

In *(quasiquotation)s*, a *(list template D)* can sometimes be confused with either an *(unquotation D)* or a *(splicing unquotation D)*. The interpretation as an *(unquotation)* or *(splicing unquotation D)* takes precedence.

7.1.5. Programs and definitions

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⟨program⟩ → ⟨command or definition⟩*
⟨command or definition⟩ → ⟨command⟩ | ⟨definition⟩
⟨definition⟩ → (define ⟨variable⟩ ⟨expression⟩)
  | (define ⟨variable⟩ ⟨def forms⟩) ⟨body⟩
  | (begin ⟨definition⟩*)
⟨def forms⟩ → ⟨variable⟩*
  | ⟨variable⟩+ . ⟨variable⟩

```

7.2. Formal semantics

This section provides a formal denotational semantics for the primitive expressions of Scheme and selected built-in procedures. The concepts and notation used here are described in [93]; the notation is summarized below:

⟨...⟩	sequence formation
$s \downarrow k$	kth member of the sequence s (1-based)
# s	length of sequence s
$s \S t$	concatenation of sequences s and t
$s \dagger k$	drop the first k members of sequence s
$t \rightarrow a, b$	McCarthy conditional “if t then a else b ”
$\rho[x/i]$	substitution “ ρ with x for i ”
x in D	injection of x into domain D
$x D$	projection of x to domain D

The reason that expression continuations take sequences of values instead of single values is to simplify the formal treatment of procedure calls and to make it easy to add multiple return values.

The boolean flag associated with pairs, vectors, and strings will be true for mutable objects and false for immutable objects.

The order of evaluation within a call is unspecified. We mimic that here by applying arbitrary permutations *permute* and *unpermute*, which must be inverses, to the arguments in a call before and after they are evaluated. This is not quite right since it suggests, incorrectly, that the order of evaluation is constant throughout a program (for any given number of arguments), but it is a closer approximation to the intended semantics than a left-to-right evaluation would be.

The storage allocator *new* is implementation-dependent, but it must obey the following axiom: if $new \sigma \in L$, then $\sigma (new \sigma | L) \downarrow 2 = \text{false}$.

The definition of \mathcal{K} is omitted because an accurate definition of \mathcal{K} would complicate the semantics without being very interesting.

If P is a program in which all variables are defined before being referenced or assigned, then the meaning of P is

$$\mathcal{E}[\llbracket (\lambda I^* P') \langle \text{undefined} \rangle \dots \rrbracket]$$

where I^* is the sequence of variables defined in P , P' is the sequence of expressions obtained by replacing every definition in P by an assignment, $\langle \text{undefined} \rangle$ is an expression that evaluates to *undefined*, and \mathcal{E} is the semantic function that assigns meaning to expressions.

7.2.1. Abstract syntax

$K \in \text{Con}$	constants, including quotations
$I \in \text{Ide}$	identifiers (variables)
$E \in \text{Exp}$	expressions
$\Gamma \in \text{Com} = \text{Exp}$	commands

$$\begin{aligned}
\text{Exp} \rightarrow & K \mid I \mid (E_0 \ E^*) \\
& \mid (\lambda I^* \ \Gamma^* \ E_0) \\
& \mid (\lambda I^* \ . \ I) \ \Gamma^* \ E_0 \\
& \mid (\lambda I \ \Gamma^* \ E_0) \\
& \mid (\text{if } E_0 \ E_1 \ E_2) \mid (\text{if } E_0 \ E_1) \\
& \mid (\text{set! } I \ E)
\end{aligned}$$

7.2.2. Domain equations

$\alpha \in L$	locations
$\nu \in N$	natural numbers
$T = \{\text{false}, \text{true}\}$	booleans
Q	symbols
H	characters
R	numbers
$E_p = L \times L \times T$	pairs
$E_v = L^* \times T$	vectors
$E_s = L^* \times T$	strings
$M = \{\text{false}, \text{true}, \text{null}, \text{undefined}, \text{unspecified}\}$	miscellaneous
$\phi \in F = L \times (E^* \rightarrow K \rightarrow C)$	procedure values
$\epsilon \in E = Q + H + R + E_p + E_v + E_s + M + F$	expressed values
$\sigma \in S = L \rightarrow (E \times T)$	stores
$\rho \in U = \text{Ide} \rightarrow L$	environments
$\theta \in C = S \rightarrow A$	command continuations
$\kappa \in K = E^* \rightarrow C$	expression continuations
A	answers
X	errors

7.2.3. Semantic functions

$$\begin{aligned}
\mathcal{K} : \text{Con} &\rightarrow \text{E} \\
\mathcal{E} : \text{Exp} &\rightarrow \text{U} \rightarrow \text{K} \rightarrow \text{C} \\
\mathcal{E}^* : \text{Exp}^* &\rightarrow \text{U} \rightarrow \text{K} \rightarrow \text{C} \\
\mathcal{C} : \text{Com}^* &\rightarrow \text{U} \rightarrow \text{C} \rightarrow \text{C}
\end{aligned}$$

Definition of \mathcal{K} deliberately omitted.

