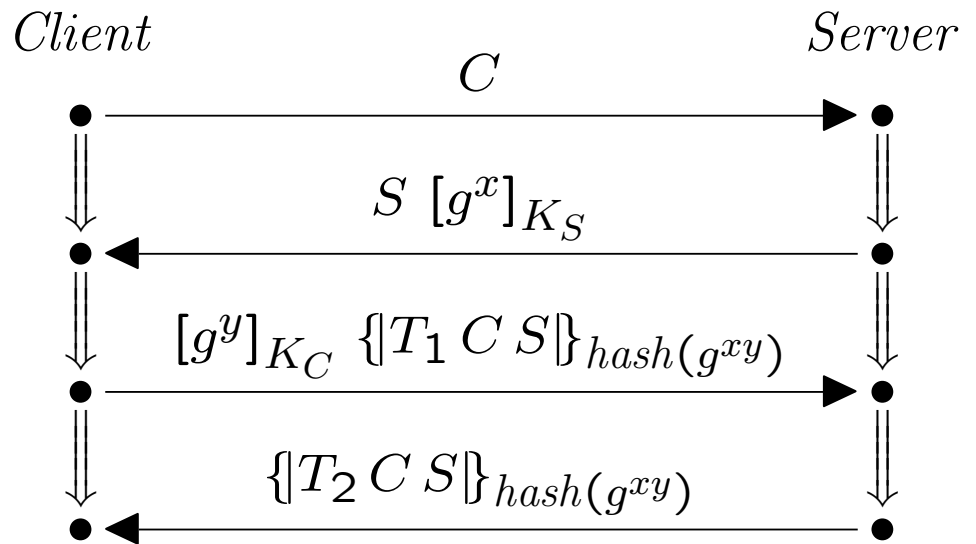


# **Incorporating Diffie-Hellman Into Strand Spaces**

**Jonathan Herzog  
The MITRE Corporation**

This work supported by the National Security Agency

# The TLS-DH Protocol (Simplified)



Three components:

1. A handshake

2. Authenticated Diffie-Hellman

–  $[M]_{K_X}$  is  $M$  signed with  $X$ 's private key,

3. Confirmation, using a hash of  $g^{xy}$  as the ke

# Incorporating Diffie-Hellman

Standard Dolev-Yao model doesn't consider Diffie-Hellman  
– How to incorporate?

One approach [ PQ '01, BB '03, MS '03]:

- Give adversary specific additional abilities
  - Multiplication, inverses, etc.
- Prove secrets not deducible

Good for finding flaws

However, lack of flaws does not imply security

- “Real” adversary may have additional power
- May be undiscovered attacks

# Proving Security

Focus on proofs of security rather than flaws

Want proof method that captures all attacks

Tempting to use “computational” model

- Everything is an algorithm
- Messages are bit-strings drawn from distribution
- Proofs use reductions
  - “If an adversary can break the protocol, adversary can break the underlying encryption”
- Relies upon intractability assumptions
  - Actual form of Diffie-Hellman assumption

# Present Work

Computational proofs extremely meaningful

- Grounded in complexity theory

However, lacks benefits of Dolev-Yao model:

- High level of abstraction
- Simplicity
- Automation
- Re-use of general theorems

This talk: best of both worlds

- High-level security proofs for protocols like
- Existence of computational proofs guarante

# General Approach

Increase expressiveness of model

- New operators
- New adversary powers

Assume Diffie-Hellman is hard

- Translate Diffie-Hellman into formal terminology
- (Introduce some strand space vocabulary)

Analyze TLS

- Assuming Diffie-Hellman to be hard

Demonstrate translation accuracy

- Show: if translation is false, Diffie-Hellman is hard

# New Operators

Randomized encryption

Signatures, also randomized

Hashing

- Turns any message into a key

Formal, free algebra abstraction of group operation

- Atomic Diffie-Hellman elements:  $d_a, d_b \in$

- Analogous to  $g^x, g^y$

- Formal Diffie-Hellman operation:  $DH(d_a,$

- Analogous to  $g^{xy}$

- Produces compound messages

- Will reserve  $g^x, g^y, g^{xy}$  for the computational

# Extending the Adversary

What additional powers to give to adversary?

