LOCAL COMPUTATION ALGORITHMS

(a tale of two computation models)

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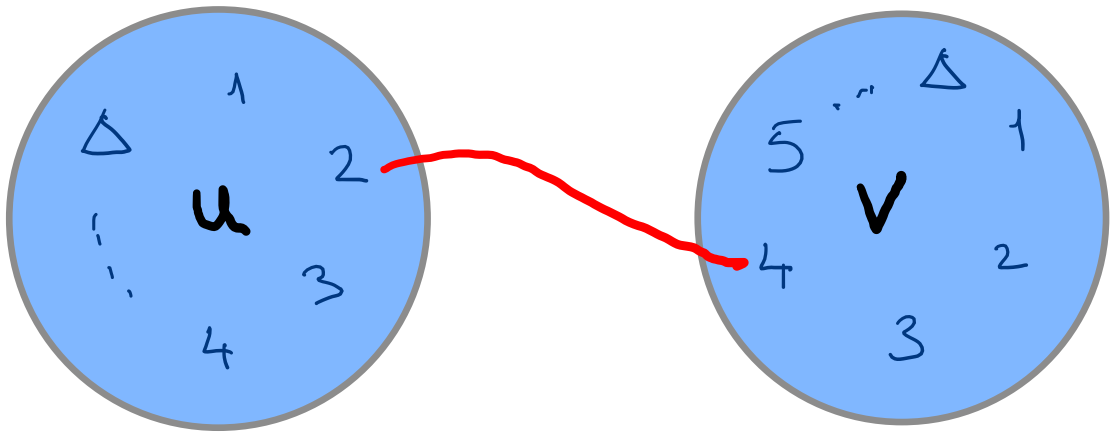
Want to solve graph problems

Labeled Graphs

* vertices have unique ID's $\in \{1, \ldots, n\}$
* edges have port numbers $\in \{1, \ldots, \Delta\}$



