Differentially Private Testing of Properties of Distributions

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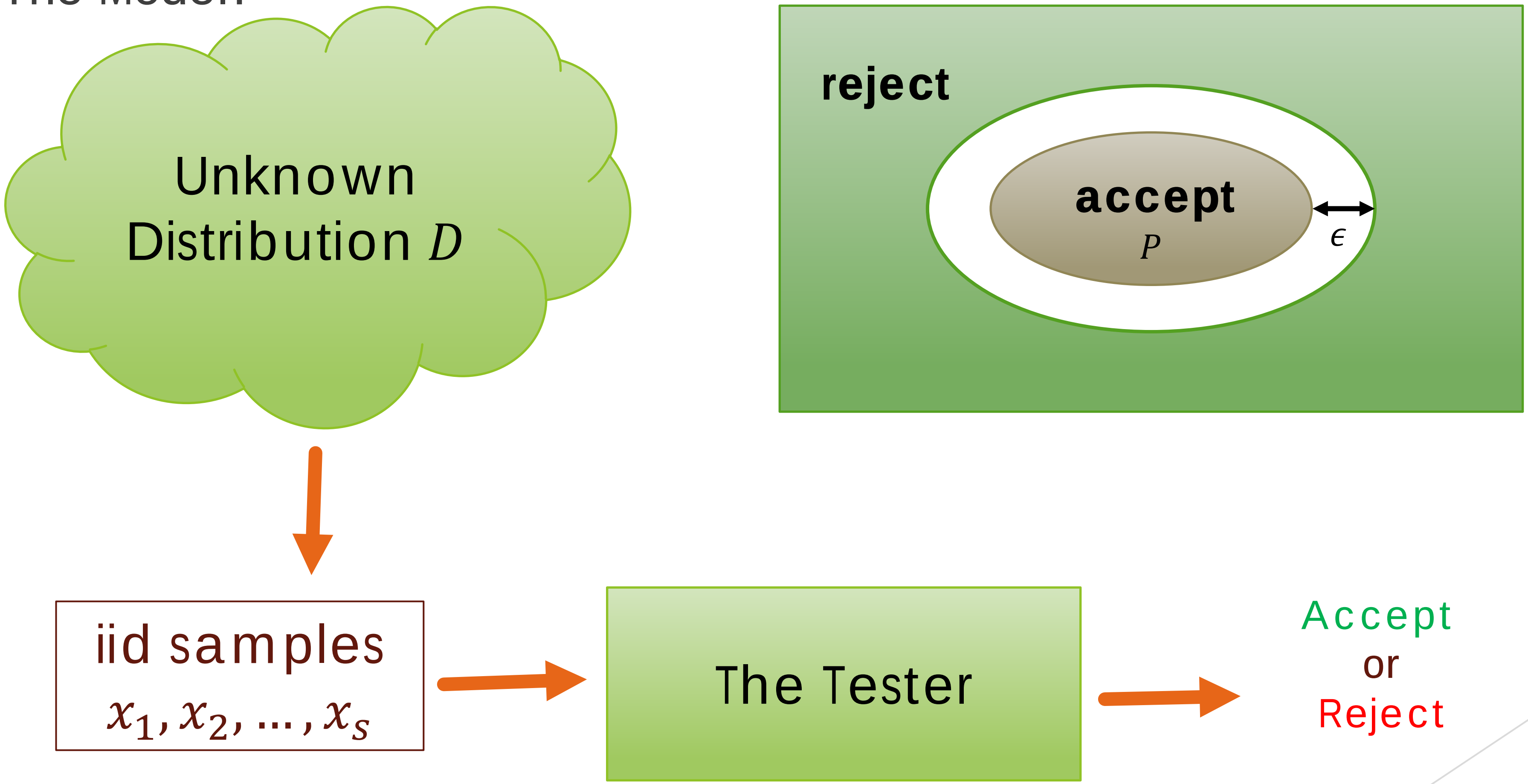
Property testing of distributions

The Model:



[Rubinfeld and Sudan'96, Goldriech and Ron'00, Batu, Fortnow, Rubinfeld, Smith, and White'00, …]

Differential privacy

- Any possible output O
- Two neighboring data set X, X' s.t. |X – X'| = 1

[Di

Main Question:

Can we test properties of distribution with respect to differential privacy? optimal sample complexity?

[Diakonikolas, Hardt, and Schmidt'15, Cai, Daskalakis, and Kamath'16, …]
[In an independent work: Acharya, Sun, Zhang'17]

Problems: Testing uniformity

iid samples $x_1, x_2, \ldots, x_s$ → Is D Uniform, or ϵ-far from being uniform? → Accept or Reject

Sample Complexity:

- **When $\boldsymbol{\epsilon = \Omega(n^{1/4})}$: $\boldsymbol{O(\sqrt{n}/\epsilon^2 + \sqrt{n}/(\epsilon\sqrt{\xi}))}$**
- General case: $\tilde{O}(\sqrt{n}/\epsilon^2 + \sqrt{n}/(\epsilon\xi) + 1/\epsilon^2\xi)$

[Paninski'08, Batu, Fortnow, Rubinfeld, Smith, and White'13, Valiant and Vaiant'14, Chan, Diakonikolas, Valiant, and Vaiant'14, Diakonikolas, Gouleakis, Peebles, and E. Price'16, …]

Problems: Testing Identity (Goodness of Fit)

iid samples
$x_1, x_2, \dots, x_s$
from distribution D

Explicitly given
Distribution q

Is D equal to q,
or ϵ-far from it?

