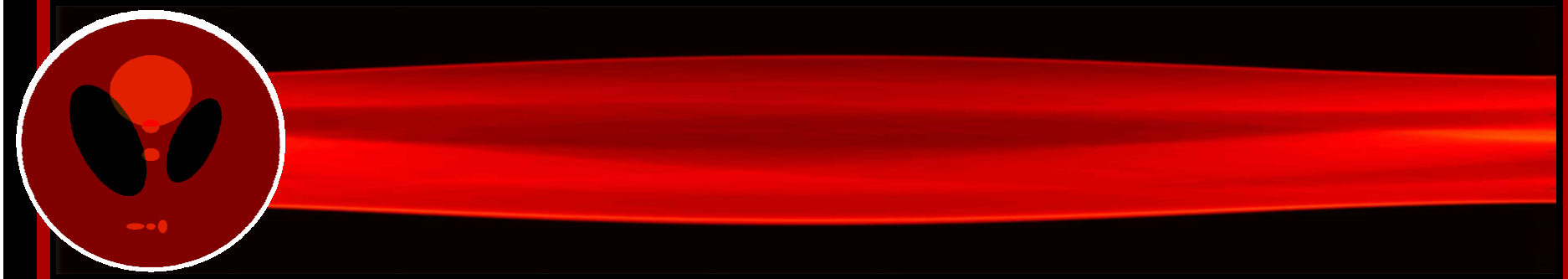
Fast Hierarchical Back Projection

ECE558 Final Project



Jason Chang
Professor Kamalabadi
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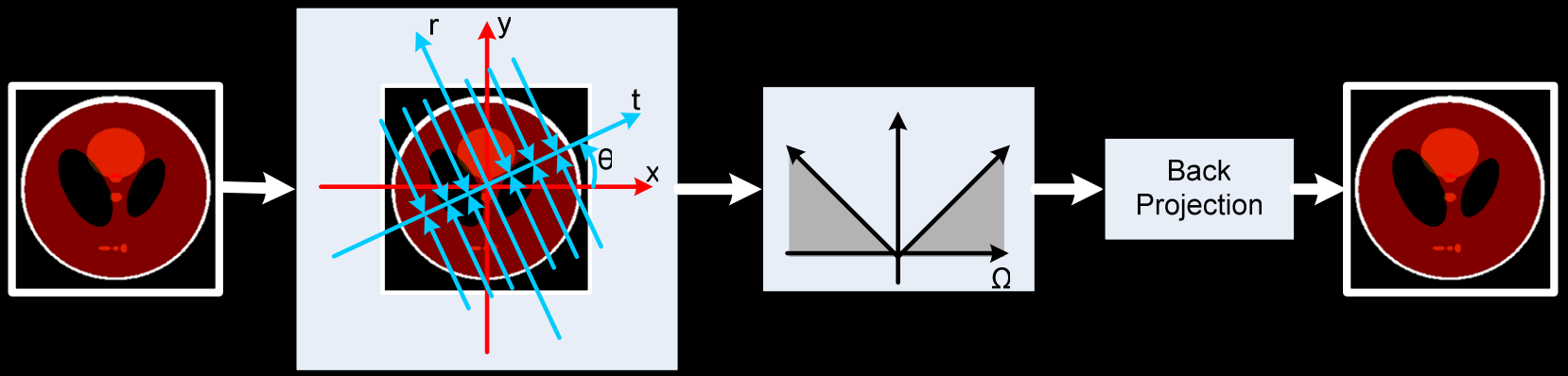
Outline

- Direct Filtered Back Projection (FBP)
- Fast Hierarchical Back Projection (FHBP)
- FHBP Results and Comparisons

Filtered Back Projection

- Applications - Medical imaging, luggage scanners, etc.
- Current Methods ~O(N^3)
- Increasing resolution demand