$$\mathcal{E}[\llbracket K \rrbracket] = \lambda \rho \kappa . \text{send}(\mathcal{K}[\llbracket K \rrbracket]) \kappa$$

$$\begin{aligned}
E[\![I]\!] &= \lambda\rho\kappa . hold\,(lookup\,\rho\,I) \\
&\quad (single(\lambda\epsilon . \epsilon = undefined \rightarrow \\
&\quad \quad wrong\, "undefined\, variable", \\
&\quad \quad send\,\epsilon\,\kappa)) \\
E[\![E_0\ E^*]\!] &= \\
&\lambda\rho\kappa . E^*(permute(\langle E_0 \rangle \S E^*)) \\
&\rho \\
&(\lambda\epsilon^*. ((\lambda\epsilon^*. applicate(\epsilon^* \downarrow 1)(\epsilon^* \uparrow 1)\,\kappa) \\
&\quad (unpermute\,\epsilon^*))) \\
E[\![\text{lambda } (I^*)\ \Gamma^*\ E_0]\!] &= \\
&\lambda\rho\kappa . \lambda\sigma . \\
&\quad new\,\sigma \in L \rightarrow \\
&\quad send\,(\langle new\,\sigma | L, \\
&\quad \quad \lambda\epsilon^*\kappa' . \#\epsilon^* = \#I^* \rightarrow \\
&\quad \quad tievals(\lambda\alpha^*. (\lambda\rho' . C[\![\Gamma^*]\!]\rho' (E[\![E_0]\!]\rho'\kappa')) \\
&\quad \quad \quad (extends\,\rho\,I^*\alpha^*)) \\
&\quad \quad \epsilon^*, \\
&\quad \quad wrong\, "wrong\, number\, of\, arguments") \\
&\quad in\,E) \\
&\quad \kappa \\
&\quad (update\,(new\,\sigma | L)\,unspecified\,\sigma), \\
&\quad wrong\, "out\, of\, memory"\,\sigma \\
E[\![\text{lambda } (I^* . I)\ \Gamma^*\ E_0]\!] &= \\
&\lambda\rho\kappa . \lambda\sigma . \\
&\quad new\,\sigma \in L \rightarrow \\
&\quad send\,(\langle new\,\sigma | L, \\
&\quad \quad \lambda\epsilon^*\kappa' . \#\epsilon^* \geq \#I^* \rightarrow \\
&\quad \quad tievalsrest \\
&\quad \quad \quad (\lambda\alpha^*. (\lambda\rho' . C[\![\Gamma^*]\!]\rho' (E[\![E_0]\!]\rho'\kappa')) \\
&\quad \quad \quad (extends\,\rho\,(I^* \S \langle I \rangle)\,\alpha^*)) \\
&\quad \quad \epsilon^*, \\
&\quad \quad (\#I^*), \\
&\quad \quad wrong\, "too\, few\, arguments") \in E) \\
&\quad \kappa \\
&\quad (update\,(new\,\sigma | L)\,unspecified\,\sigma), \\
&\quad wrong\, "out\, of\, memory"\,\sigma \\
E[\![\text{lambda } I\ \Gamma^*\ E_0]\!] &= E[\![\text{lambda } (. I)\ \Gamma^*\ E_0]\!] \\
E[\![\text{if } E_0\ E_1\ E_2]\!] &= \\
&\lambda\rho\kappa . E[\![E_0]\!]\rho\,(single(\lambda\epsilon . truish\,\epsilon \rightarrow E[\![E_1]\!]\rho\kappa, \\
&\quad E[\![E_2]\!]\rho\kappa)) \\
E[\![\text{if } E_0\ E_1]\!] &= \\
&\lambda\rho\kappa . E[\![E_0]\!]\rho\,(single(\lambda\epsilon . truish\,\epsilon \rightarrow E[\![E_1]\!]\rho\kappa, \\
&\quad send\,unspecified\,\kappa))
\end{aligned}$$

Here and elsewhere, any expressed value other than *undefined* may be used in place of *unspecified*.

