because the sides are perpendicular to the rough face, the p values for each orientation must be considered together. The mixed convection data-sets were run with various assignments of p to orientations. Vertical and upward-facing natural convections being larger than downward-facing convection, it quickly became clear that $p = 2$ for upward-facing h_* in Figure 6 and vertical h_V in Figure 8. In Figure 4 for the downward-facing plate (h_R), $p \neq 2$. The ℓ^3 -norm is a better fit than ℓ^2 -norm, but the ℓ^4 -norm, which is midway between the two “ ℓ^4 Mixed Exp. Uncertainty” curves, fits best.

$$h_* = k \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_*}{L_*} \right\|_2 \quad (10)$$

$$h_V = k \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_V}{L_V} \right\|_2 \quad (11)$$

$$h_R = k \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_R}{R} \right\|_4 \quad (12)$$

The measurements leading to equations (10), (11), and (12) were performed with horizontal forced flow, perpendicular to the natural convective flows from the plate. The horizontal-vertical mixing of equation (11) might be due to vector addition of the forced and natural flows or just the ℓ^2 -norm of the two magnitudes. In the vector case a drop in convection would be expected when a weak forced flow opposed the fluid rising due to convection from a vertical plate. The apparatus was modified to perform this experiment, producing Figure 10, which shows that neither was the case! It is ℓ^4 -norm below $\text{Re}=7500$ and ℓ^2 -norm above. Equation (13) is a blending function implementing the two norms with a sharp transition between them:

$$B_\downarrow(h_F, h_N) = \left\{ h_F^4 + 2h_F^2 h_N^2 \left[1 - \frac{16 h_N^{16}}{15 h_N^{16} + h_F^{16}} \right] + h_N^4 \right\}^{1/4} \quad (13)$$

$$h_\downarrow = B_\downarrow \left(\frac{k \text{Nu}_F}{L_F}, \frac{k \text{Nu}_V}{L_V} \right)$$

A vertical plate with forced upward convection was also surprising. In Figure 12 the convections aid each other as the ℓ^2 -norm except around $\text{Re}=8500$ where they mix as the ℓ^4 -norm. This could be explained by the reduced velocity shear creating less turbulent mixing around $\text{Re}=8500$. The aiding transition (14) is much gentler than the opposing transition (13):

$$B_\uparrow(h_F, h_N) = \left\{ h_F^4 + 2h_F^2 h_N^2 \left[1 - \frac{4 h_N^2 h_F^2}{3 h_N^4 + h_F^4} \right] + h_N^4 \right\}^{1/4} \quad (14)$$

$$h_\uparrow = B_\uparrow \left(\frac{k \text{Nu}_F}{L_F}, \frac{k \text{Nu}_V}{L_V} \right)$$

The Reynolds numbers of the transitions between the ℓ^2 -norm and ℓ^4 -norm depend only on the relative strength of the natural and forced convective components h_N and h_F . Instead of Reynolds and Rayleigh numbers, the blend functions operate directly on h_N and h_F , simplifying their form.

Correlations (10), (12), (11) are of the form recommended by Churchill and Usagi [6] and Lin, Yu, and Chen [7]. The formulas in equations (14) and (13) are ad-hoc, but dimensionally correct and match measurements within a couple percent.

7. Natural Convection from an Inclined Plate

Following the approach of Fujii and Imura [10], the Ra_V argument to the vertical correlation (3) is scaled by $|\cos \theta|$ in order to model the reduced vertical mode of convection on a tilted plate. Similarly, the Ra_* argument to the upward correlation (2) is scaled by $|\sin \theta|$. Although it has negligible effect on the correlation, the Ra_R argument to the downward correlation (4) is also scaled by $|\sin \theta|$ so that the combination of formulas (15) and (16) is continuous around $\theta = 0$.

When $V = 0$, if $\Delta T > 0$ and $\theta > 0$ or if $\Delta T < 0$ and $\theta < 0$ (downward convection), then:

$$h = k \max \left(\frac{Nu_V(Ra_V |\cos \theta|)}{L_V}, \frac{Nu_R(Ra_R |\sin \theta|)}{R} \right) \quad (15)$$

Otherwise (upward convection):

$$h = k \max \left(\frac{Nu_V(Ra_V |\cos \theta|)}{L_V}, \frac{Nu_*(Ra_* |\sin \theta|)}{L_*} \right) \quad (16)$$

G. D. Raithby and K. G. T. Hollands writing in chapter 4 of *Handbook of heat transfer* [11] recommend

$$h = k \max \left(\frac{Nu_V(Ra_V |\cos \theta|)}{L_V}, \frac{Nu_*(Ra_* |\sin \theta|)}{L_*}, \frac{Nu_R(Ra_R)}{L_R} \right)$$

where Nu_R uses an unscaled Ra_R . Currently achievable measurement uncertainties are much larger than the difference expected between these two approaches.

Given the many uncertainties of natural convection measurements and the coarse assumptions of the sides model equation developed in [1], a graph of natural convection h versus angle θ is a speculative venture. All but two points in Figure 1 are within the measurement uncertainties of formulas (15) and (16).

Three Ra values are involved in generating each of the curves labeled ‘‘Theory’’ in Figure 1. Ra_R and Ra_* are taken from the Rayleigh numbers during measurements at $\theta = +90^\circ$ and $\theta = -90^\circ$, respectively. Ra_V is derived from the geometric mean of Ra_R and Ra_* .