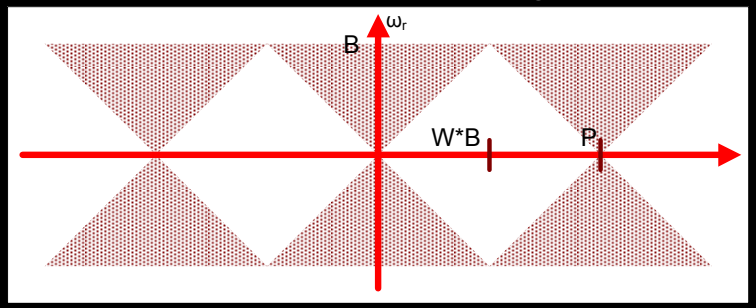


N: Size of Image θ: Projection angle t: Point in projection

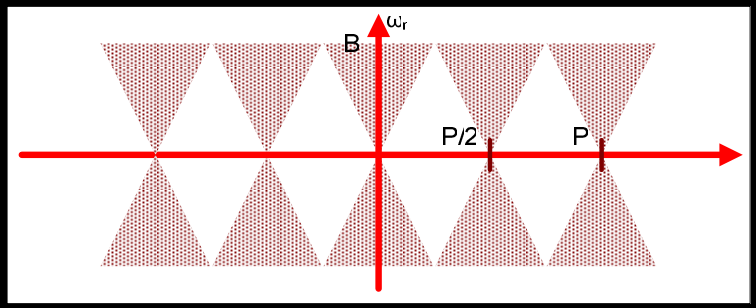
FHBP Background (1),(2)

- Intuitively, Smaller image = Less projections
- From [2], DTFT[Radon] is ≈ bow-tie

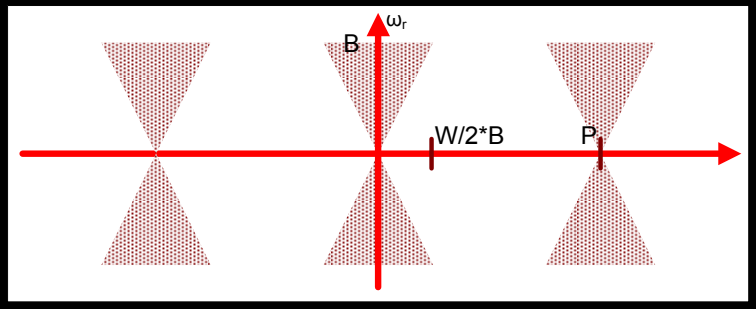
DTFT[Radon] N, P (full image)



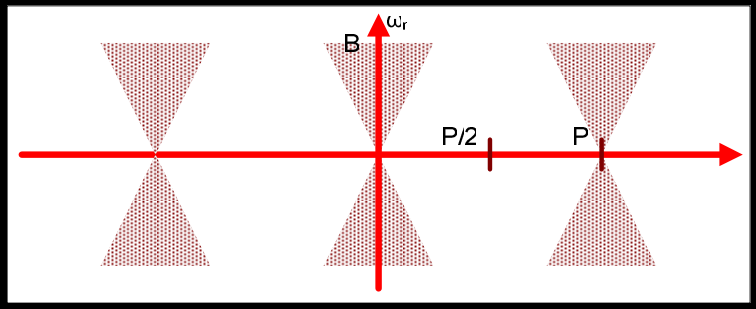
DTFT[Radon] N/2, P/2 (downsampled sub image)



DTFT[Radon] N/2, P (sub image)