$$\begin{aligned}
E[\![\text{set! } I\ E]\!] &= \\
&\lambda\rho\kappa . E[\![E]\!]\rho\,(single(\lambda\epsilon . assign\,(lookup\,\rho\,I) \\
&\quad \epsilon \\
&\quad (send\,unspecified\,\kappa)))
\end{aligned}$$

$$\begin{aligned}
E^*[\] &= \lambda\rho\kappa . \kappa\langle \rangle \\
E^*[\![E_0\ E^*]\!] &= \\
&\lambda\rho\kappa . E[\![E_0]\!]\rho\,(single(\lambda\epsilon_0 . E^*[\![E^*]\!]\rho\,(\lambda\epsilon^*. \kappa\,(\langle\epsilon_0 \rangle \S \epsilon^*)))) \\
C[\] &= \lambda\rho\theta . \theta \\
C[\![\Gamma_0\ \Gamma^*]\!] &= \lambda\rho\theta . E[\![\Gamma_0]\!]\rho\,(\lambda\epsilon^*. C[\![\Gamma^*]\!]\rho\theta)
\end{aligned}$$

7.2.4. Auxiliary functions

$$\begin{aligned}
lookup &: U \rightarrow Ide \rightarrow L \\
lookup &= \lambda\rho I . \rho I \\
extends &: U \rightarrow Ide^* \rightarrow L^* \rightarrow U \\
extends &= \lambda\rho I^*\alpha^*. \#I^* = 0 \rightarrow \rho, \\
&\quad extends\,(\rho[(\alpha^* \downarrow 1)/(I^* \downarrow 1)])\,(I^* \uparrow 1)\,(\alpha^* \uparrow 1) \\
wrong &: X \rightarrow C \quad [implementation-dependent] \\
send &: E \rightarrow K \rightarrow C \\
send &= \lambda\epsilon\kappa . \kappa\langle\epsilon\rangle \\
single &: (E \rightarrow C) \rightarrow K \\
single &= \lambda\psi\epsilon^*. \#\epsilon^* = 1 \rightarrow \psi(\epsilon^* \downarrow 1), \\
&\quad wrong\, "wrong\, number\, of\, return\, values" \\
new &: S \rightarrow (L + \{error\}) \quad [implementation-dependent] \\
hold &: L \rightarrow K \rightarrow C \\
hold &= \lambda\alpha\kappa\sigma . send(\sigma\alpha \downarrow 1)\kappa\sigma \\
assign &: L \rightarrow E \rightarrow C \rightarrow C \\
assign &= \lambda\alpha\epsilon\theta\sigma . \theta(update\,\alpha\epsilon\sigma) \\
update &: L \rightarrow E \rightarrow S \rightarrow S \\
update &= \lambda\alpha\epsilon\sigma . \sigma[\langle\epsilon, true\rangle/\alpha] \\
tievals &: (L^* \rightarrow C) \rightarrow E^* \rightarrow C \\
tievals &= \lambda\psi\epsilon^*\sigma . \#\epsilon^* = 0 \rightarrow \psi\langle \rangle\sigma, \\
&\quad new\,\sigma \in L \rightarrow tievals\,(\lambda\alpha^*. \psi((new\,\sigma | L) \S \alpha^*)) \\
&\quad (\epsilon^* \uparrow 1) \\
&\quad (update\,(new\,\sigma | L)(\epsilon^* \downarrow 1)\sigma), \\
&\quad wrong\, "out\, of\, memory"\,\sigma \\
tievalsrest &: (L^* \rightarrow C) \rightarrow E^* \rightarrow N \rightarrow C \\
tievalsrest &= \lambda\psi\epsilon^*\nu . list\,(dropfirst\,\epsilon^*\nu) \\
&\quad (single(\lambda\epsilon . tievals\,\psi\,((takefirst\,\epsilon^*\nu) \S \langle\epsilon\rangle))) \\
dropfirst &= \lambda ln . n = 0 \rightarrow l, dropfirst\,(l \uparrow 1)(n - 1) \\
takefirst &= \lambda ln . n = 0 \rightarrow \langle \rangle, \langle l \downarrow 1 \rangle \S (takefirst\,(l \uparrow 1)(n - 1)) \\