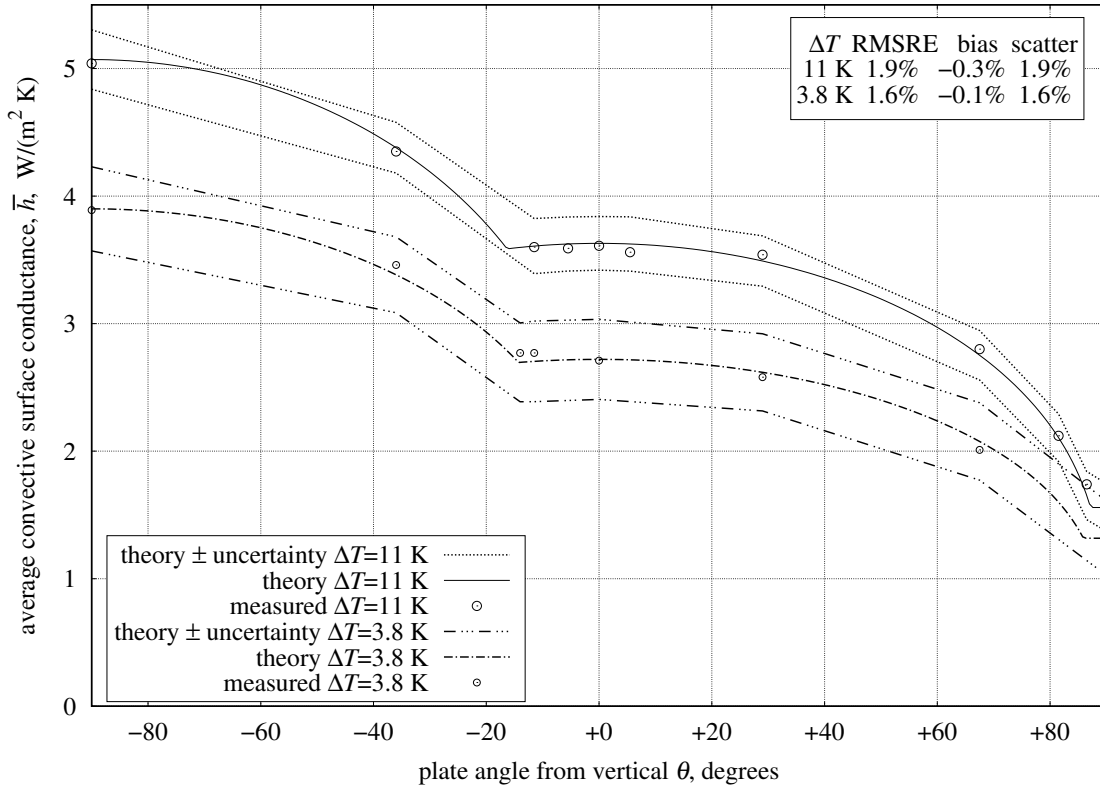


Figure 1 natural convection vs angle

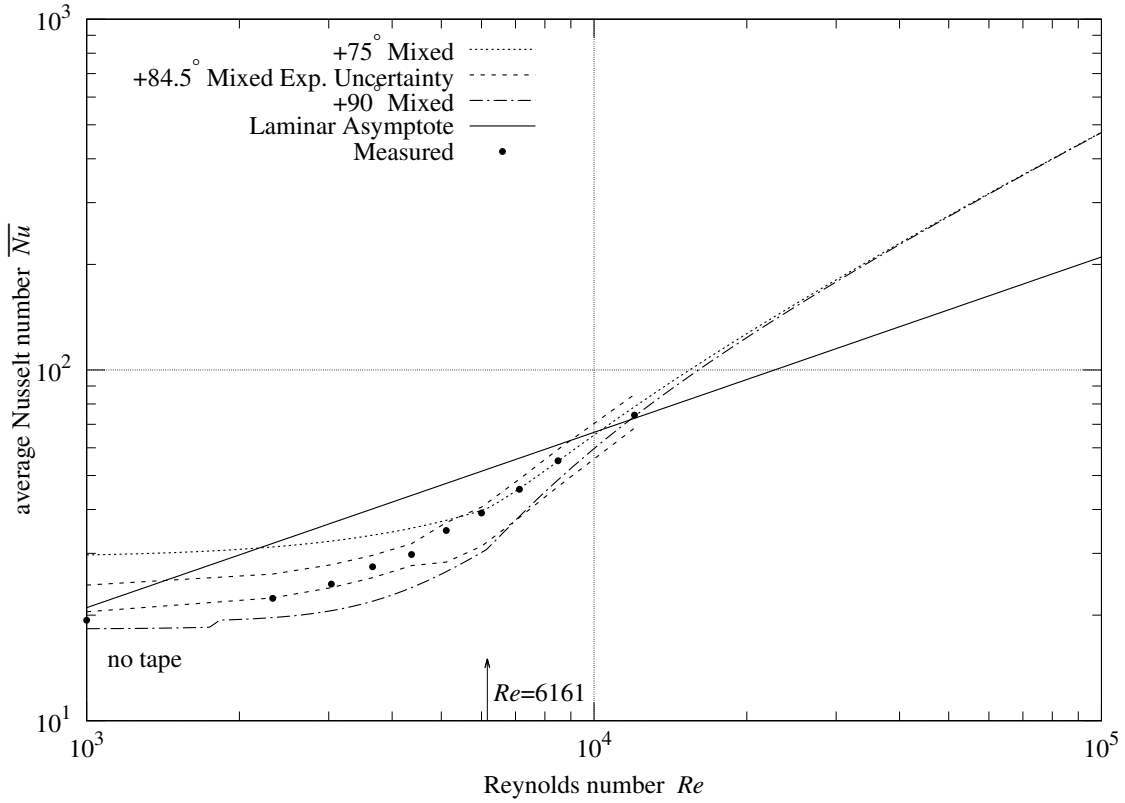


Figure 2 opposing mixed convection from inclined plate $\theta = +84.5^\circ$, 1 mm roughness, $\Delta T = 10$ K

