Breaking the Linear-Memory Barrier in Massively Parallel Computing

Fast MIS on Trees with n^{ε} Memory per Machine

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joint work with Sebastian Brandt and Jara Uitto

Model:

Sublinear-Memory MPC

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Sublinear-Memory MPC

large-scale data

does not fit onto a single machine

centralized algorithms not applicable

parallel computation framework

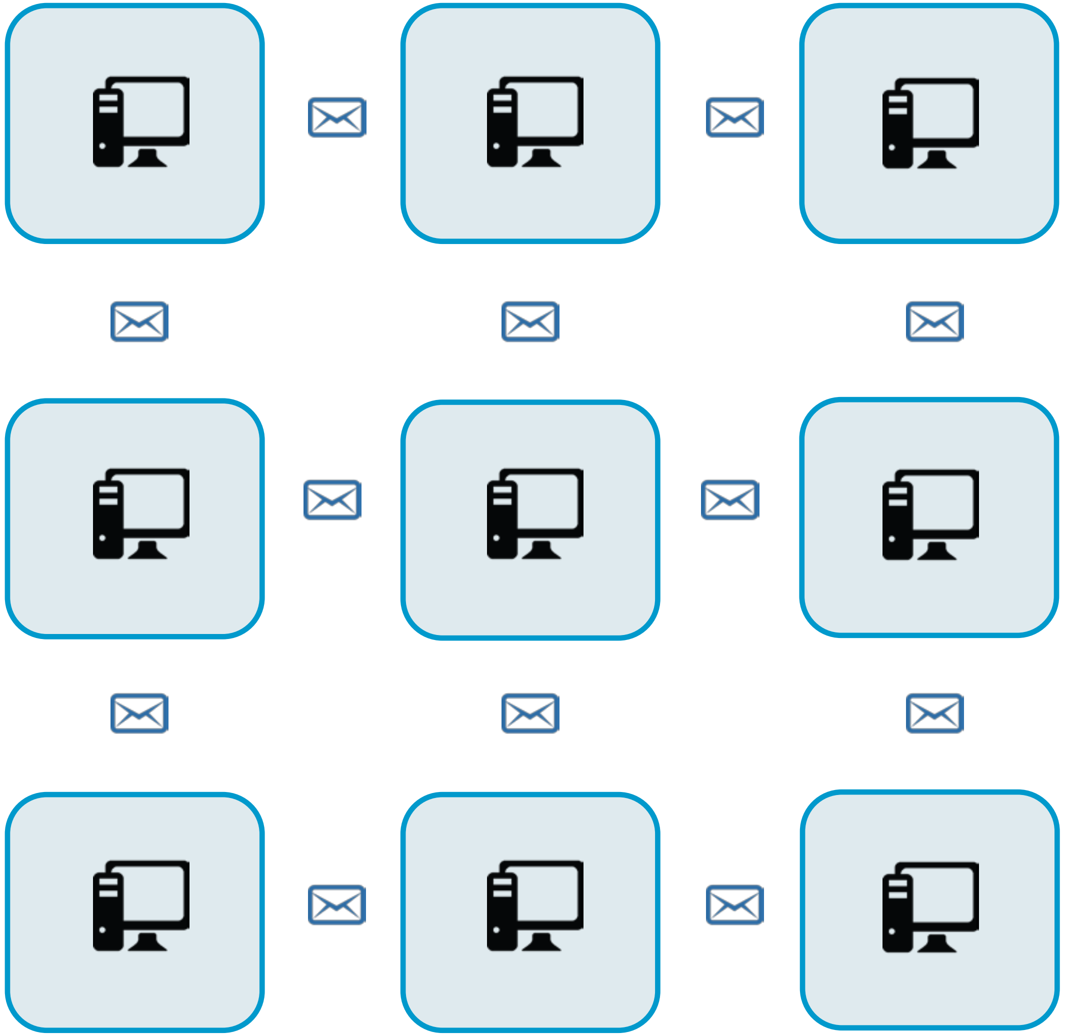
Massively Parallel Computation

Karloff, Suri, Vassilvitskii [SODA'10]

