Testing local properties via spherical queries

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WOLA 2018, MIT

Local properties

- **Monotonicity** is a **2-local** property.
  - Sequence $S_1, \ldots, S_n$ monotone $\iff \forall i: \; S_i \leq S_{i+1}$.

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$$5 \; 7 \; \underbrace{9 \; 1} \; 3 \text{ is not monotone}$$

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Property of d-dimensional arrays (1D sequences, 2D images,...) is **k-local**

$$\Updownarrow$$

defined by a set of allowed $k \times \ldots \times k$ consecutive subarrays.

- **Lipschitz continuity** is also 2-**local**.
  - $S_1, \ldots, S_n$ Lipschitz-continuous $\iff$ $\forall i: \; |S_{i+1} - S_i| \leq 1$.
- Problems in **pattern matching** are typically $o(n)$-**local**.

The result

Any k-local property of d-dimensional arrays over **any*** alphabet has 1-sided non-adaptive ϵ-test with number of queries

- $O(k\mathbf{log}(\epsilon\mathbf{n}/k))$ for $d = 1$.
- $O(c_d k\epsilon^{-1/d}\mathbf{n^{d-1}})$ for $d > 1$

Tight across whole range!
(and matches monotonicity in $1D$ even for 2-sided adaptive [Fischer 04, Chakrabarty-Seshadhri 14, Belovs 18])

Test based on querying **spheres** of varying sizes.