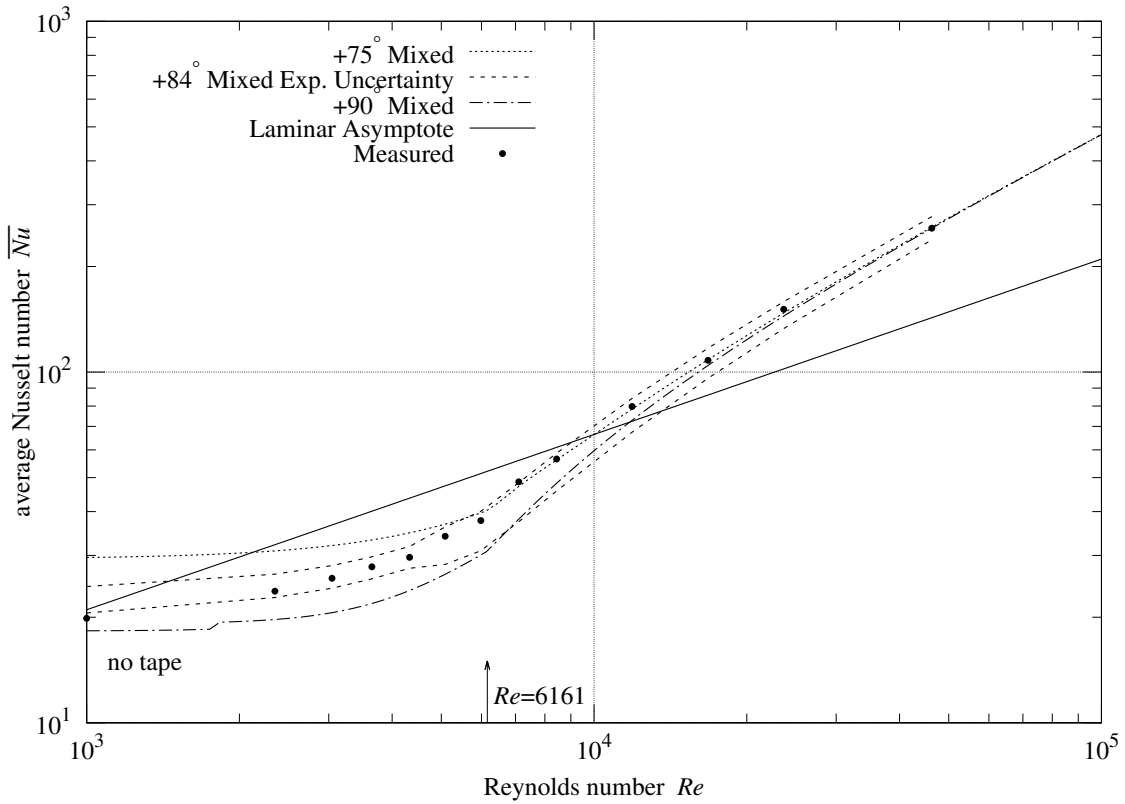


Figure 3 aiding mixed convection from inclined plate $\theta = +84.5^\circ$, 1 mm roughness, $\Delta T = 11$ K

8. Mixed Convection from an Inclined Plate

Combining the natural and mixed horizontal flow equations so that the mixed equations (10), (11), and (12) result when θ is an integer multiple of 90° , and the natural equations (15) and (16) result when $V = 0$, leads to formula (17) for mixing with horizontal flow:

$$\begin{aligned} h &= k \max \left(\left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_V (\text{Ra}_V |\cos \theta|)}{L_V} \right\|_2, \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_R (\text{Ra}_R |\sin \theta|)}{R} \right\|_4 \right) & 0^\circ \leq \theta \leq +90^\circ \\ h &= k \max \left(\left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_V (\text{Ra}_V |\cos \theta|)}{L_V} \right\|_2, \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_* (\text{Ra}_* |\sin \theta|)}{L_*} \right\|_2 \right) & -90^\circ \leq \theta \leq 0^\circ \end{aligned} \quad (17)$$

Let $0^\circ \leq \psi \leq 180^\circ$ be the angle of the forced flow² from the zenith. The blend function for the vertical convective mode (18) is the ℓ^2 -norm except for the vertical downward component of the forced flow which transitions from the ℓ^4 -norm to the ℓ^2 -norm as the forced flow increases, and the upward component of the forced flow which is the ℓ^2 -norm except where the convections are close in value, which use the ℓ^4 -norm:

$$\begin{aligned} M(h_F, h_N, \psi) &= \max(0, \cos \psi) \frac{4 h_N^2 h_F^2}{3 h_N^4 + h_F^4} - \min(0, \cos \psi) \frac{16 h_N^{16}}{15 h_N^{16} + h_F^{16}} \\ B(h_F, h_N, \psi) &= \left\{ [h_F^2 + h_N^2]^2 - 2 h_N^2 h_F^2 M(h_F, h_N, \psi) \right\}^{1/4} \end{aligned} \quad (18)$$

$$\begin{aligned} h &= k \max \left(B \left(\frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_V (\text{Ra}_V |\cos \theta|)}{L_V}, \psi \right), \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_R (\text{Ra}_R |\sin \theta|)}{R} \right\|_4 \right) & 0^\circ \leq \theta \leq +90^\circ \\ h &= k \max \left(B \left(\frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_V (\text{Ra}_V |\cos \theta|)}{L_V}, \psi \right), \left\| \frac{\text{Nu}_F}{L_F}, \frac{\text{Nu}_* (\text{Ra}_* |\sin \theta|)}{L_*} \right\|_2 \right) & -90^\circ \leq \theta \leq 0^\circ \end{aligned} \quad (19)$$

While equation (19) collapses to validated equations (10), (11), (12), (13), and (14) when θ is an integer multiple of 90° and to (15) and (16) when $V = 0$, that does not prove that (19) is valid for mixed convection from an inclined plate. Figure 2 and Figure 3 show mixed convection measured at a plate angle of $+84.5^\circ$. $+84.5^\circ$ was chosen because it is in the range of orientations where convection is most sensitive to θ . The curves for $\theta = +90^\circ$ and $\theta = +75^\circ$ are shown for comparison.

Figure 2 has forced flow opposing its natural convection, $\psi = 95.5^\circ$. The obstruction of the plume by the wind-tunnel is made worse at low flow rates, resulting in lower than expected convection.

Figure 3 has forced flow aiding its natural convection, $\psi = 84.5^\circ$.

² The forced flow is parallel to the plane of the test surface.

9. Discussion

Mixed convection from a plate encompasses a variety of interactions between convective modes. Conduction is amplified ($\ell^{1/2}$ -norm) by natural convection from vertical and upward-facing surfaces (Jaffer [5]). Territorial competition develops between turbulent and laminar flows in forced convection. Natural and forced convection cooperate as the ℓ^2 -norm or ℓ^4 -norm in mixed convection (Section 6). Mixed convection modes compete on inclined plates (Section 8).

The present work is restricted to flat rectangular surfaces with at least one horizontal edge because convection from other shapes and orientations can't be accurately measured in the Convection Machine wind-tunnel.

The restriction that the the external flow alone would be turbulent simplifies the system such that the mixed convective surface conductance depends only on the isolated forced and natural conductances, thermal conductivity k and the plate orientation (θ, ψ) . Furthermore, this work finds that the (turbulent) mixed convection is bounded by the ℓ^4 -norm and ℓ^2 -norm of the convective components:

$$(|h_F|^4 + |h_N|^4)^{1/4} \leq h_M \leq (|h_F|^2 + |h_N|^2)^{1/2} \quad (20)$$

These simplifications do not apply when the mixed convection is laminar. In *Convection Measurement Apparatus and Methodology* [1], Jaffer presents some laminar flow measurements violating inequality (20).

In the case of forced horizontal flow parallel to a vertical plate, convection mixes as the ℓ^2 -norm as shown in Figure 8; this is not surprising as the magnitude of the sum of perpendicular vectors is the ℓ^2 -norm.

Fig. 14(f) from Fujii and Imura [10] shows that upward-facing convection draws fluid from the sides of the plate into a central plume. As horizontal forced flow increases, it will move the plume leeward; there will be more convection from the windward side and less from the leeward side. While not as intuitive as the vertical plate case, the ℓ^2 -norm also applies to this combination of perpendicular flows (Figure 6).