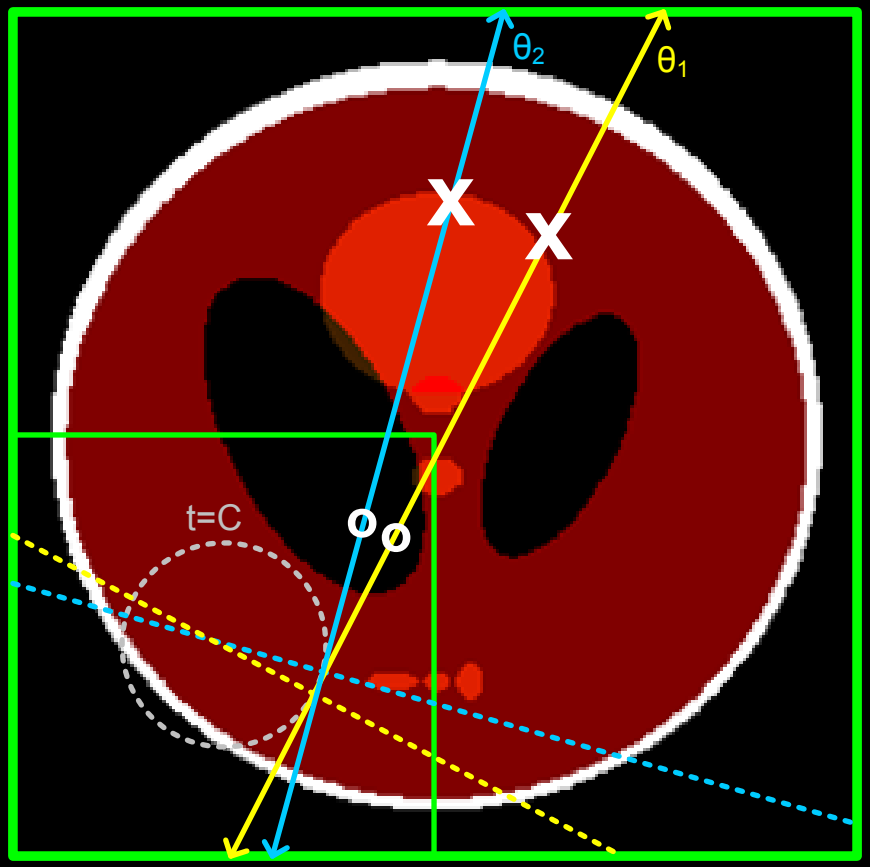
DTFT[Radon] N/2, P/2 (downsampled LPF sub image)



W: Radial Support of Image B: Bandwidth of Image P: Number of Projection Angles N: Size of Image

FHBP Background

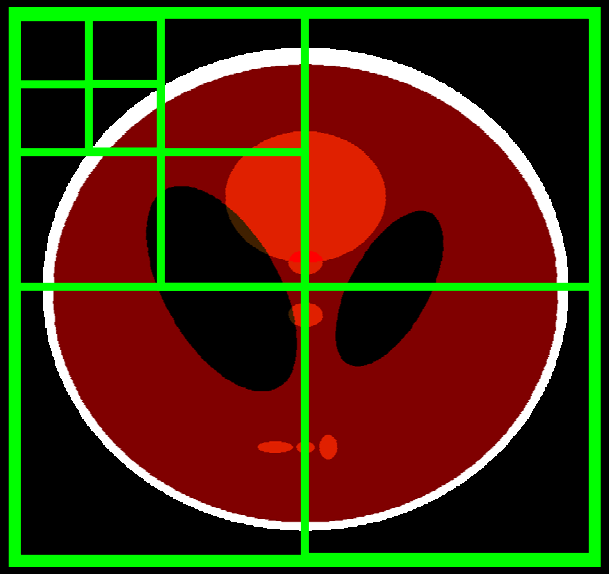
Spatial Domain Downsampling & LPF Analysis

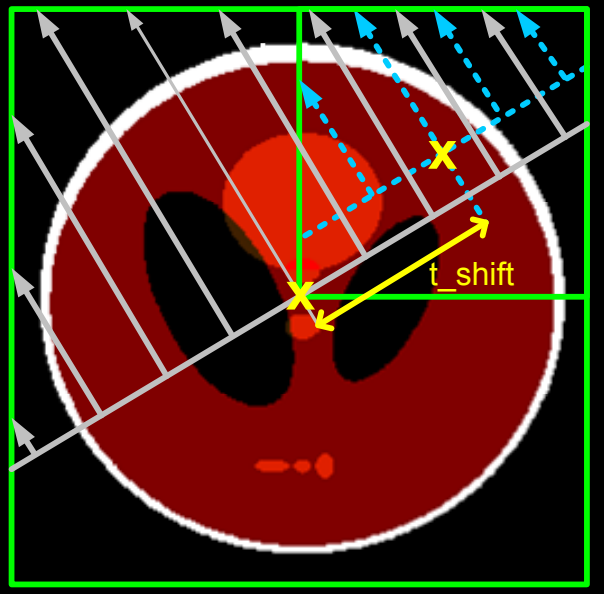


o – low frequency **x** – high frequency

FHBP Overview (1)