Accept
or
Reject

Sample Complexity:

- **When $\boldsymbol{\epsilon = \Omega(n^{1/4})}$: $\boldsymbol{O(\sqrt{n}/\epsilon^2 + \sqrt{n}/(\epsilon\sqrt{\xi}))}$**
- General case: $\boldsymbol{\tilde{O}(\sqrt{n}/\epsilon^2 + \sqrt{n}/(\epsilon\xi) + 1/\epsilon^2\xi)}$

Problems: Testing Closeness (Equivalence)

iid samples
$x_1, x_2, \ldots, x_s$
from distribution p

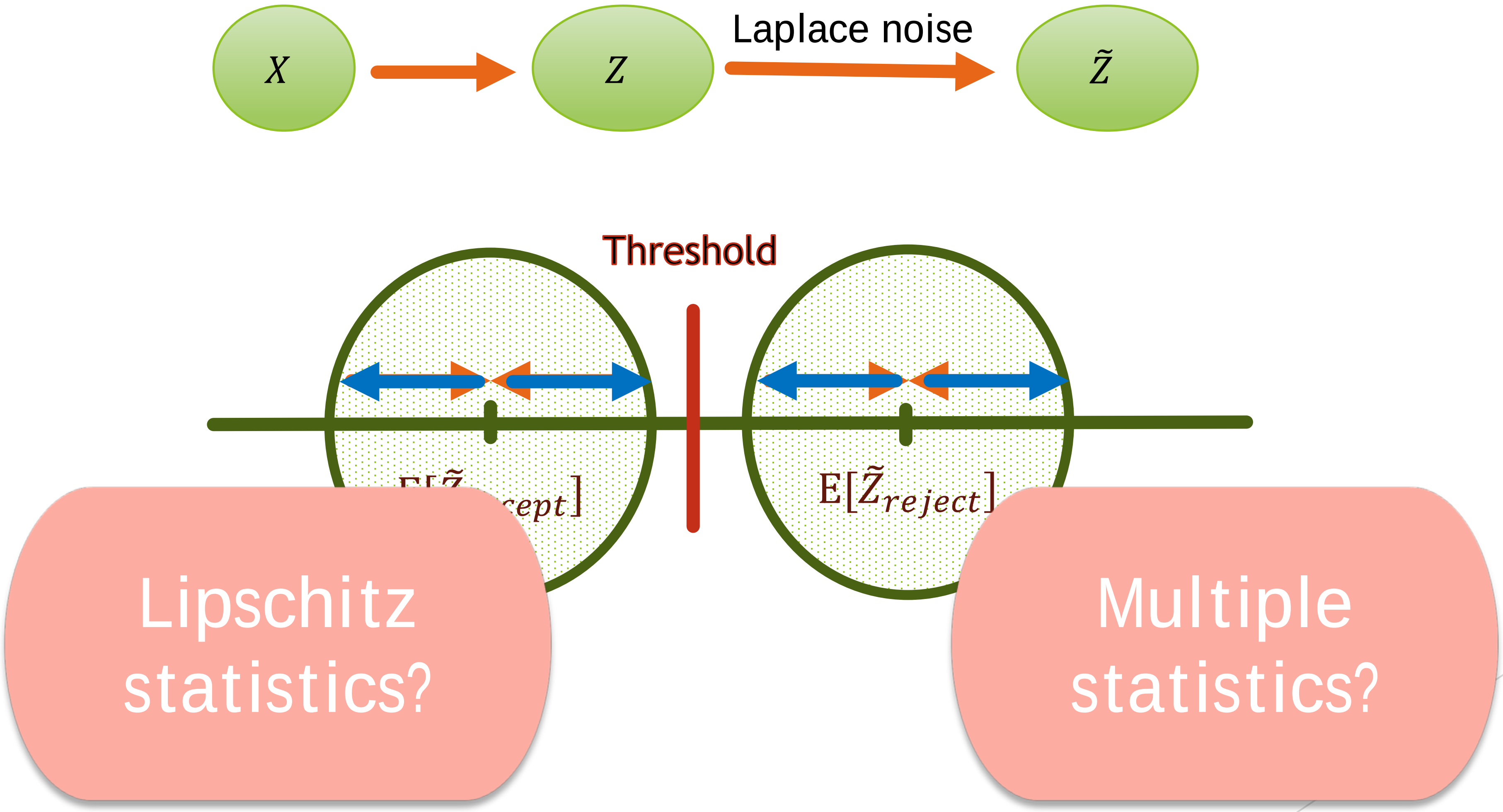
iid samples
$y_1, y_2, \ldots, y_s$
from distribution q

Is p equal to q,
or ϵ-far from it?

Accept
or
Reject

- Sample Complexity: $\boldsymbol{O}(\boldsymbol{n}^{2/3}\boldsymbol{\epsilon}^{4/3} + \sqrt{\boldsymbol{n}}/\boldsymbol{\epsilon}^2 + \sqrt{\boldsymbol{n}}/(\boldsymbol{\epsilon}\sqrt{\boldsymbol{\xi}}) + \mathbf{1}/\boldsymbol{\epsilon}^2\boldsymbol{\xi})$

General Framework



Thank you