Fig. 14(c) from Fujii and Imura [10] shows that downward-facing convection has the fluid flowing out from opposite sides just below the convecting surface. A slow horizontal forced flow will oppose half of the natural flow and aid the rest. Downward flow adjacent to a heated vertical plate is also in conflict at low flow rates. The ℓ^4 -norm applies over the Nu-versus-Re curve for downward-facing (Figure 4), and the lower part of the vertical plate curve in Figure 10 (the rest is ℓ^2 -norm). In both cases the Nu-versus-Re curves are monotonically increasing.

Velocity shear drives turbulence, but velocity shear for a vertical plate with forced upward flow is reduced when the aiding convections are close in magnitude, resulting in a small range of ℓ^4 -norm mixing (with the rest ℓ^2 -norm) in Figure 12.

10. Conclusion

When mixed convection is turbulent, the formulas presented here provide a straightforward framework for predicting convection in terms of the natural and forced convective surface conductances.

The natural and forced convective modes cooperate as the ℓ^2 -norm or ℓ^4 -norm depending on orientation and their relative strengths. In the absence of laminar forced flow, mixed convection is never less than the least of the natural and the forced components and is bounded by their ℓ^2 -norm and ℓ^4 -norm:

$$(|h_F|^4 + |h_N|^4)^{1/4} \leq h_M \leq (|h_F|^2 + |h_N|^2)^{1/2}$$

Horizontal and vertical convective regimes compete when θ is not an integer multiple of 90° . The resulting convection is the maximum (which is the L^∞ -norm) of each natural-forced mixture. The winner is not always the regime with the greatest natural convection because the downward regime mixes as ℓ^4 -norm while others mix as ℓ^2 -norm.

Acknowledgments

Thanks to Martin Jaffer and Roberta Jaffer for their assistance and problem-solving suggestions.

11. Nomenclature

Pr	$0 < \text{Pr}$	Prandtl number
Re	$0 \leq \text{Re}$	Reynolds number of external flow
Ra	$0 \leq \text{Ra}$	Rayleigh number
θ	$-90^\circ \leq \theta \leq 90^\circ$	Surface angle from vertical (-90° is face up)
ψ	$0^\circ \leq \psi \leq 180^\circ$	Angle of in-plane fluid flow from vertical
L_F	$0 < L_F$	Length of plate in direction of flow (m)
L_V	$0 < L_V$	Width of plate (m)
k	$0 < k$	Fluid thermal conductivity (W/(m · K))
h_M	$0 < h_M$	Mixed convective surface conductance (W/(m ² · K))
U_M	$0 < U_M$	Mixed convective conductance (W/K)

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13. Appendix: Measurements of Mixed Convection in Horizontal and Vertical Orientations

Convection Measurement Apparatus and Methodology [1] and graphs of the 102-minute time-series producing each measurement below are available at:

<http://people.csail.mit.edu/jaffer/convect>

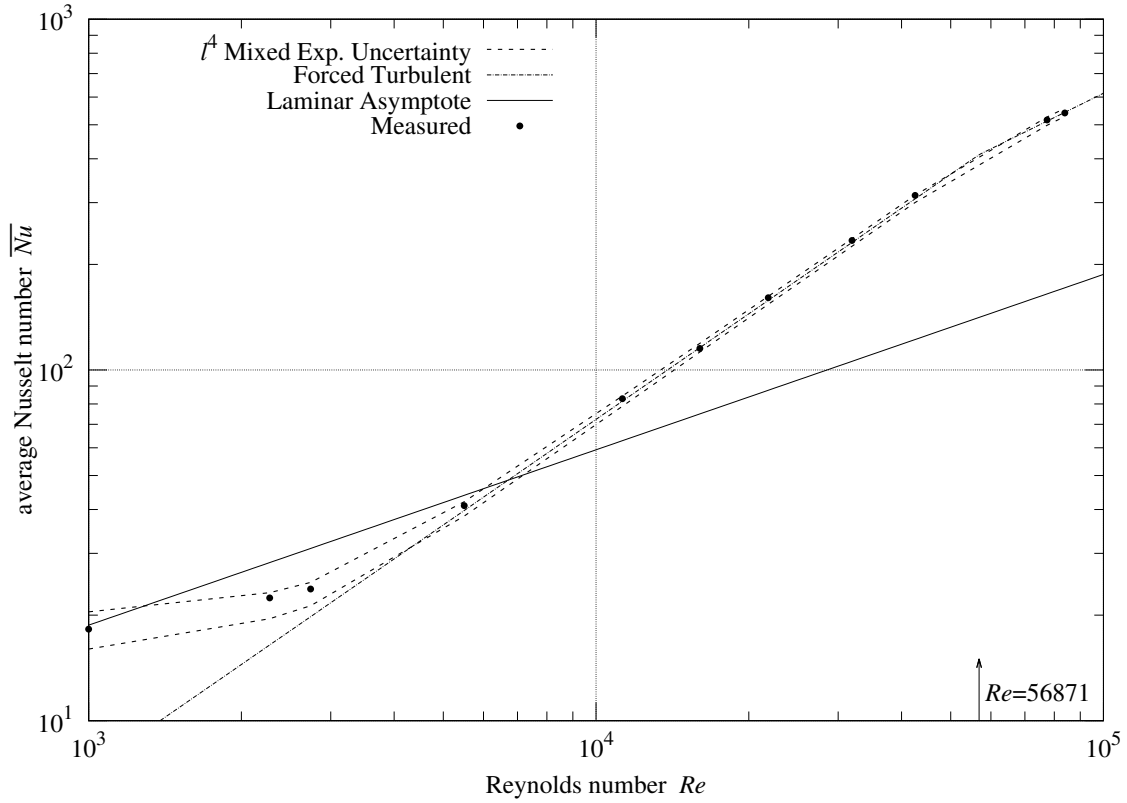


Figure 4 mixed convection from downward facing plate, 3 mm roughness, $\Delta T = 10$ K