Want to prove security against any efficient adversary

Might as well give the adversary all reasonable powers

– Adversary can perform every tractable function

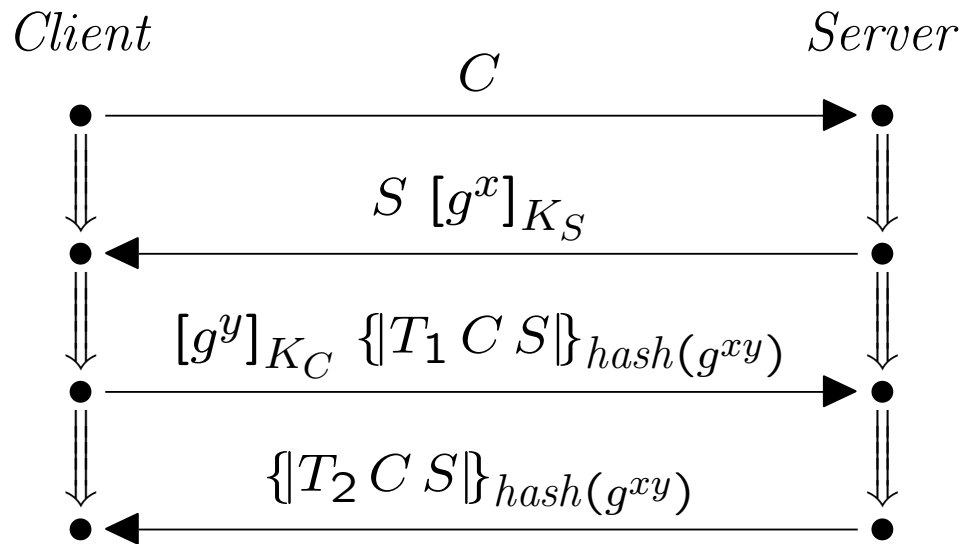
$$f : A^* \rightarrow D$$

( $A$  is any message)

Other techniques free to consider smaller sets

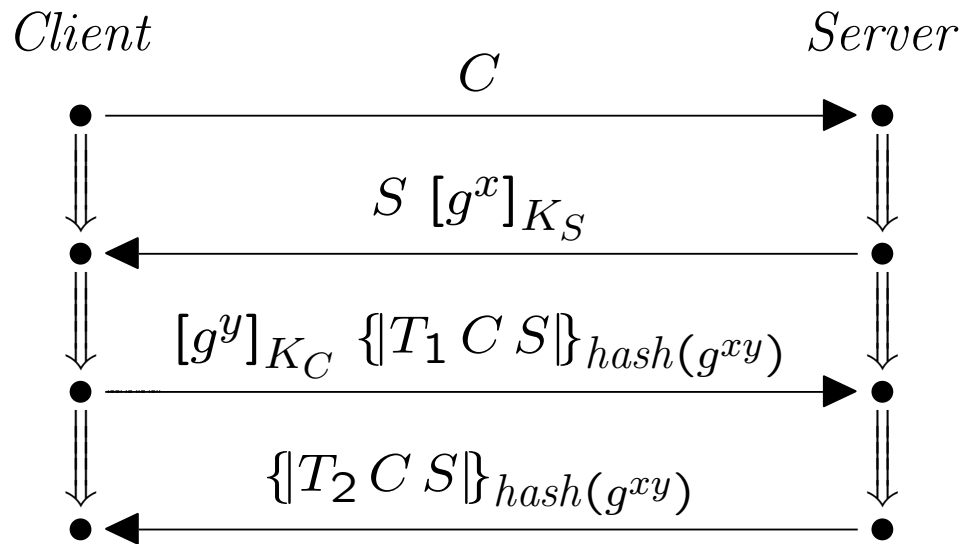


# Strand Space Terminology



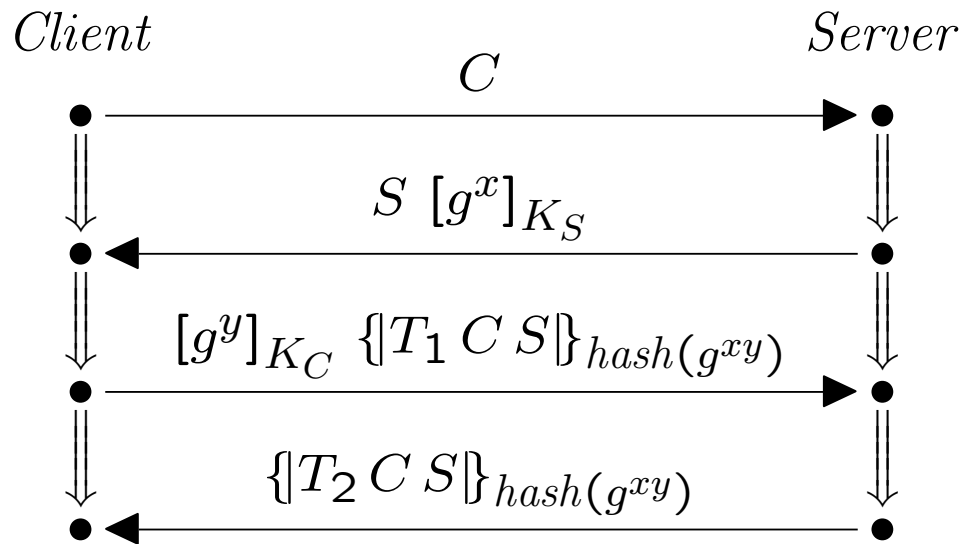
**Regular Participant:** One who follows the protocol  
As opposed to adversary

