Variable Density PRM Waypoint Generation and Connection Radii for Energy-Efficient Flight through Wind Fields

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***Abstract*— This paper presents a method for planning low-cost paths for an unmanned aerial vehicle (UAV) in a spatially varying wind field using a biased sampling-based path search technique. The objective of the path planning problem is to minimise the energy consumption of a UAV by finding time-efficient paths through the available flight region. A Gaussian process (GP) model of the wind field is constructed from sparse samples of the wind, and the gradient of this model over the entire sample space is used to guide the waypoint sampling density and connection radius across the flight region. For a UAV planning problem in strong winds the proposed method is demonstrated to produce paths with comparable cost to a uniformly-sampled technique whilst using fewer than one tenth of the number of nodes and edges.**

I. Introduction

Prior knowledge of the wind can be used to aid energy-efficient path planning for UAVs. Planning to exploit favourable wind currents for small UAVs is particularly important when the airspeed of the aircraft is less than the maximum wind speeds in the flight region. This work explores the use of a roadmap planner to generate time and energy-efficient paths for point-to-point UAV flights in wind.

This work builds on the probabilistic roadmap (PRM) planner first described in [1] and the subsequent PRM* algorithm, which was shown in [2] to be asymptotically optimal. While different biased waypoint sampling techniques introduced previously have been shown to improve path planning performance over uniform random sampling with the same number of waypoints [3]–[9], these existing methods are targeted towards planning around obstacles and rely on knowledge of the free space geometry to generate the biased waypoint sampling scheme. In the problem considered in this paper, the flight region represents the vehicle workspace and, in the tests shown here, is entirely free space (obstacle-free). However, the wind field represents a transformation of the cost space affecting the cost of traversing edges in the graph. For a UAV flying in wind, rather than a simple distance-based cost, the time or energy required to traverse an edge is a function of the distance, wind speed and wind direction along the edge. In some cases, such as where the headwind is greater than the maximum vehicle airspeed, the edge becomes impossible to traverse. Thus, given a limited number of waypoints in a PRM, existing biased sampling techniques cannot be applied to this problem to aid path planning. This paper proposes a new method for using available wind data to direct waypoint distribution across the free space and also use this information to guide the PRM graph connections.

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TABLE I: Partial Derivatives of the Wind

Input dimension	GP Wind Parameter	
	u	**v**
North	$\partial u/\partial d_n$	$\partial v/\partial d_n$
East	$\partial u/\partial d_e$	$\partial v/\partial d_e$

II. Sampling-Based Planning

The proposed approach is to bias waypoint sampling according to the gradients of the wind field to sample more densely in regions of large changes and more sparsely in areas where the wind is relatively uniform. We generate a GP model of the wind field using sparse samples of the wind and extract the gradients of these models, shown in Table I, to form the probability density functions from which to draw waypoint samples. The PRM is then formed using variable connection radii computed by scaling the sampling density term in the PRM* radius equation (1) with the same density function produced by the GP wind gradients.

$$r^*_{\text{PRM}} := r > \sqrt{\frac{6}{\pi} \times \textit{free space area} \times \frac{\log n}{n}}, \tag{1}$$

A. Biased Waypoint Distribution

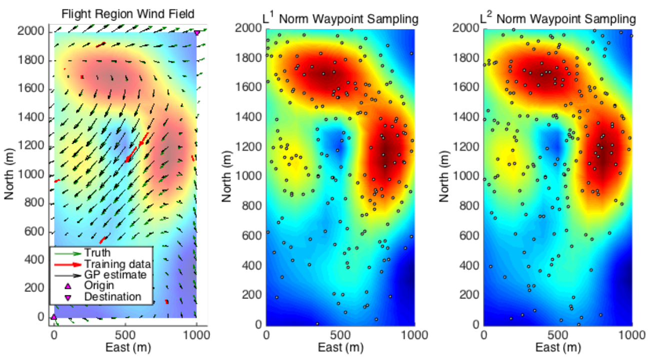
Two different methods are trialled for combining the four partial derivatives. The first is a simple summation which represents the L^1 norm of the combined vector,

$$L^1\text{norm} = \left|\frac{\partial u}{\partial d_n}\right| + \left|\frac{\partial v}{\partial d_n}\right| + \left|\frac{\partial u}{\partial d_e}\right| + \left|\frac{\partial v}{\partial d_e}\right|; \tag{2}$$

the second method takes the L^2 norm,

$$L^2\text{norm} = \sqrt{\left(\frac{\partial u}{\partial d_n}\right)^2 + \left(\frac{\partial v}{\partial d_n}\right)^2 + \left(\frac{\partial u}{\partial d_e}\right)^2 + \left(\frac{\partial v}{\partial d_e}\right)^2}. \tag{3}$$

Examples of biased waypoint sampling for the flight region from Fig. 1a are shown in Fig. 1b and Fig. 1c. The underlying sampling densities generated from the L^1 and L^2 norms of the wind gradient vectors are also shown as the coloured surfaces beneath the waypoints.