- v is 2'nd neigh. of u
- u is 4'th neigh of v

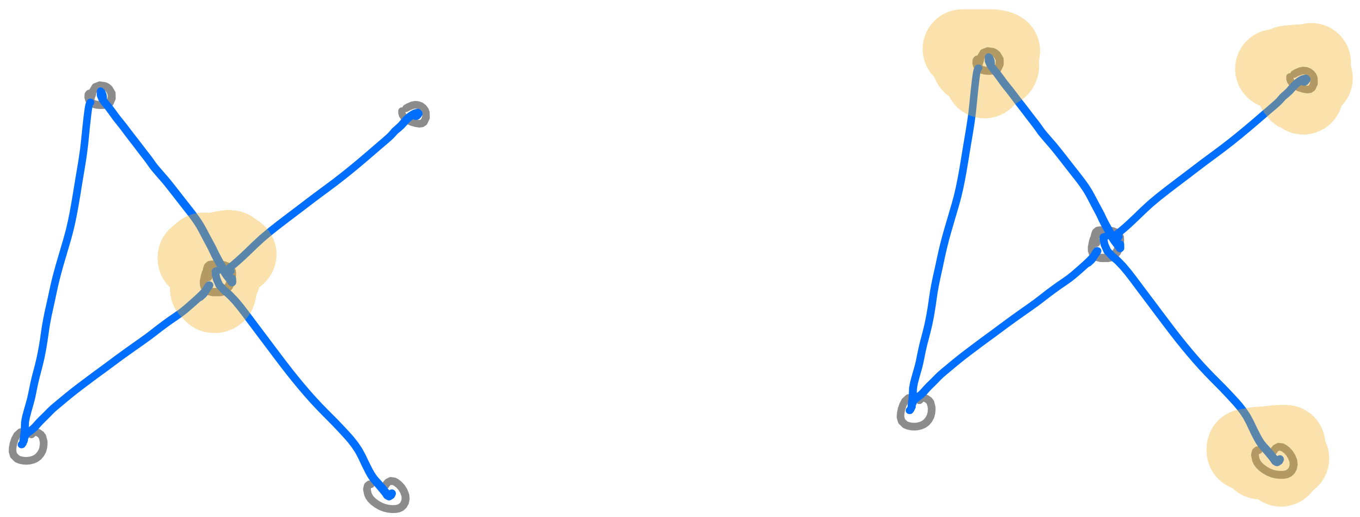
Access Model to Graph

probe(v) :
return list of all neighbors of v

[Δ times more expensive than

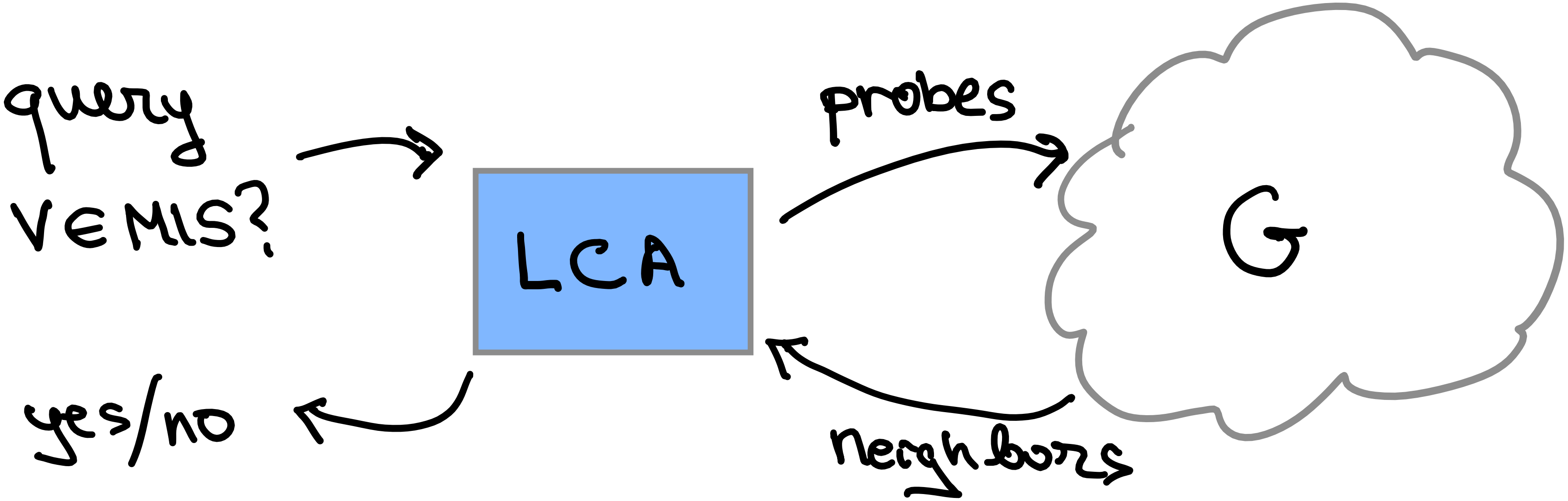
probe(v, i) = return i'th neigh. of v]

Labeled graph G & fixed MIS in G



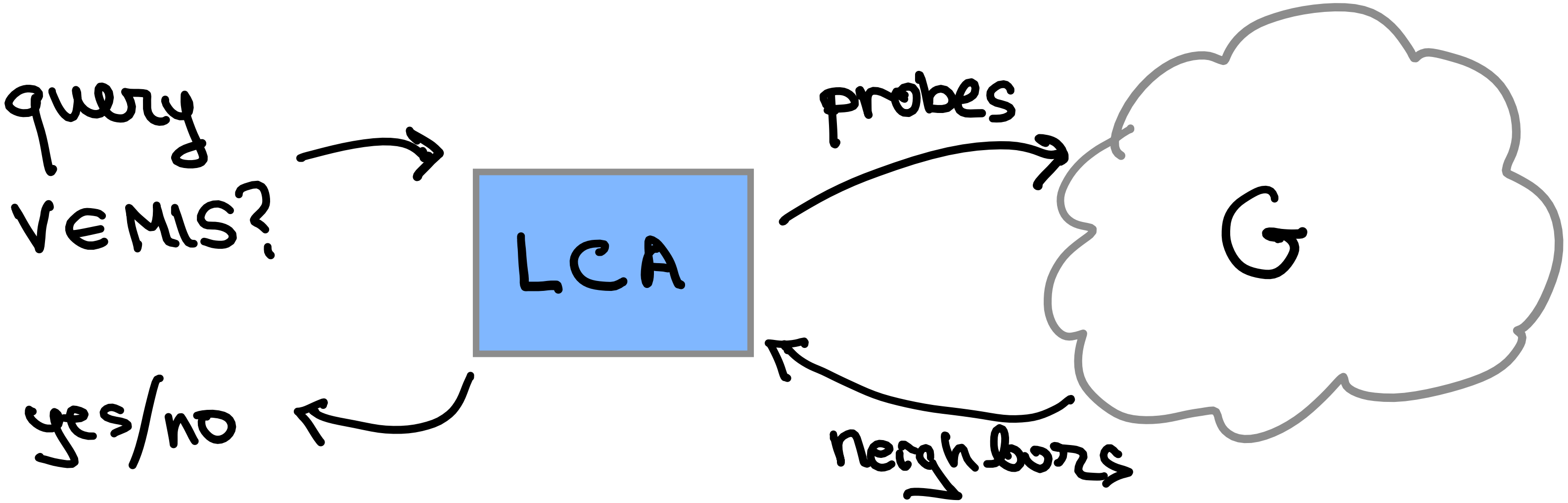
MIS = maximal ind. set

Local Computation Alg (for MIS)



* LCA generates a seq. of probes
* LCA always answers correctly

(stateless det version of RTVX11)