truish &: E \rightarrow T \\
truish &= \lambda\epsilon . \epsilon = false \rightarrow false, true \\
permute &: Exp^* \rightarrow Exp^* \quad [implementation-dependent] \\
unpermute &: E^* \rightarrow E^* \quad [inverse\, of\, permute] \\
applicate &: E \rightarrow E^* \rightarrow K \rightarrow C \\
applicate &= \lambda\epsilon\epsilon^*\kappa . \epsilon \in F \rightarrow (\epsilon | F \downarrow 2)\epsilon^*\kappa, wrong\, "bad\, procedure" \\
onearg &: (E \rightarrow K \rightarrow C) \rightarrow (E^* \rightarrow K \rightarrow C) \\
onearg &= \lambda\zeta\epsilon^*\kappa . \#\epsilon^* = 1 \rightarrow \zeta(\epsilon^* \downarrow 1)\kappa, \\
&\quad wrong\, "wrong\, number\, of\, arguments" \\
twoarg &: (E \rightarrow E \rightarrow K \rightarrow C) \rightarrow (E^* \rightarrow K \rightarrow C) \\
twoarg &= \lambda\zeta\epsilon^*\kappa . \#\epsilon^* = 2 \rightarrow \zeta(\epsilon^* \downarrow 1)(\epsilon^* \downarrow 2)\kappa, \\
&\quad wrong\, "wrong\, number\, of\, arguments"
\end{aligned}$$

list : $E^* \rightarrow K \rightarrow C$

list =

$\lambda \epsilon^* \kappa . \# \epsilon^* = 0 \rightarrow send\ null\ \kappa,$

$list(\epsilon^* \uparrow 1)(single(\lambda \epsilon . cons(\epsilon^* \downarrow 1, \epsilon) \kappa))$

cons : $E^* \rightarrow K \rightarrow C$

cons =

$twoarg(\lambda \epsilon_1 \epsilon_2 \kappa \sigma . new\ \sigma \in L \rightarrow$

$(\lambda \sigma' . new\ \sigma' \in L \rightarrow$

$send(\langle new\ \sigma | L, new\ \sigma' | L, true \rangle$

$in\ E)$

κ

$(update(new\ \sigma' | L) \epsilon_2 \sigma'),$

wrong “out of memory” σ')

$(update(new\ \sigma | L) \epsilon_1 \sigma),$

wrong “out of memory” $\sigma)$

less : $E^* \rightarrow K \rightarrow C$

less =

$twoarg(\lambda \epsilon_1 \epsilon_2 \kappa . (\epsilon_1 \in R \wedge \epsilon_2 \in R) \rightarrow$

$send(\epsilon_1 | R < \epsilon_2 | R \rightarrow true, false) \kappa,$

wrong “non-numeric argument to <”)

add : $E^* \rightarrow K \rightarrow C$

add =

$twoarg(\lambda \epsilon_1 \epsilon_2 \kappa . (\epsilon_1 \in R \wedge \epsilon_2 \in R) \rightarrow$

$send((\epsilon_1 | R + \epsilon_2 | R) \text{ in } E) \kappa,$

wrong “non-numeric argument to +”)

car : $E^* \rightarrow K \rightarrow C$

car =

$onearg(\lambda \epsilon \kappa . \epsilon \in E_p \rightarrow hold(\epsilon | E_p \downarrow 1) \kappa,$

wrong “non-pair argument to car”)

cdr : $E^* \rightarrow K \rightarrow C$ [similar to *car*]

setcar : $E^* \rightarrow K \rightarrow C$

setcar =

$twoarg(\lambda \epsilon_1 \epsilon_2 \kappa . \epsilon_1 \in E_p \rightarrow$

$(\epsilon_1 | E_p \downarrow 3) \rightarrow assign(\epsilon_1 | E_p \downarrow 1)$

ϵ_2

(send unspecified κ *,*

wrong “immutable argument to set-car!”,

wrong “non-pair argument to set-car!”)