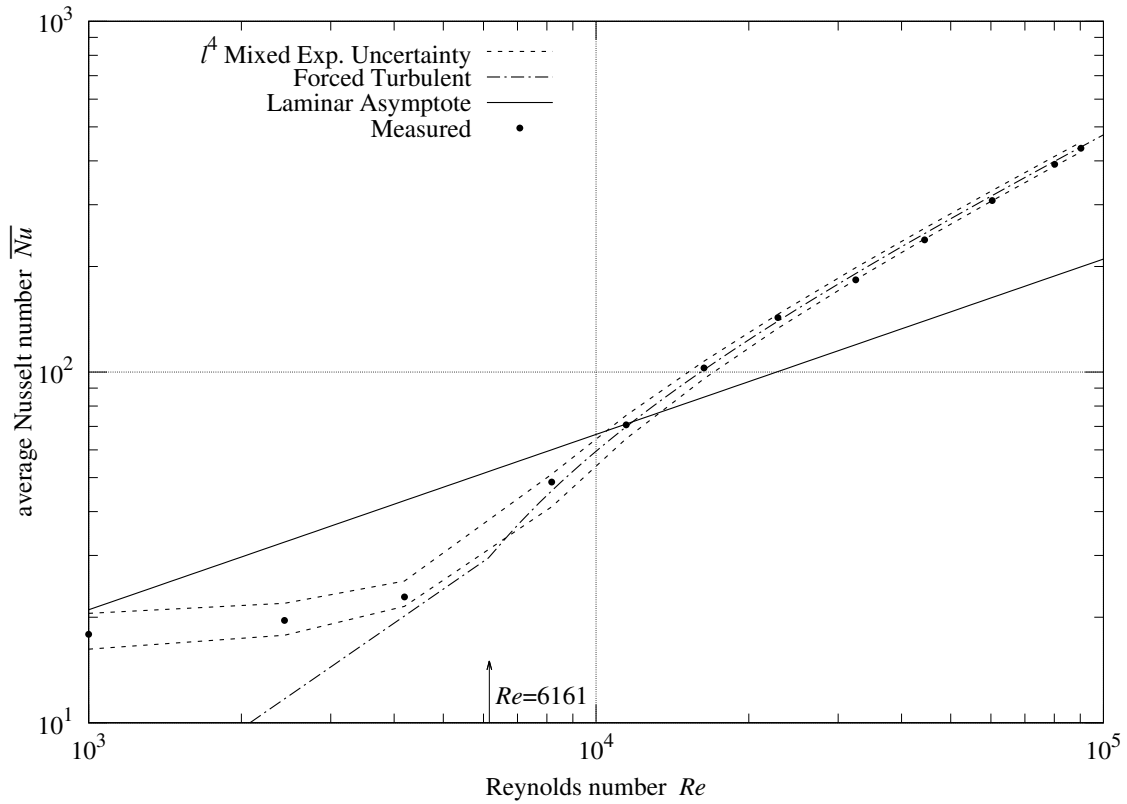


Figure 5 mixed convection from downward facing plate, 1 mm roughness, $\Delta T = 11$ K

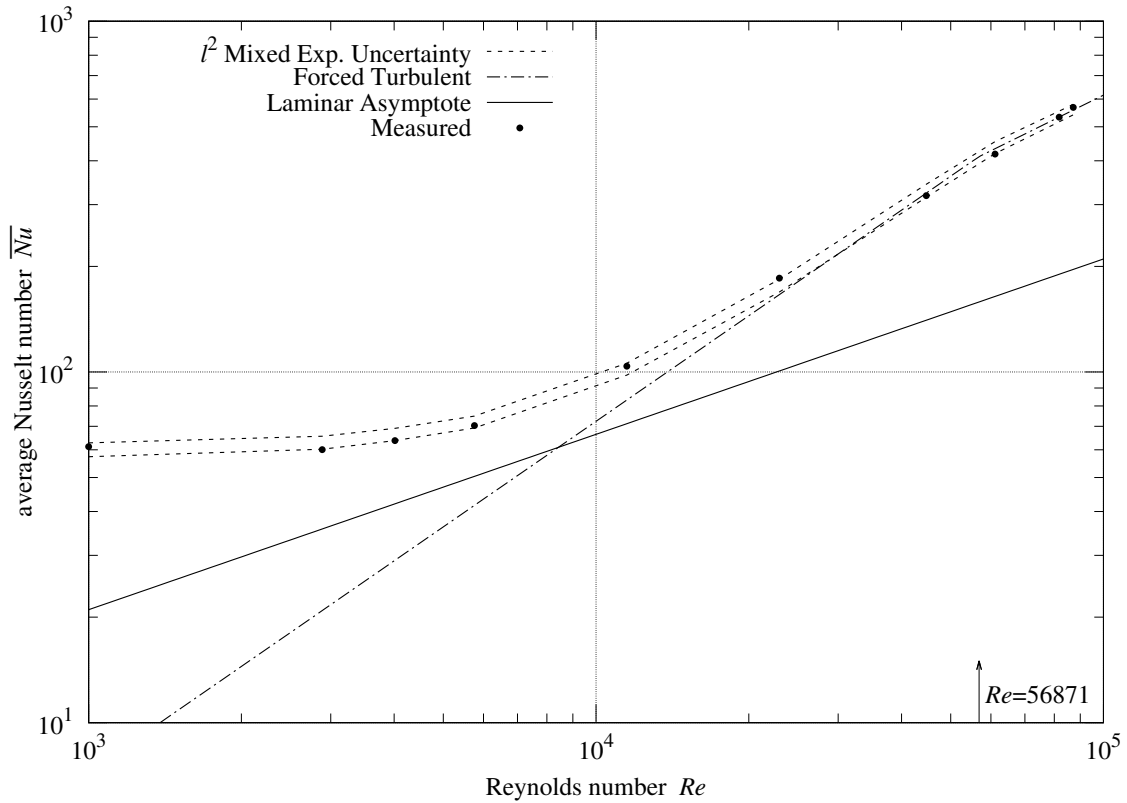


Figure 6 mixed convection from upward facing plate, 3 mm roughness, $\Delta T = 10$ K