# Strand Space Terminology



- Strand:** Sequence of messages sent, received
- Regular strand: trace of one particular execution
- Adversary strand: single operation
  - Link together to form more complex operations

# Strand Space Terminology



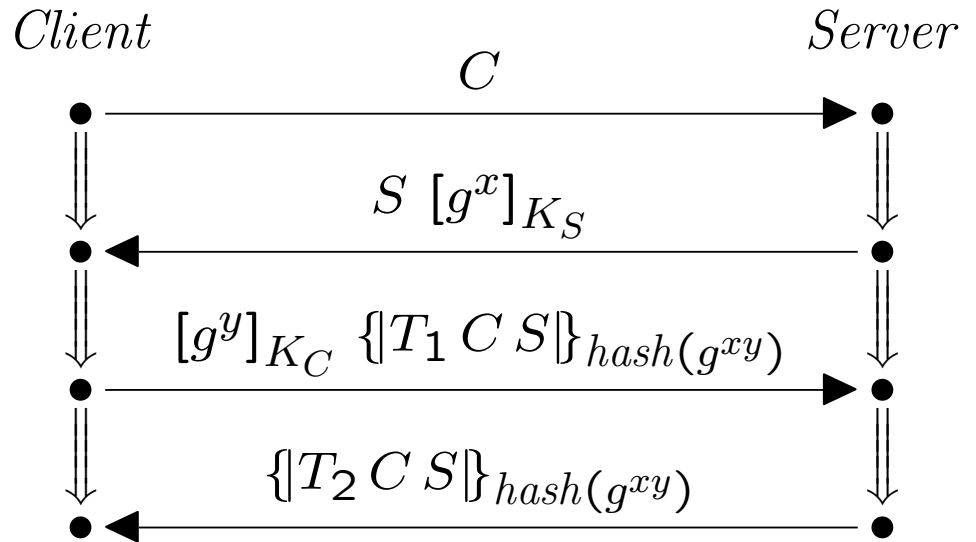
**Bundle:** Collection of communicating strands

Who says what to who

Global view of all conversations

Could be different from intended conversation

# Strand Space Terminology



**Origination:** Strand utters value it never heard

“First” time value is used

No origination  $\rightarrow$  secret

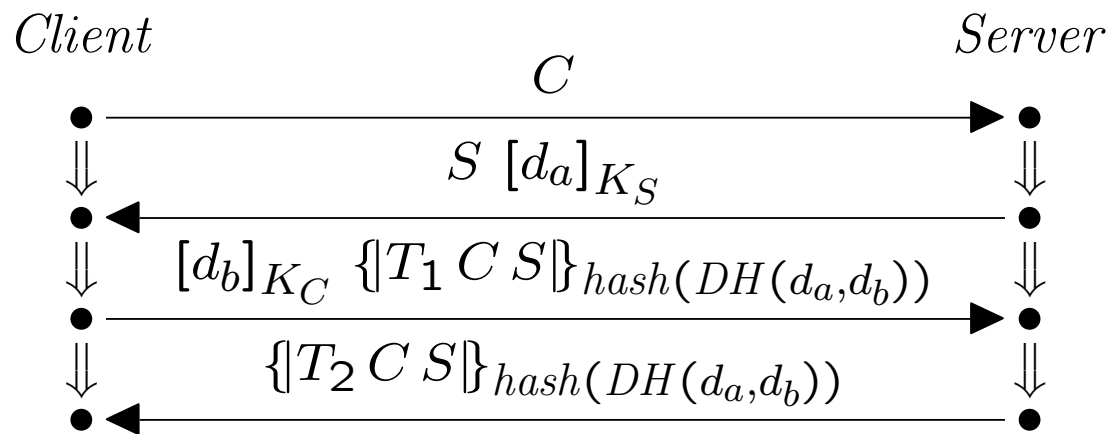
Note: value does not originate when used a

# Formal Diffie-Hellman Condition

- If**
1.  $g^x$  and  $g^y$  are created only by honest participants  
 $d_a$  and  $d_b$  originate only on regular strands
  2.  $g^{xy}$  is not uttered by honest participants  
 $DH(d_a, d_b)$  does not originate on regular strands

**Then**  $g^{xy}$  is not emitted by adversary either  
 $DH(d_a, d_b)$  does not originate at all

# Proof Sketch of Security



Assumption:  $d_a, d_b$  originate only on regular str

No value  $DH(d_1, d_2)$  originates on regular node

Therefore  $DH(d_a, d_b)$  does not originate:

- secrecy

Thus  $hash(DH(d_a, d_b))$  does not originate

Encrypted with secret key → emitted by regular s

- authentication

# Deriving the Diffie-Hellman Condition

How to justify such a condition?