equ : $E^* \rightarrow K \rightarrow C$

equ =

$twoarg(\lambda \epsilon_1 \epsilon_2 \kappa . (\epsilon_1 \in M \wedge \epsilon_2 \in M) \rightarrow$

$send(\epsilon_1 | M = \epsilon_2 | M \rightarrow true, false) \kappa,$

$(\epsilon_1 \in Q \wedge \epsilon_2 \in Q) \rightarrow$

$send(\epsilon_1 | Q = \epsilon_2 | Q \rightarrow true, false) \kappa,$

$(\epsilon_1 \in H \wedge \epsilon_2 \in H) \rightarrow$

$send(\epsilon_1 | H = \epsilon_2 | H \rightarrow true, false) \kappa,$

$(\epsilon_1 \in R \wedge \epsilon_2 \in R) \rightarrow$

$send(\epsilon_1 | R = \epsilon_2 | R \rightarrow true, false) \kappa,$

$(\epsilon_1 \in E_p \wedge \epsilon_2 \in E_p) \rightarrow$

$send((\lambda p_1 p_2 . ((p_1 \downarrow 1) = (p_2 \downarrow 1) \wedge$

$(p_1 \downarrow 2) = (p_2 \downarrow 2)) \rightarrow true,$

$false)$

$\kappa,$

$(\epsilon_1 | E_p)$

$(\epsilon_2 | E_p))$

$(\epsilon_1 \in E_v \wedge \epsilon_2 \in E_v) \rightarrow \dots,$

$(\epsilon_1 \in E_s \wedge \epsilon_2 \in E_s) \rightarrow \dots,$

$(\epsilon_1 \in F \wedge \epsilon_2 \in F) \rightarrow$

$send((\epsilon_1 | F \downarrow 1) = (\epsilon_2 | F \downarrow 1) \rightarrow true, false)$

$\kappa,$

send false $\kappa)$

apply : $E^* \rightarrow K \rightarrow C$

apply =

$twoarg(\lambda \epsilon_1 \epsilon_2 \kappa . \epsilon_1 \in F \rightarrow valueslist(\epsilon_2)(\lambda \epsilon^* . applicate \epsilon_1 \epsilon^* \kappa),$

wrong “bad procedure argument to apply”)

valueslist : $E^* \rightarrow K \rightarrow C$

valueslist =

$onearg(\lambda \epsilon \kappa . \epsilon \in E_p \rightarrow$

$cdr(\epsilon)$

$(\lambda \epsilon^* . valueslist$

ϵ^*

$(\lambda \epsilon^* . car(\epsilon)(single(\lambda \epsilon . \kappa(\langle \epsilon \rangle \ \epsilon^*))))),$

$\epsilon = null \rightarrow \kappa \langle \rangle,$

wrong “non-list argument to values-list”)

cwcc : $E^* \rightarrow K \rightarrow C$ [call-with-current-continuation]

cwcc =

$onearg(\lambda \epsilon \kappa . \epsilon \in F \rightarrow$

$(\lambda \sigma . new\ \sigma \in L \rightarrow$

$apply \epsilon$

$\langle new\ \sigma | L, \lambda \epsilon^* \kappa' . \kappa \epsilon^* \rangle \text{ in } E)$

κ

(update (new σ | L)

unspecified

$\sigma),$

wrong “out of memory” $\sigma),$

wrong “bad procedure argument”)

7.3. Derived expression types

This section gives rewrite rules for the derived expression types. By the application of these rules, any expression can be reduced to a semantically equivalent expression in which only the primitive expression types (literal, variable, call, lambda, if, set!) occur.

```
(cond ((test) (sequence))
      (clause2) ...)
  ≡ (if (test)
        (begin (sequence))
        (cond (clause2) ...))

(cond ((test))
      (clause2) ...)
  ≡ (or (test) (cond (clause2) ...))

(cond ((test) => (recipient))
      (clause2) ...)
  ≡ (let ((test-result (test))
           (thunk2 (lambda () (recipient)))
           (thunk3 (lambda () (cond (clause2) ...))))
      (if test-result
          ((thunk2) test-result)
          (thunk3)))
```