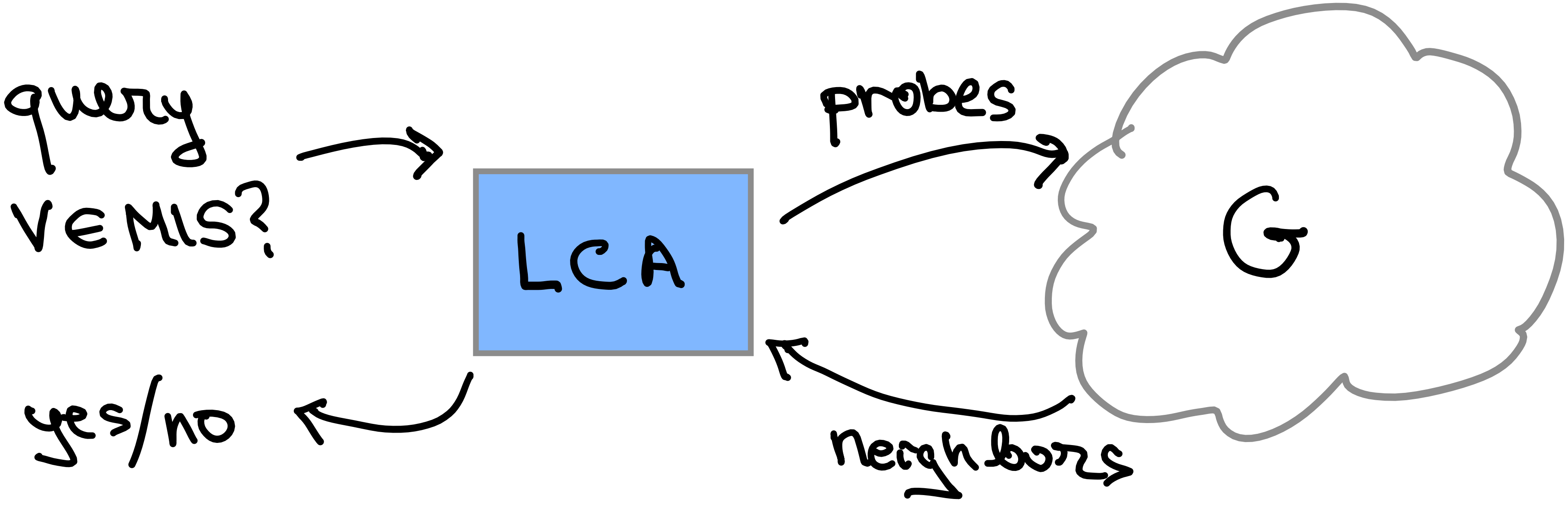


GOAL: MIN # probes. ($o(n)$ probes per query)

Rules:

consistent: $\exists$ MIS $\forall q$ $LCA(q) = I_{MIS}(q)$

stateless: LCA can't store previous queries, probes, neigh, answers, MIS ...



