Tunable-Risk Sampling-Based Path Planning Using a Cost Hierarchy

Tixiao Shan and Brendan Englot

***Abstract*— We propose a new sampling-based path planning algorithm, the Optimal Minimum Risk Rapidly-Exploring Random Tree (MR-RRT*), that plans minimum risk paths in accordance with primary and secondary cost criteria. The primary cost criterion is a user-defined measure of accumulated risk, which may represent proximity to obstacles, exposure to threats, or similar. Risk is only penalized in areas of the configuration space where it exceeds a user-defined threshold, causing many graph nodes to achieve identical primary cost. The algorithm uses a secondary cost criterion to break ties in primary cost. The proposed method affords the user the flexibility to tune the relative importance of the alternate cost criteria, while adhering to the requirements for asymptotically optimal planning with respect to the primary cost.**

I. INTRODUCTION

Sampling-based path planning algorithms, in addition to being highly successful in solving problems of high dimension [1], are capable of planning under a variety of challenging costs and constraints. Algorithms such as the probabilistic roadmap (PRM) [2], rapidly-exploring random tree (RRT) [3], and their optimal variants PRM* and RRT* [4], have been adapted to curb robot state uncertainty in the objective [5], [6] and constraints [7], [8], maximize information-gathering under energy constraints [9], minimize distance traveled under task-based [10] and risk-based [11] constraints, efficiently explore narrow passages [12], and formulate multi-objective Pareto fronts [13].

In this effort, we focus on the minimization of risk, which may entail maintaining a safe distance from obstacles, avoiding exposure to threats, or other related objectives. Transition-based rapidly-exploring random trees (T-RRTs) produce generalized low-risk solutions by probabilistically rejecting samples that are drawn in high-risk regions of the configuration space [11]. This algorithm can be used to maintain clearance from obstacles, or to avoid exposure to threats. A property common to both applications is the dependency of the risk function on a single robot configuration. Although quantities such as state estimate uncertainty and collision probability may also serve as measures of risk, such measures depend on the robot's initial error covariance, and measurement and action histories [8], lying outside the scope of the risks discussed in this work.

T-RRT*, a recent extension of T-RRT, inherits the property of asymptotic optimality from RRT* while achieving clearance from obstacles by rejection sampling [14]. However, to maintain a high safety margin, this algorithm must avoid high-risk regions throughout the entire configuration space. It is challenging to plan to or through a high-risk location while also maintaing safety elsewhere in the graph.

To address such issues, we propose the Optimal Minimum Risk Rapidly Exploring Random Tree (MR-RRT*), which can enter high risk regions when they are a critical component of feasibility, while penalizing them in general. Like T-RRT*, the level of risk avoidance is tunable, except risk is addressed in the objective rather than through a sample rejection constraint. A risk penalty is only imposed on regions of the configuration space that lie above a user-designated threshold. As a result, many nodes in the tree achieve identical values of primary cost, and a secondary cost function, such as accumulated distance along the path, may be used to break ties in the primary, risk-based cost. In areas of the configuration space where risk is not a priority, paths may be constructed with respect to the secondary cost criterion instead, allowing a user to tune the relative importance of the two cost criteria. Unlike prior multiobjective approaches that inherit the optimality properties of RRT* through a weighted sum of costs [6], [15], we believe this is the first approach that leverages a cost hierarchy instead.

T. Shan and B. Englot are with the Department of Mechanical Engineering, Stevens Institute of Technology, Castle Point on Hudson, Hoboken NJ 07030 USA, tshan3@stevens.edu, benglot@stevens.edu.