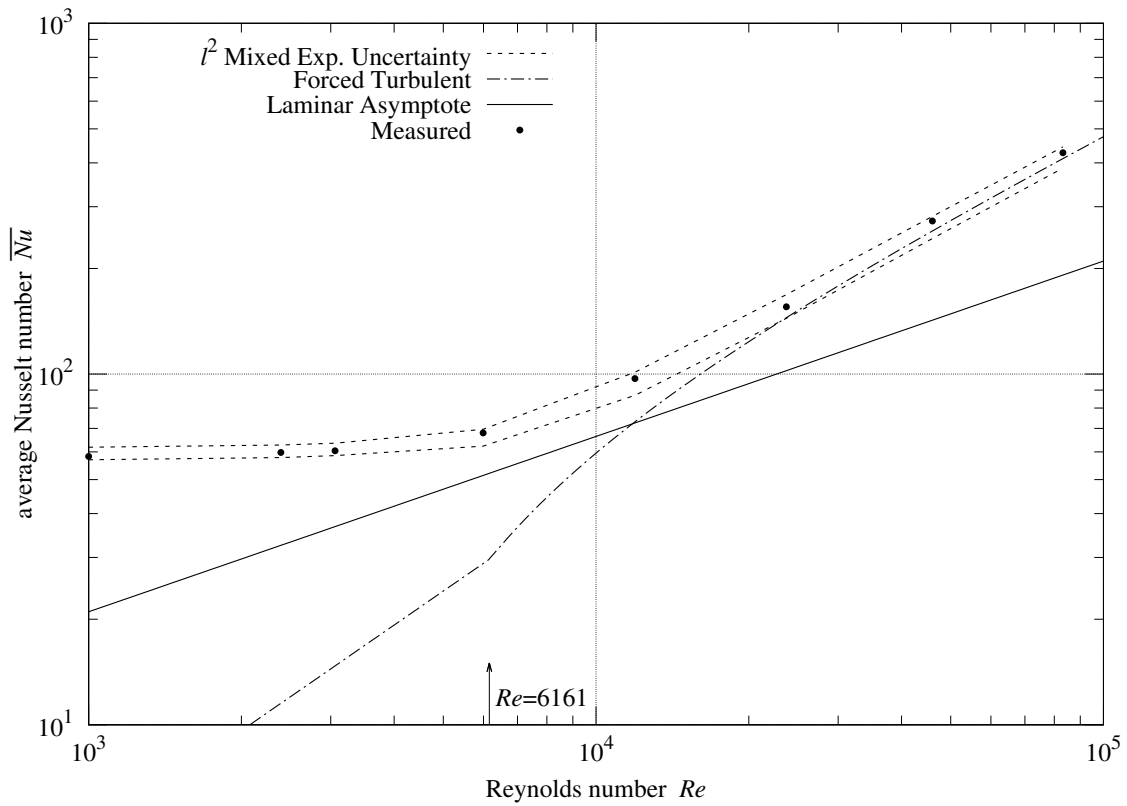


Figure 7 mixed convection from upward facing plate, 1 mm roughness, $\Delta T = 10.3$ K