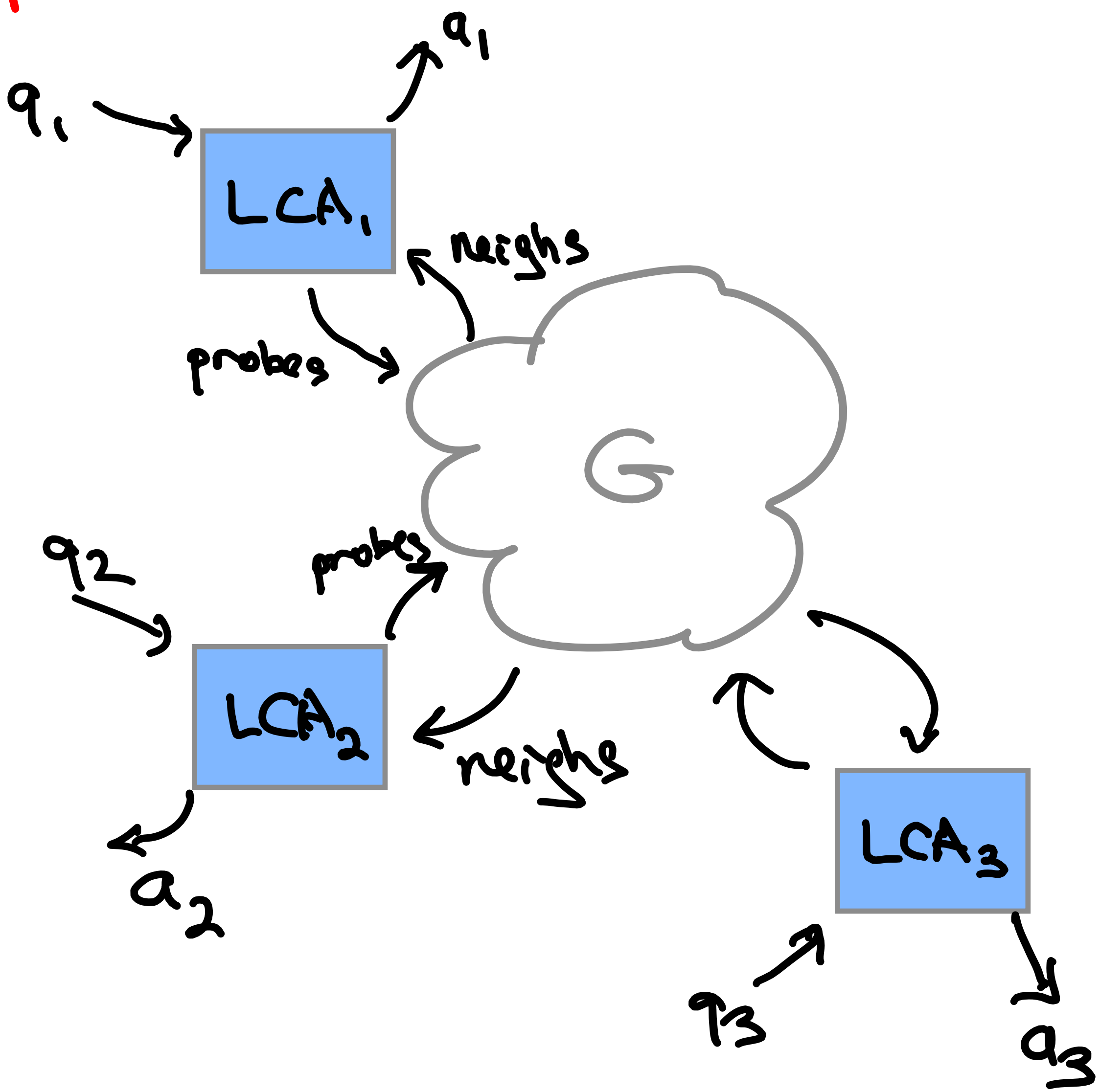
LCA for MIS ?!

* answer yes unless conflicts prev. answer.
  requires storing answers.
* can't store MIS becaus did not read G.

LCA - application

- uncoord. servers
  no communication
  between servers
- all consist. wrt
  to same MIS

[NO-08, YYI-12]
use "LCA" to apx VC



LCA Variations

1) Randomized (use a random seed)
[NO-08, RTVX-11]

Never wrong, but

$$\Pr[\text{too many probes}] < \frac{1}{\text{poly}(n)}$$

2) With states (to generate a random graph [ELMR, BRY'17]

(LCA vs.) Distributed LOCAL

* distributed model in which output of v is determined by

$$\text{Ball}(v, \text{radius}=r)$$

* Equiv. r rounds of communication with unbounded msg lengths.

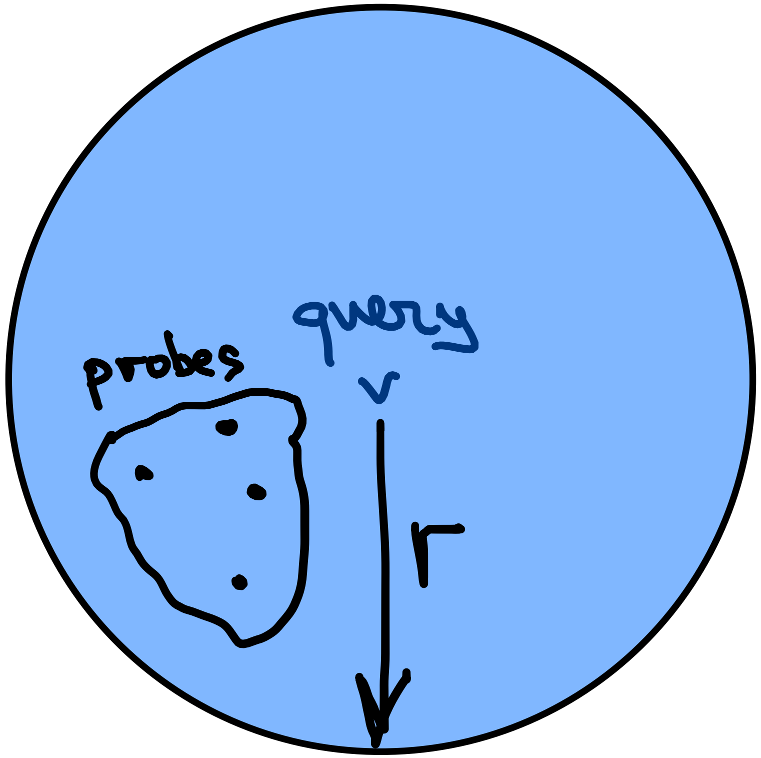
* running time is unbounded.

focus on local info.

goal: MIN # rounds.