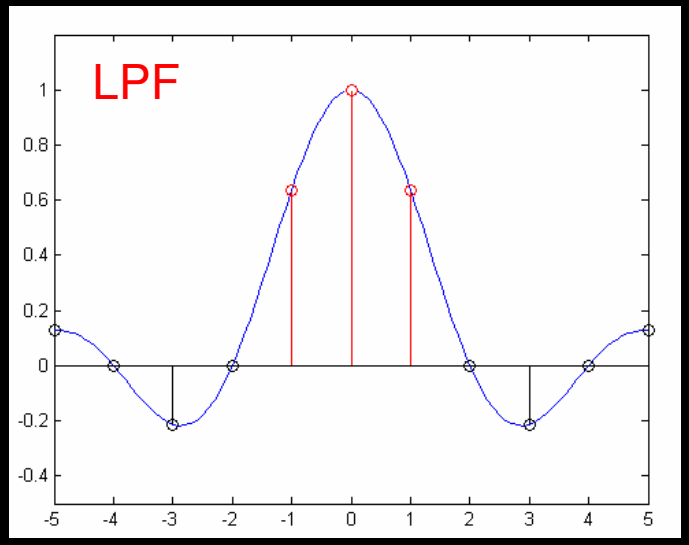
1) Divide image into 4 sub-images
2) Re-center projections to sub-image
3) ↓2 & LPF the projection angles
4) Back Project sub-image recursively

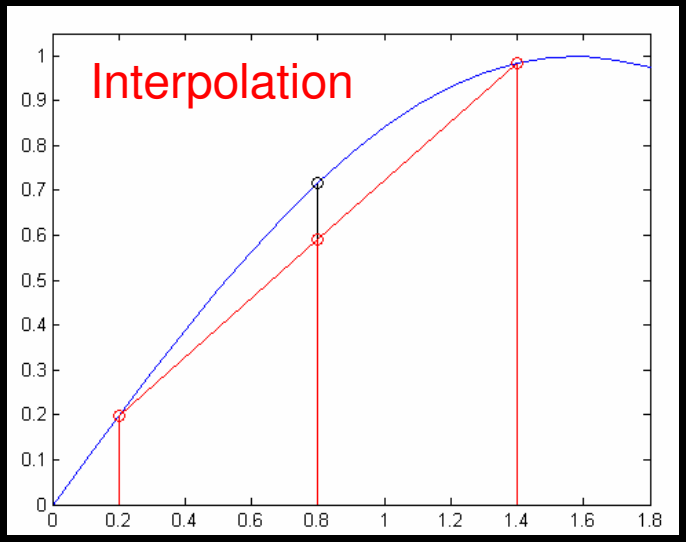




FHBP as Approximation

- DTFT[Radon] not perfect bow-tie
- LPF Filter not ideal
- Interpolation of discrete Radon





analog ideal digital ideal actual

Improving FHBP Parameters

- Begin downsampling at level Q of recursion
- Increase LPF length
- Interpolate Np (radially) by M prior to backprojecting