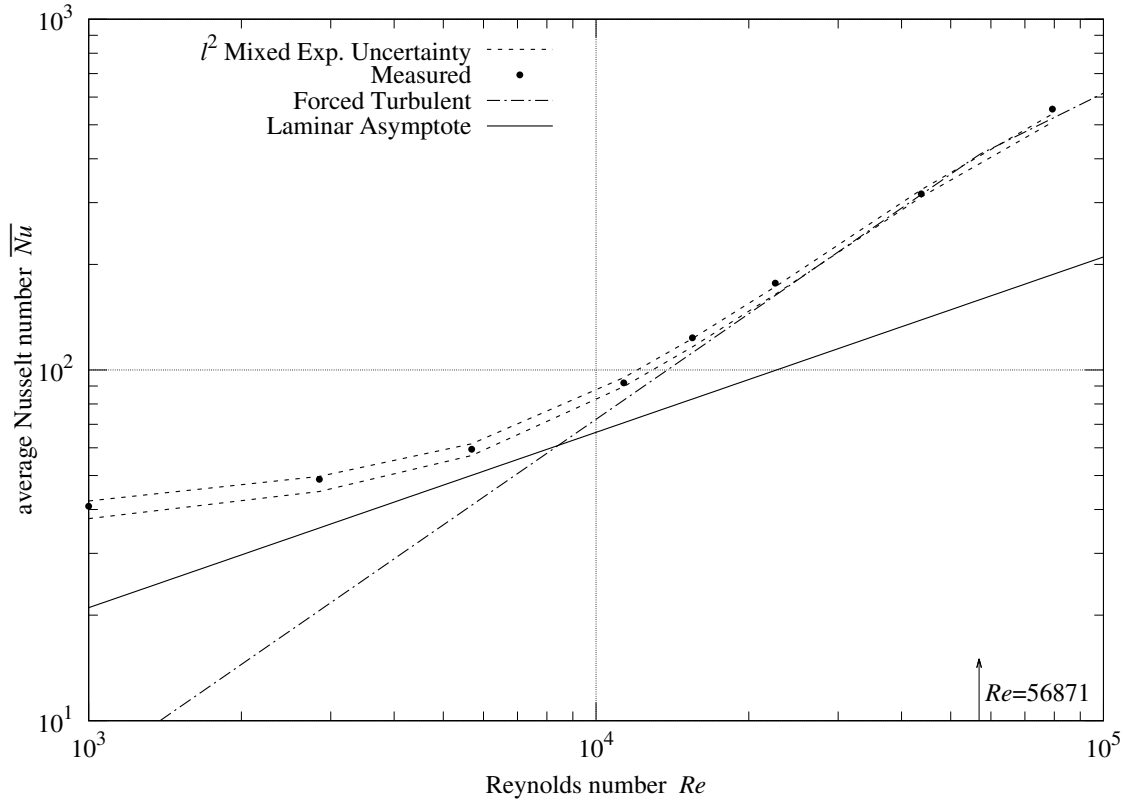


Figure 8 mixed convection from vertical plate, 3 mm roughness, $\Delta T = 10$ K

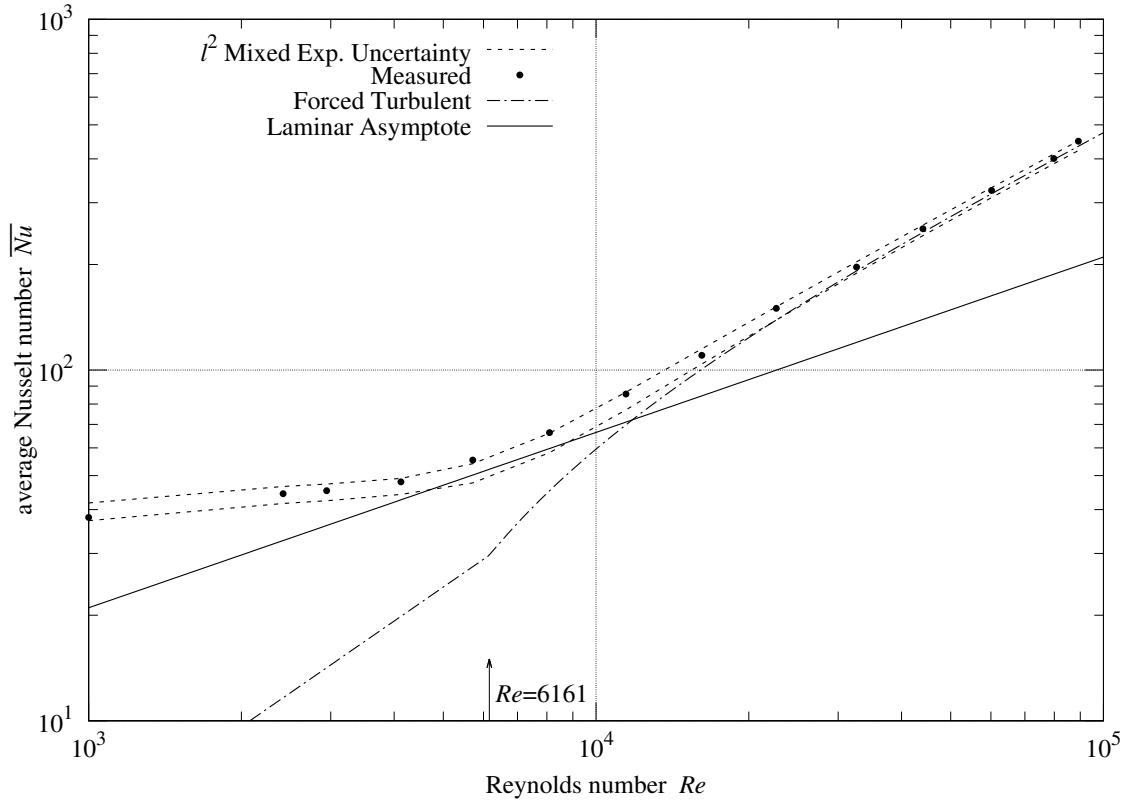


Figure 9 mixed convection from vertical plate, 1 mm roughness, $\Delta T = 11$ K

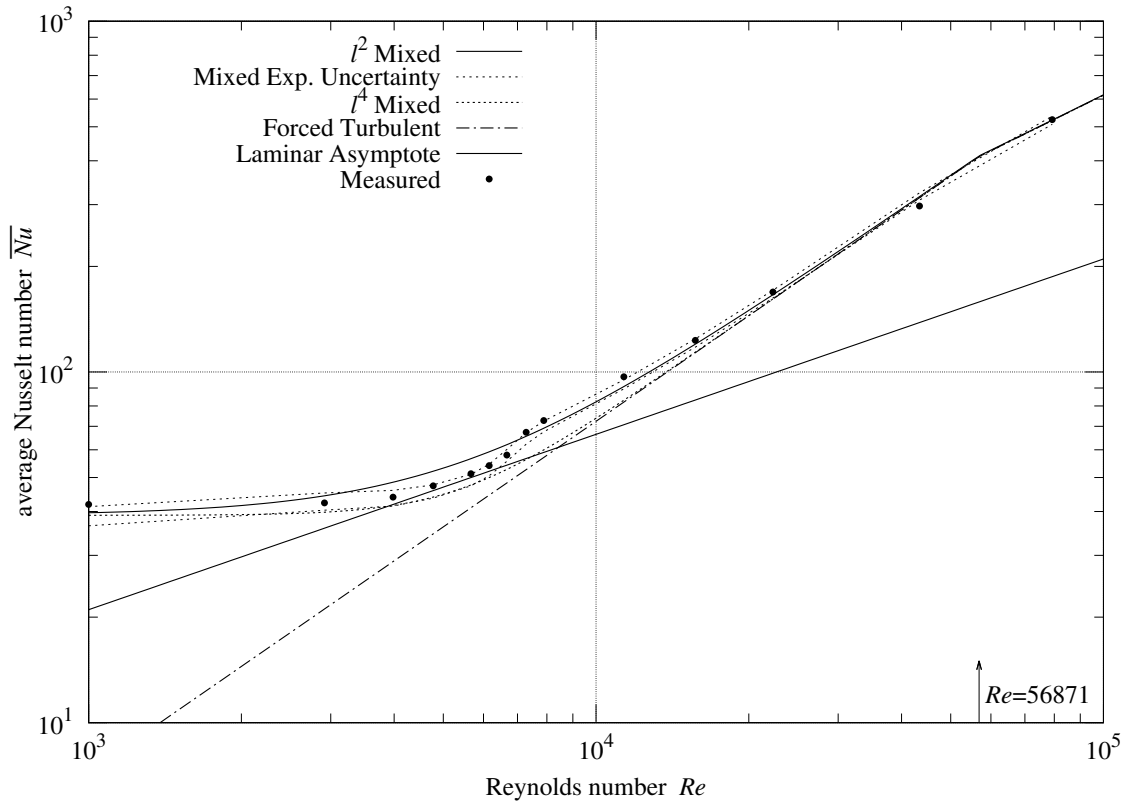


Figure 10 opposing mixed convection vertical plate, 3 mm roughness, $\Delta T = 10$ K

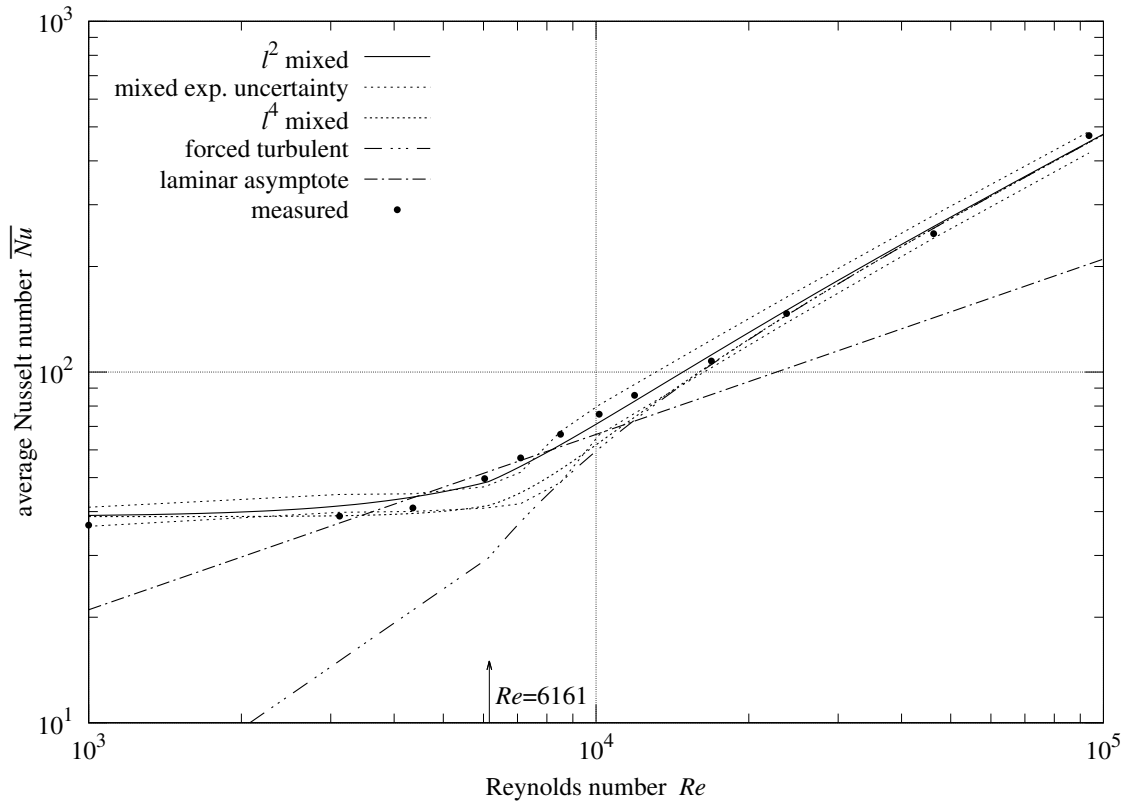


Figure 11 opposing mixed convection vertical plate, 1 mm roughness, $\Delta T = 10.5$ K