Easy Reductions

LCA with probe radius $\leq r$ $\implies$ LOCAL with r rnds.

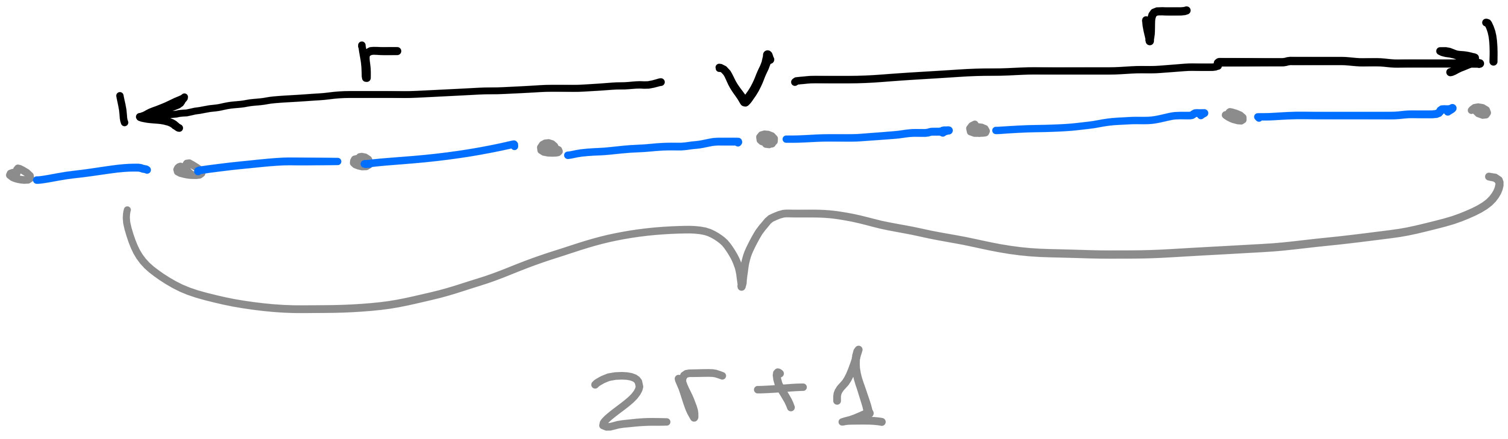


Easy Reductions

Det. distributed LOCAL with r rounds $\longrightarrow$ LCA with Δ^r probes

But, $\Delta = 2$:

$\#\text{probes} \leq 2r+1$



Classic Parallel Challenges

MIS, Vertex Cover (2-apx), Maximal Matching, $\Delta+1$ color

Greedy Seq. Alg.

1) fix a vertex ordering $v_1, v_2, \dots, v_n$

2) FOR $i=1$ TO n:

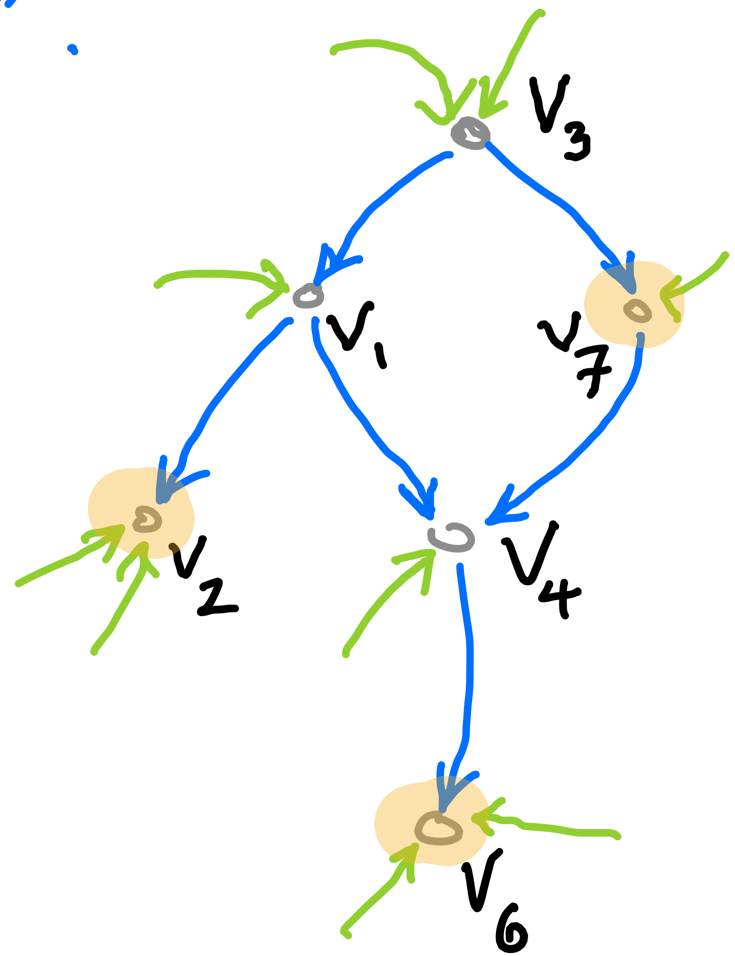
   add v_i to MIS if $MIS \cap N(v_i) = \emptyset$

Q: Simulate by LCA, LOCAL, PRAM...