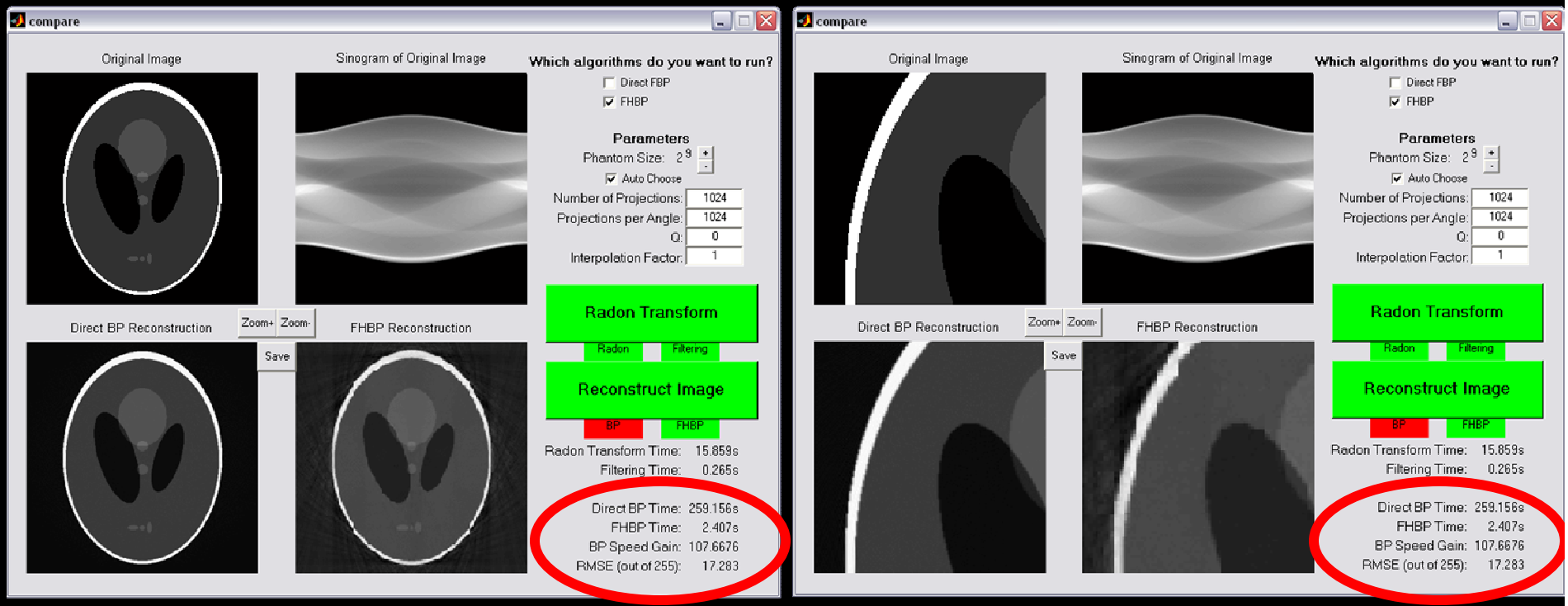
Notes [3]:

- Q changes speed exponentially
- M changes speed linearly

Q: Number of non-downsampling levels Np: Number of points per projection M: Radial interpolation factor