II. PROBLEM DEFINITION

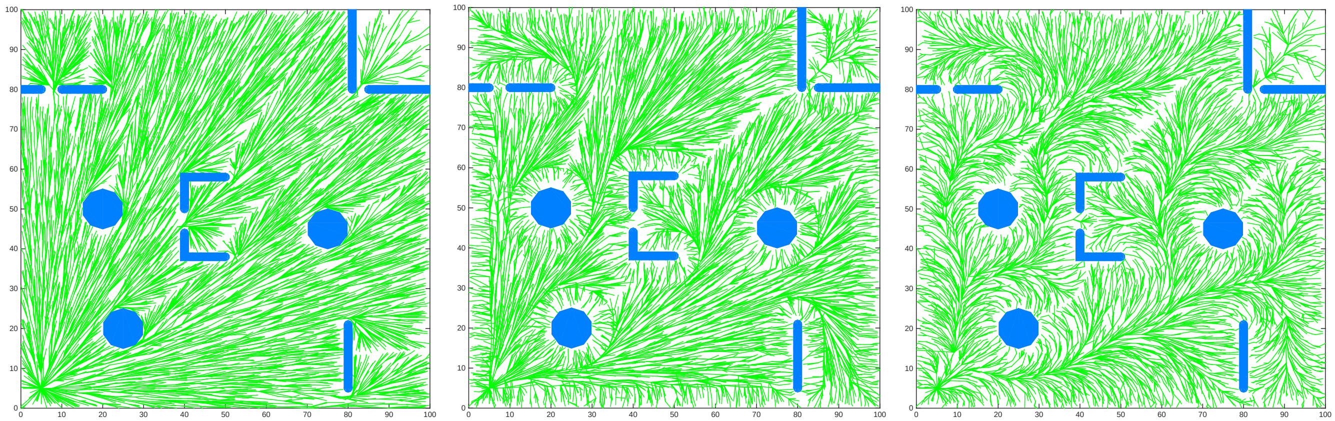
Let $\mathcal{C} \subset \mathbb{R}^d$ be a robot's configuration space. $x \in \mathcal{C}$ represents the robot's position and volume occupied in $\mathcal{C}$. $\mathcal{C}_{obst} \subset \mathcal{C}$ denotes the set of obstacles in $\mathcal{C}$ that will cause collision with the robot. $\mathcal{C}_{free} = cl(\mathcal{C} \backslash \mathcal{C}_{obst})$, in which $cl()$ represents the closure of an open set, denotes the space that is free of collision in $\mathcal{C}$. We will assume that given an initial configuration $x_{init} \in \mathcal{C}_{free}$, the robot must reach a goal region $X_{goal} \subset \mathcal{C}_{free}$. Let a *path* be a continuous function $\sigma : [0, 1] \rightarrow \mathbb{R}^d$ of finite length. We will assume the robot moves through $\mathcal{C}_{free}$ along paths obtained from a directed graph $G(V, E)$, with vertices V and edges E.

Our proposed algorithm uses the following primary cost function, which penalizes the risk accumulated along a path that is derived from G:

$$c_{Risk}(\sigma) := \int_{\sigma(0)}^{\sigma(1)} Risk(\sigma(s)) ds \tag{1}$$

$$Risk(x) := \begin{cases} \mathcal{R}(x), & \text{if } \mathcal{R}(x) > RiskThreshold \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where the function $Risk : \mathcal{C}_{free} \rightarrow \mathbb{R}_0^+$ evaluates the risk at an invididividual robot configuration. We penalize a robot's risk using the tunable risk threshold $RiskTheshold \in \mathbb{R}^+$. $\mathcal{R} : \mathcal{C}_{free} \rightarrow \mathbb{R}^+$ represents the strictly positive underlying risk at a robot configuration, which is evaluated against $RiskThreshold$ and returned if the threshold is exceeded.



(a) $RiskThreshold = \infty$ (b) $RiskThreshold = 0.2$ (c) $RiskThreshold = 0$

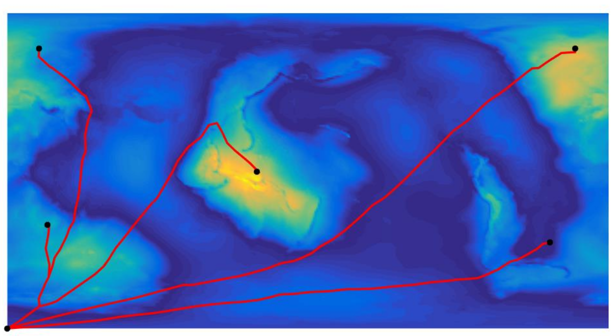
Fig. 1: Trees generated by MR-RRT* under different values of $RiskThreshold$, when $\mathcal{R}(x)$ is the inverse distance transform.