PLAN: LCA Simulation of Seq. Greedy

1) compute acyclic orientation of G

2) Sim. Greedy by "DFS".



query: $v_3 \in MIS$?

probe all vertices reachable from v_3

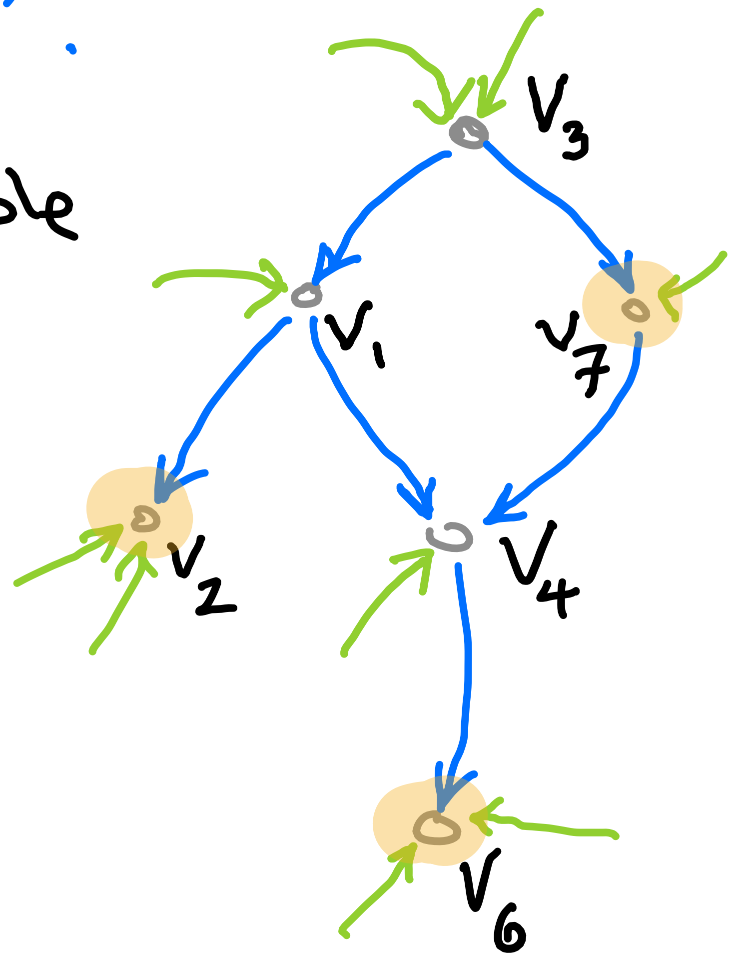
THM: DFS version agrees with greedy wrt topological ordering

PLAN: LCA Simulation of Seq. Greedy

1) compute acyclic orientation of G

2) Sim. Greedy by "DFS".

#probes = # vertices reachable from v_3.



GOAL: find orientation such that

$$\max_v |\text{reachable}(v)|$$

is small.

Acyclic Orientation

1) use ID's : $id(u) > id(v)$

$$\Downarrow$$

$$u \longrightarrow v$$

may induce long paths :

$$1 \to 2 \to 3 \to \cdots \to n$$

$$|reach(1)| = n$$

means n probes.

Acyclic Orientations

2) Random Permutations [NO-08, YYI-12, ARVX-12, MRVX-12, RV-16]

$\{r(v)\}_v$ i.i.d. unif $[0,1]$

$$u \xrightarrow{r(u)>r(v)} v$$

THM:

$$\mathbb{E}_{r,v}\left[\,|\mathrm{reach}(v)|\,\right] \leq e^{\Delta} \qquad \text{[NO-08]}$$

$$\Pr_r\left[\max_v |\mathrm{reach}(v)| > 2^{\Delta}\cdot\log n\right] < \frac{1}{n^2} \qquad \text{[RV16]}$$

$\Rightarrow$ State & # probes $\geq \Omega(\log n)$

Acyclic Orientation

3) Vertex Coloring

$\text{color}(u) > \text{color}(v) \Longrightarrow u \longrightarrow v$

reach radius $\lesssim$ # colors

$|\text{reach. set}| \lesssim \Delta^{\#\text{colors}}$

lots of distributed vertex coloring algs [L92, BE09, ...] achieve

poly(Δ) colors

poly(Δ) + $\log^* n$ rnds

Recap: LCA sim. of Seq. Greedy

1) acyclic orientation (via vertex color)
2) run DFS version.

colors computed on-the-fly during DFS.