- Does it diminish the computational soundness of the model?

Derivation:

1. Give computational semantics to Strand Space

2. Then show:

“If a bundle violates the formal Diffie-Hellman condition, it maps to an efficient algorithm that solves Diffie-Hellman”

# Derivation Sketch

Give bit-string value to every message in bundle

Every atomic term represents random variable

- Atomic terms given random value
- Compound terms built up from atomic ones

Adversary strands all tractable functions

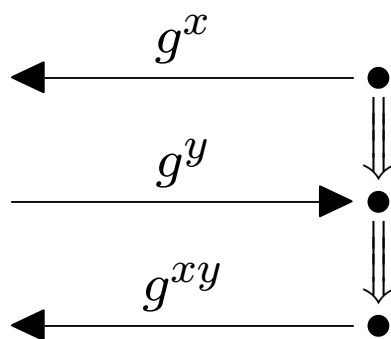
Regular strands may not be

- Regular participants unconstrained
- Might represent intractable computations

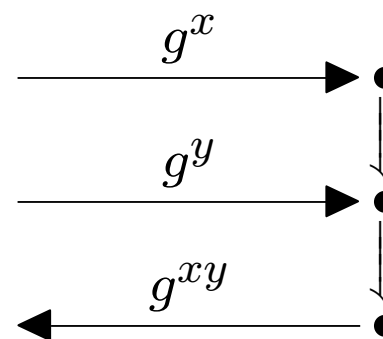


# Tractable and Intractable Regular Strands

Tractable regular strand



Intractable regular strand



# Tractable Regular Strands

Want to avoid intractable strands

Details highly strand-specific

- Also specialized for TLS

General idea:

- Invoke traits of protocols like TLS

- Real participants know secret exponents
- Don't utter secret values, but hash into

- Together, make regular strands tractable

Details in paper

# Deriving Formal Diffie-Hellman conditions

Suppose some bundle violates security property

- $d_a, d_b$  originate on regular nodes
- $DH(d_a, d_b)$  originates only on adversary node
- Regular strands tractable

Turn it into algorithm

- $g^x, g^y$  given as inputs, assign to  $d_a, d_b$
- Choose values for all other atomic messages
- Each strand easy to compute
- Compose individual computations according to bundle structure
- Node for  $DH(d_a, d_b)$  now has value for  $g^{xy}$ .
- Case analysis:  $g^{xy}$  appears unencrypted

# Conclusion

Diffie-Hellman incorporated into Strand Spaces

- Does not diminish computational soundness

Probably can be used by automated Strand Spaces

Areas for generalization:

- “Common protocol traits” based on TLS
  - Group key protocols likely have other traits
- Approach possibly applicable to other formalisms
- Also probably applicable to other primitives

# Backup slides

# Randomized Encryption

Encryption explicitly takes randomness as argument

$$Enc : \mathcal{A} \times Key \times Rand \rightarrow \mathcal{A}$$

$$Enc(M, K, r) = \{M\}_K^r$$

Signatures similar

# Common Protocol Traits

Real protocol participants don't solve Diffie-Hellman problem

- Won't calculate  $g^{xy}$  unless they know  $x$  or  $y$
- Presumably, regular participants choose  $g^x$  picking  $x$ .
- Def: regular strands are *conservative* if they use  $DH(d_1, d_2)$  unless  $d_1$  or  $d_2$  originates on node

Also, honest participants don't commonly "say"

- Def: regular strands are *silent* if no  $DH(d_1, d_2)$  originates on regular strands
- Still allows regular strands to use  $DH(d_1, d_2)$  key
- All such keys are produced by hashing

# Side-stepping Diffie-Hellman

If hashing is strong, a hash of  $DH(d_1, d_2)$  has the same distribution as random value

Hence, no need to calculate pre-image to hash

- Pick random values instead
- If this changes anything, then hashing is not strong
  - Proof uses conservativeness of regular strands

No longer need to solve Diffie-Hellman to calculate regular strands

All strands efficiently computable



# Computational Soundness of Dolev-Yao

Work in progress

Backes, Pfitzmann, Waidner

- Universally Composable Cryptographic Libra

Lincoln, Mitchell, Mitchell, Scedrov

- Incorporating poly-time indistinguishability i  
cess calculi

More direct approaches

- Abadi and Rogaway
- Bogdan
- Myself

Probably will be settled in next five years

# Comparison with Millen, Shmatikov

Them	Me
Finds flaws	Produces proofs
May not find all flaws	May not produce proofs for all correct protocols
Untyped	Typed
Limited adversary powers w.r.t. Diffie-Hellman	Unlimited adversary powers w.r.t. Diffie-Hellman
Decision procedure	Pretty sure results can be incorporated into tools