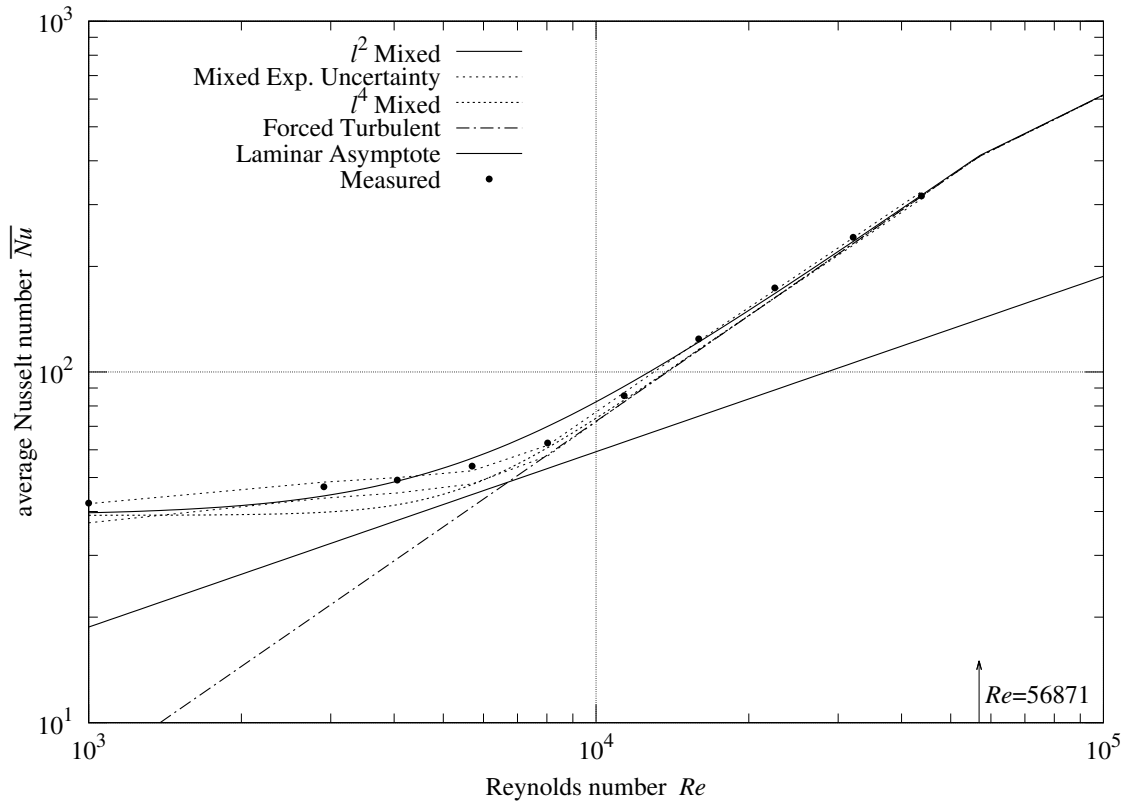


Figure 12 aiding mixed convection vertical plate, 3 mm roughness, $\Delta T = 10$ K

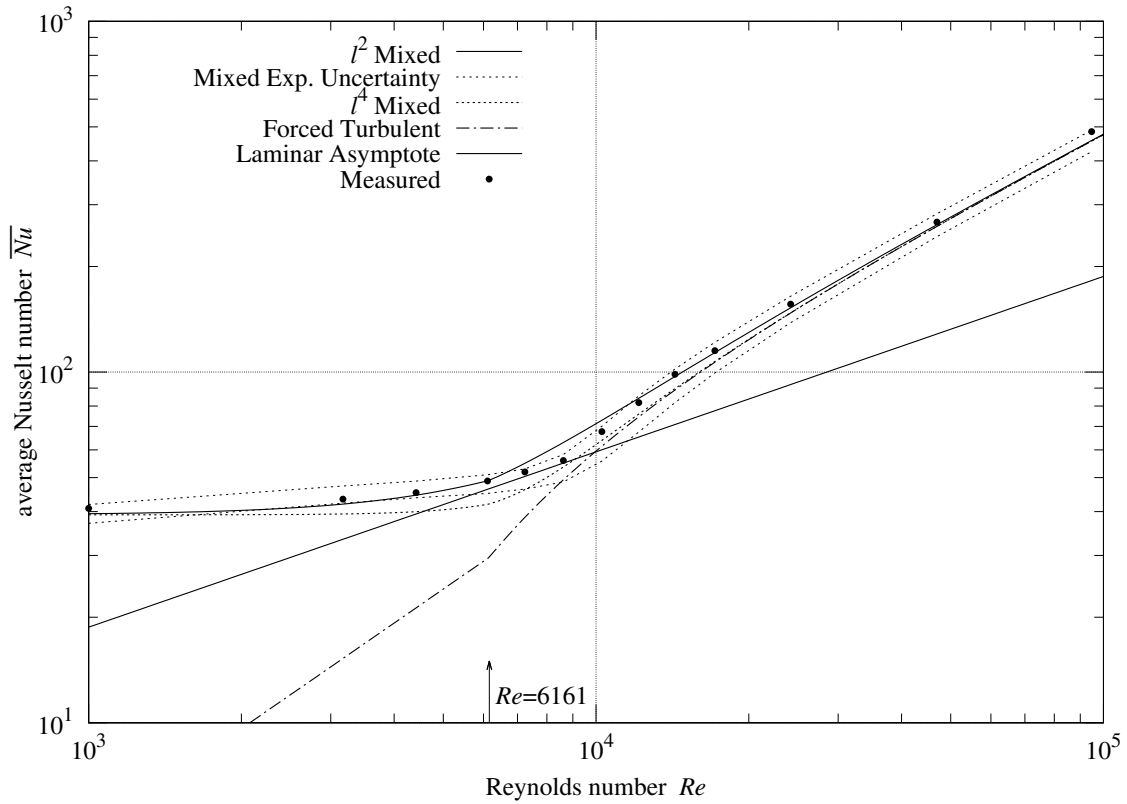


Figure 13 aiding mixed convection vertical plate, 1 mm roughness, $\Delta T = 10.5$ K