probe radius $\leq$ # colors + rnds$\left(\begin{matrix}\text{distributed} \\ \text{LOCAL}\end{matrix}\right)$

$\Rightarrow$ # probes $\leq \Delta^{\text{probe rad.}}$

E.G. # probes $\leq \Delta^{\text{poly}(\Delta) + \lg^* n}$

LCA Simulation of Seq. Greedy (MIS)

THM: $\#\text{probes} \leq \Delta^2 \cdot \Delta^4 \cdot \log^* n$

$\text{probe radius} \leq \Delta^2 + \log^* n$

holds: MIS, $(\Delta+1)$-coloring, maximal Match., 2 apx VC

need to describe how to compute vertex coloring in LCA

(Slightly) Better LCA vertex coloring

THM: $O(\Delta^2)$ colors with

$\#\text{probes} \leq \Delta^4 \cdot \log^* n$

$\text{probe rad} \leq \log^* n$

proof:

0) First design dist. LOCAL alg.

1) Partition E to Δ^2 disjoint paths/cycles.

$$E_{i,j} \triangleq \left\{ (v,u) \;\middle|\; \begin{array}{l} u = \text{neighbor}(v,i) \\ v = \text{neighbor}(u,j) \end{array} \right\}$$

$(\deg_{E_{i,j}} \leq 2)$ zero rnds [Kuhn 09]

(Slightly) Better LCA vertex coloring

THM: $O(\Delta^2)$ colors with

$\#$probes $\leq \Delta^4 \cdot \log^* n$

probe rad $\leq \log^* n$

proof:

1) partition $E = \bigcup_{i,j} E_{i,j}$ (paths/cycles)

2) 3-color every $E_{i,j}$ ($O(\lg^* n)$ rnds [PR])

$\Longrightarrow$ 3^{Δ^2} colors for G

$\text{color}(v) = \{c_{ij}(v)\}_{i,j}$, $c_{ij}(v) \in \{1,2,3\}$

(Slightly) Better LCA vertex coloring

THM: $O(\Delta^2)$ colors with

$\#\text{probes} \le \Delta^4 \cdot \log^* n$

$\text{probe rad} \le \log^* n$

proof:

2) 3^{Δ^2}-coloring

3) reduce # colors using Linial 1-rnd algs.

$$3^{\Delta^2} \xrightarrow[\text{1-rnd}]{} 5\Delta^2 \log(3^{\Delta^2}) \xrightarrow[\text{1-rnd}]{} O(\Delta^3) \xrightarrow[\text{1-rnd}]{} O(\Delta^2)$$

(Slightly) Better LCA vertex coloring

THM: $O(\Delta^2)$ colors with

$\#$probes $\leq \Delta^4 \cdot \log^* n$

probe rad $\leq \log^* n$

proof:

Sim. LOCAL coloring alg by LCA:

path/cycle: $\log^* n$ rnds $\mapsto 2 \cdot \log^* n$ probes.

$\forall$ vertex incident to $\leq \Delta$ paths/cycles

$\Rightarrow 2\Delta \cdot \log^* n$ probes.

3 distributed rnds $\Rightarrow \Delta^3 \cdot 2\Delta \cdot \log^* n$ probes!

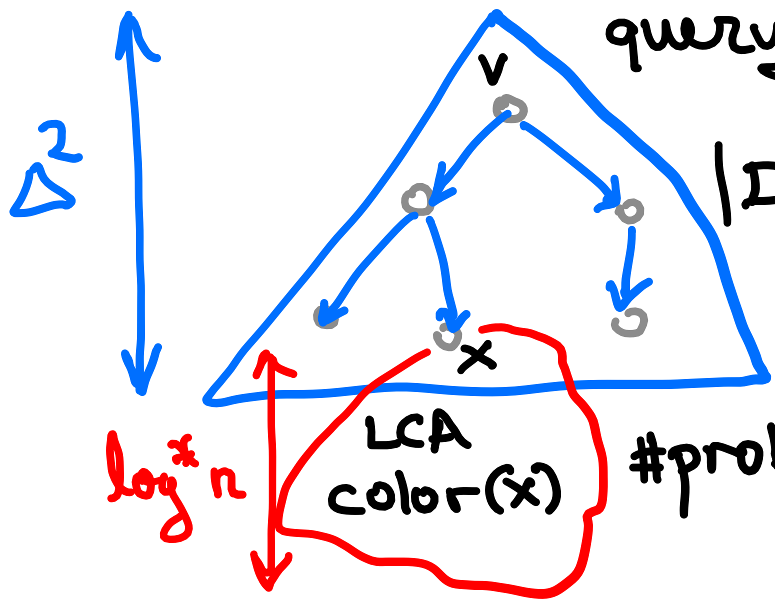
LCA Simulation of Seq. Greedy (MIS)