(a) (b) (c)

Fig. 1: (a) True and estimated 2D wind field across the flight region. The gradient of the estimated wind field is given by the underlying surface plot. A set of 200 random waypoint samples drawn from the (b) L^1 norm and (c) L^2 norm gradient distributions. Samples are shown as circles.

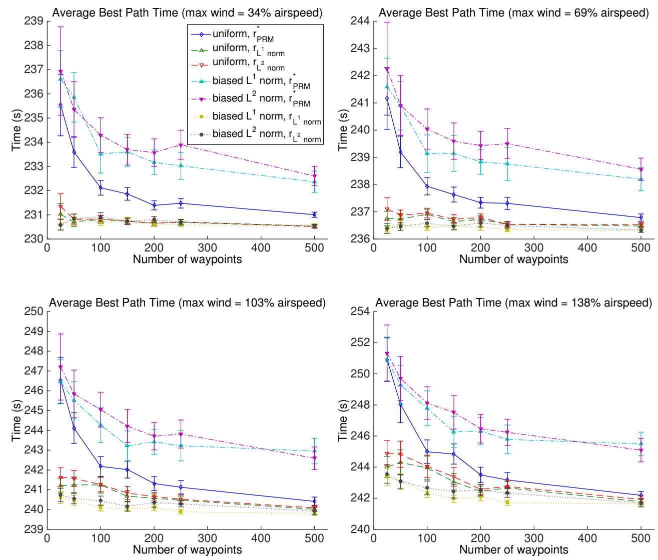


Fig. 2: Fastest time to travel from the origin node to the destination node across the four tested wind fields averaged over 100 random waypoint generations. 7 PRMs were tested, using either uniform or biased random sampling to select waypoints, and either a constant connection radius, defined as the lower bound of the PRM* radius in (1), or a variable connection radius, computed from (4), to connect the waypoints in the graph.

B. Variable Connection Radii

A natural extension to performing variable density waypoint sampling is to create the search graph using variable density-based connection radii. With biased waypoint sampling, regions of sparse sampling will struggle to form connections if a constant connection radius is used, while regions of dense sampling will likely form redundant edges, that is, where near-collinear nodes lying along a similar bearing are densely connected. In short, a constant connection radius cannot take full advantage of the biased sampling along the wind gradients.

The sampling density, $n/\textit{free space area}$, and the connection radius share an inverse square root relationship as shown in (1). Since samples are now drawn from a variable density distribution, the sampling density can no longer be considered constant in this equation. To reflect this variable sampling density in the connection radii, we propose scaling the $n/\textit{free space area}$ factor in (1) with the mean-shifted density values computed from (2) and (3). This gives a connection radius,

$$r_{var}(i) = \sqrt{\frac{6}{\pi} \times \frac{\textit{free space area}}{\textit{mean-shifted sampling density}_i} \times \frac{\log n}{n}}, \tag{4}$$

for each waypoint $i \in \{1, 2, \ldots, n\}$. In other words, the actual sampling density used to select each potential waypoint is used in (4) to compute that waypoint's connection radius. By using the mean-shifted sampling density, (4) is consistent with the original formulation of the PRM* radius, which can be easily shown since the mean-shifted sampling density for uniform random sampling is 1 everywhere. Furthermore, we hypothesise that given this consistency, the theoretical guarantees of PRM* are also applicable in this PRM formulation.

It is worth noting here that using variable connection radii will introduce asymmetry in the edge connections. That is, a waypoint with a long connection radius may connect to another waypoint whose radius is such that it does not connect back to the original waypoint. To overcome this asymmetry, we simply allow any connections made in the graph to be bidirectional.

III. Results

The fastest time to travel from the origin to the destination through the wind field was logged for each generated PRM graph. Averaged results with 95% confidence intervals are shown in Fig. 2. These results show that the paths found using biased waypoint sampling with variable connection radii typically out-performed all the other tested methods, that is, these paths were better at avoiding headwinds and exploiting favourable tailwinds to reach the destination faster. Furthermore, between the two methods for computing the sample density function, the biased L^1 norm sampling with corresponding connection radii, $r_{L^1\ \mathrm{norm}}$, appears to find lower-cost paths than the algorithm using the L^2 norm of the gradient vector. Paths found using PRMs generated from uniform random sampling with variable connection radii also show improved performance over those found using the standard PRM* algorithm, however biased sampling waypoints connected via the PRM* radius resulted in paths with the highest cost.

The plots show a general trend towards a greater difference in the path cost as the maximum wind velocity was increased. This is to be expected since the penalty for choosing a poor path through the wind field is magnified when the magnitude of the wind is increased, while the converse is true for finding a path that exploits strong tailwinds.

For all of the tested wind fields, it can be seen that using biased sampling with variable connection radii consistently found low-cost paths even when there were very few waypoints in the graph. In contrast, the standard PRM* formulation required at least 10 times more waypoints to achieve the same performance when compared to the biased sampling with variable radii PRM that was generated using 25 waypoints.

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