In addition, we define a secondary cost c_{second} as follows:

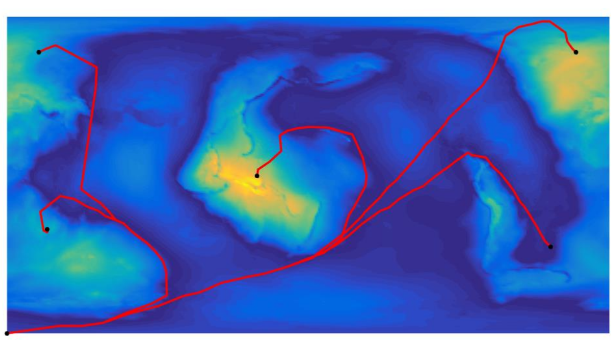
$$c_{second}(\sigma) := \int_{\sigma(0)}^{\sigma(1)} Second(\sigma(s))ds \tag{3}$$

where the function $Second : \mathcal{C}_{free} \rightarrow \mathbb{R}^{+}$ represents a strictly positive cost, ensuring that ties in secondary cost do not occur as they do for primary cost.

III. ALGORITHM DESCRIPTION AND RESULTS



(a) T-RRT*, parameterized so all goal regions are reached consistently.



(b) MR-RRT*, parameterized to penalize high elevations.

Fig. 2: T-RRT* and MR-RRT* are compared over a terrain map in which $\mathcal{R}(x)$ is the robot's altitude.

The algorithm proceeds similarly to RRT* [4], except that ties in primary cost may occur both when adding new nodes to the tree and in rewiring the existing nodes of the tree. When this occurs, ties are broken by evaluating the secondary costs of all nodes with identical primary cost. The value of $RiskThreshold$ determines the influence of the primary and secondary cost functions in the resulting tree. Let us assume that the inverse distance transform is adopted as $\mathcal{R}(x)$, and our secondary cost function is represented by the accumulated distance along a path. When $RiskTheshold$ is infinite, risk is penalized nowhere in the configuration space. In this case, MR-RRT* reduces to RRT* and produces minimum-distance paths. When $RiskThreshold$ is positive and finite, the regions within a designated distance of the obstacles are considered dangerous, and risk is penalized. When $RiskThreshold$ is zero, risk is penalized everywhere and the secondary cost function does not play a role. The paths generated by MR-RRT* follow the medial axis of the free space before heading toward their respective goal regions. Figure 1 gives examples of these situations.

We claim that the proposed algorithm is asymptotically optimal with respect to its primary cost function, which implies that the cost function is bounded and monotonic. To achieve boundedness when the inverse distance transform is adopted as $\mathcal{R}(x)$, we must impose a ceiling on this function so it cannot take on infinite values in the close vicinity of obstacles. The definition of (1) as an integral cost function ensures that the cost function is also monotonic. However, a unique feature of MR-RRT* is that the cost function is not strictly positive over a path; it can be zero-valued in areas where risk falls below the value of $RiskThreshold$. We also claim that MR-RRT* is characterized by the same worst-case $\mathcal{O}(n\ log(n))$ complexity as the original RRT* algorithm, due to a constant-factor increase in the number of cost function evaluations required over the edges of the tree.

Preliminary results are given in Figure 2 comparing MR-RRT* with T-RRT* over a terrain map where risk is equivalent to the robot's altitude, representing its exposure to threats. The robot must reach five small, high-risk goal regions and the results depicted are obtained from trees constructed over 10,000 samples. T-RRT*'s rejection sampling is parameterized to be as risk-averse as possible while consistently reaching all five goal regions.

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