THM: $\#\text{probes} \leq \Delta^2 \cdot \Delta^4 \cdot \log^* n$

$\text{probe radius} \leq \Delta^2 + \log^* n$

holds: MIS, $(\Delta+1)$-coloring, maximal Match., 2 apx VC

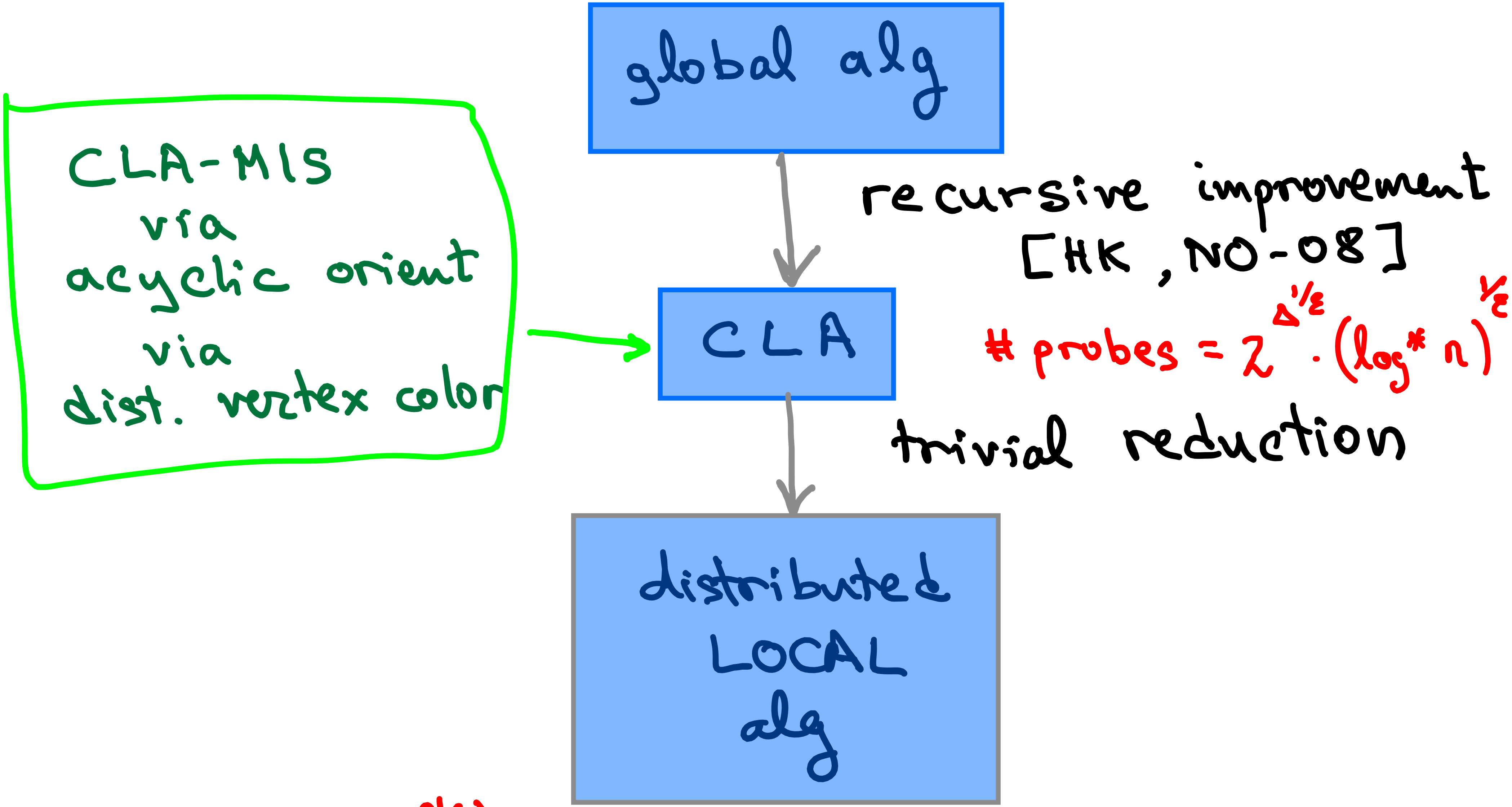
proof:



$|DFS| \leq \Delta^{(\Delta^2)}$

$\#\text{probes} \leq \Delta^4 \cdot \log^* n$

$(1-\varepsilon)$-apx max. match. in distributed LOCAL



#rnds = $\Delta^{O(1/\varepsilon)} + \varepsilon^{-2} \cdot \log^* n$

MORE LCA's etc.

1) random graph generation
   * pref. attach. [ELMR]
   * $G(n,p)$ with random/next neigh...

2) $(2+\varepsilon)$-apx max. matching with
   $\Delta^{\Delta} \cdot \log^* n$ probes [Fischer 17]

3) SLOCAL: generalize greedy seq
   to Balls of radius r. [GKM 16]