Comparisons & Results

N=512 P=1024 Np=1024 Q=0 M=1

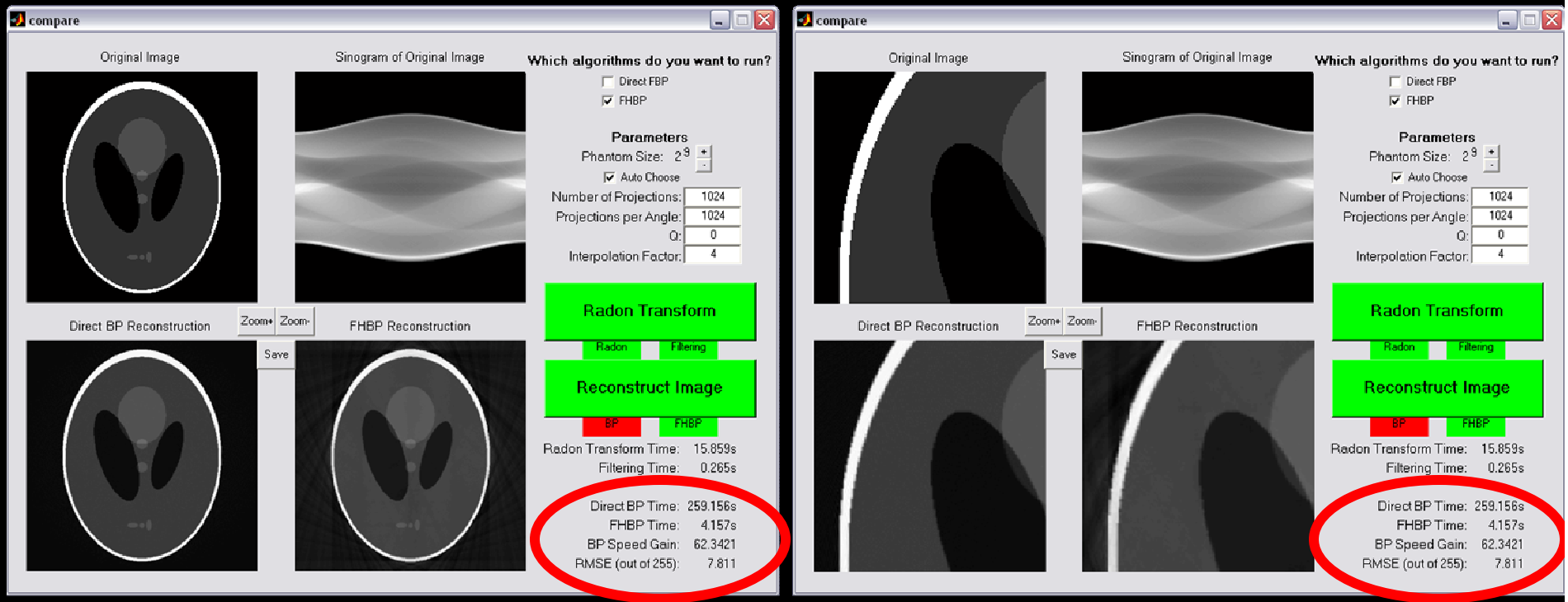


N: Size of Image P: Number of Projection Angles Np: Number of points per projection

Q: Number of non-downsampling levels M: Radial interpolation factor

Comparisons & Results

N=512 P=1024 Np=1024 Q=0 M=4

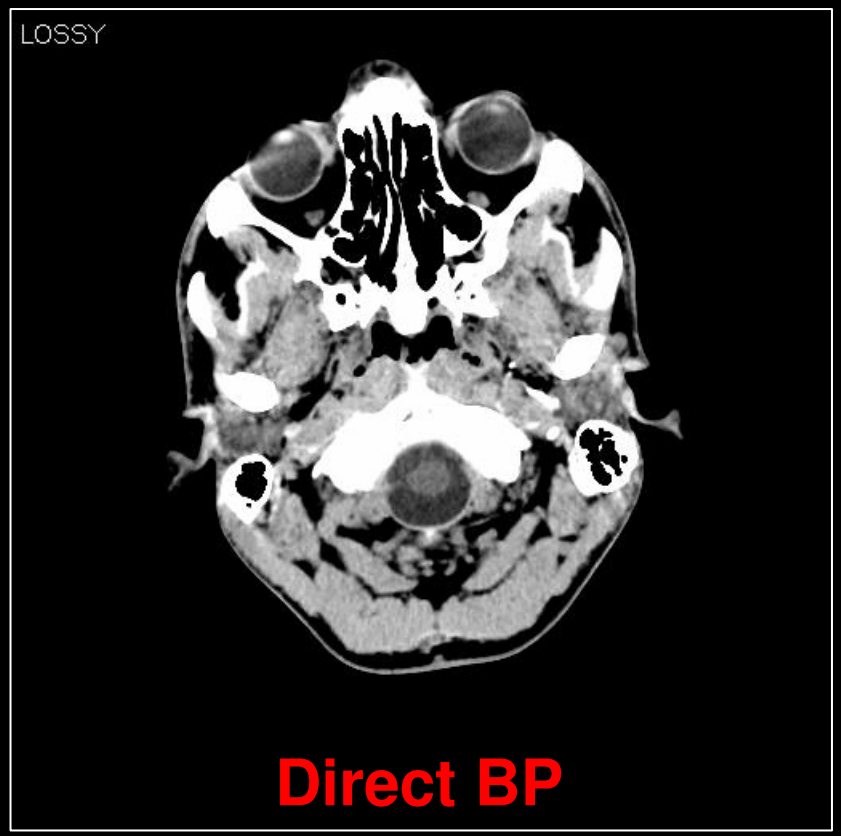


N: Size of Image P: Number of Projection Angles Np: Number of points per projection

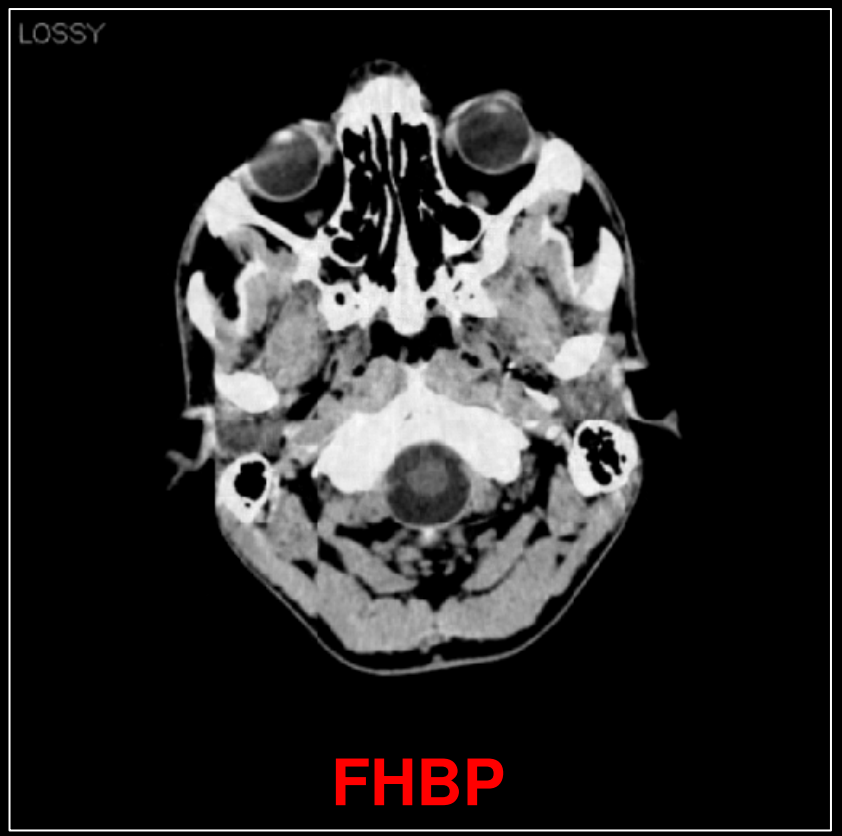
Q: Number of non-downsampling levels M: Radial interpolation factor

Comparison & Results

N=512 P=1024 Np=1024 Q=2 M=4



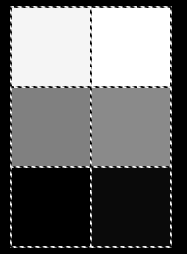
Direct BP



FHBP

Comparisons & Results

N	P	Np	Q	M	Gain	RMSE
256	512	512	0	1	41.4	21.4
256	512	512	0	4	28.4	8.9
256	512	512	3	1	9.2	21.3
256	512	512	3	4	6.8	6.6
512	1024	1024	0	1	107.7	17.3
512	1024	1024	0	4	62.3	7.8
512	1024	1024	3	1	18.7	16.8
512	1024	1024	3	4	13.0	5.5
1024	2048	2048	0	1	132.7	13.3
1024	2048	2048	0	4	93.3	7.0
…	…	…	…	…	…	…



Conclusions & Future Work

- FHBP is much faster without much loss
- Parameters allow for specific applications
  - LPF taps, Q, M
- Implementation speed
  - Matlab vs. C++ vs. Assembly

References

[1] S. Basu and Y. Bresler, "$O(N^2 \log_2 N)$ Filtered Backprojection Reconstruction Algorithm for Tomography," *IEEE Trans. Image Processing*, vol. 9, pp. 1760-1773, October 2000.

[2] P. A. Rattey and A. G. Lindgren, "Sampling the 2-D Radon Transform," *IEEE Trans. Acoustic, Speech, and Signal Processing*, vol. ASSP-29, pp. 994-1002, October, 1981.

[3] S. Basu and Y. Bresler, "Error Analysis and Performance Optimization of Fast Hierarchical Backprojection Algorithms," *IEEE Trans. Image Processing*, vol. 10, pp. 1103-1117, July 2001.

Thanks for coming!

Questions?