Goodrich, Sitchinava, Zhang [ISAAC'11]
Beame, Koutris, Suciu [PODS'14]
Andoni, Nikolov, Onak, Yaroslavtsev [STOC'14]
Beame, Koutris, Suciu [JACM'17]
Czumaj, Łacki, Madry, Mitrović, Onak, Sankowski [arXiv'17]

Massively Parallel Computation (MPC) Model

M machines with S words local memory



synchronous rounds consisting of

- **local computation**
  - in parallel at every machine
  - unbounded computational power
- **global communication**
  - all-to-all
  - for every machine:
    sent messages $\ll S$
    received messages $\ll S$

complexity: number of rounds

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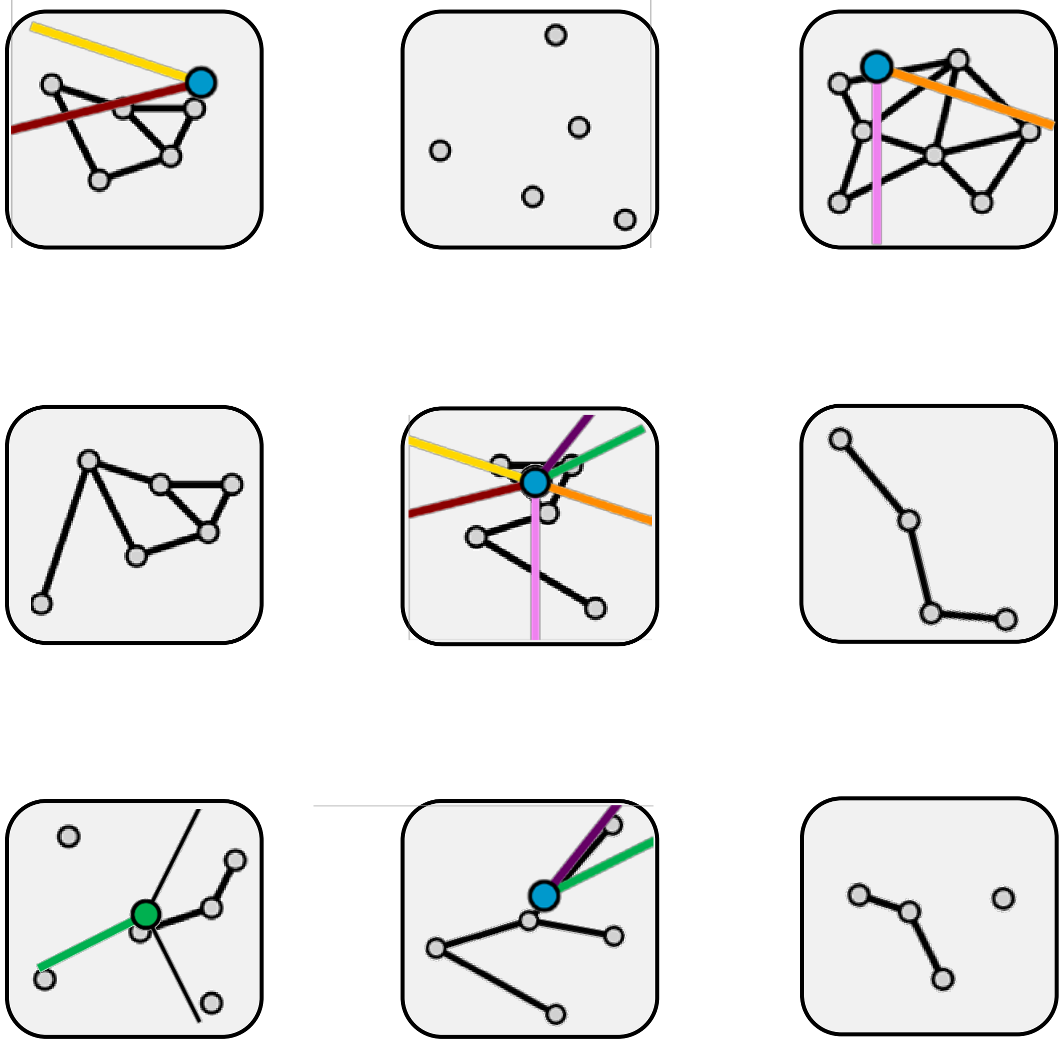
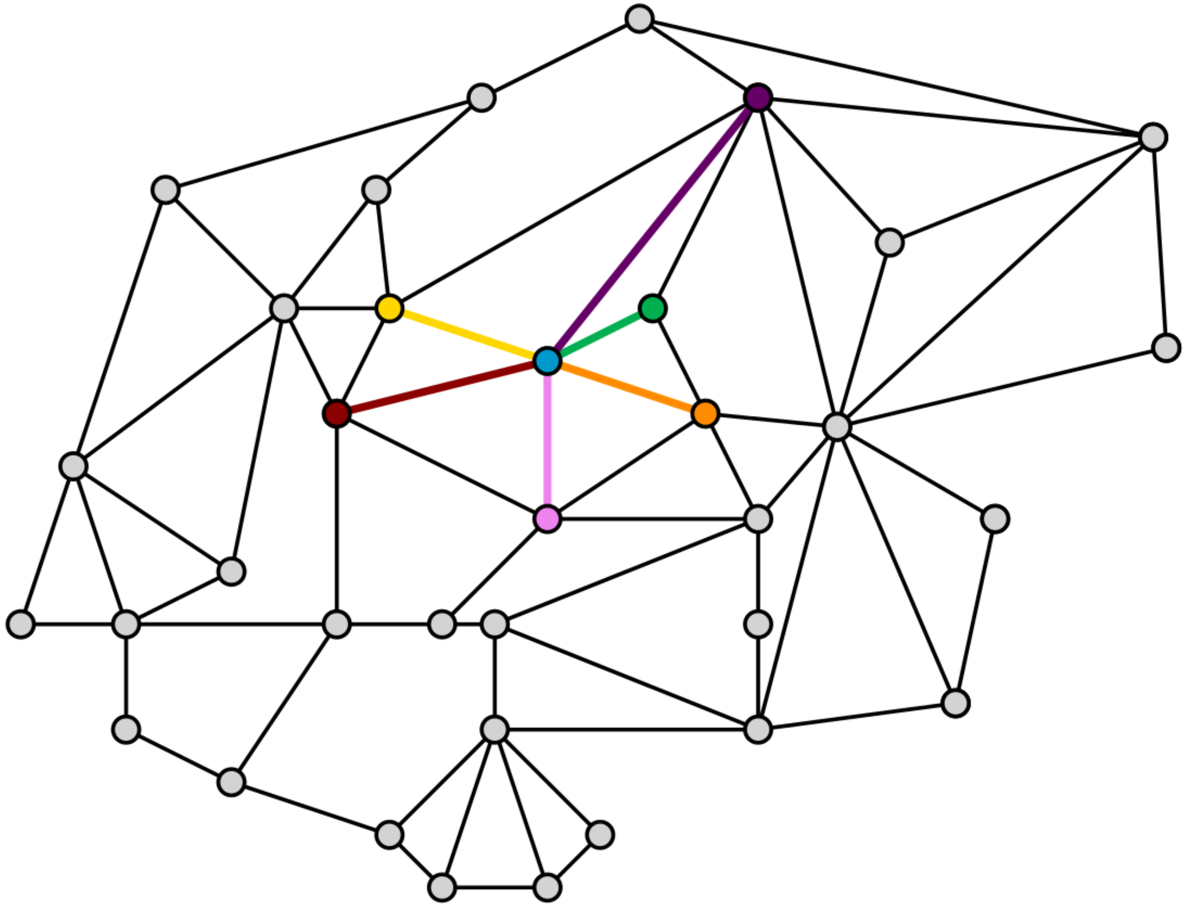




n nodes, m edges, maximum degree Δ

Note: edge may appear on two machines!

Model:

Sublinear-Memory MPC

Model:

<u>Sublinear-Memory</u> MPC

Parameter Choice for MPC

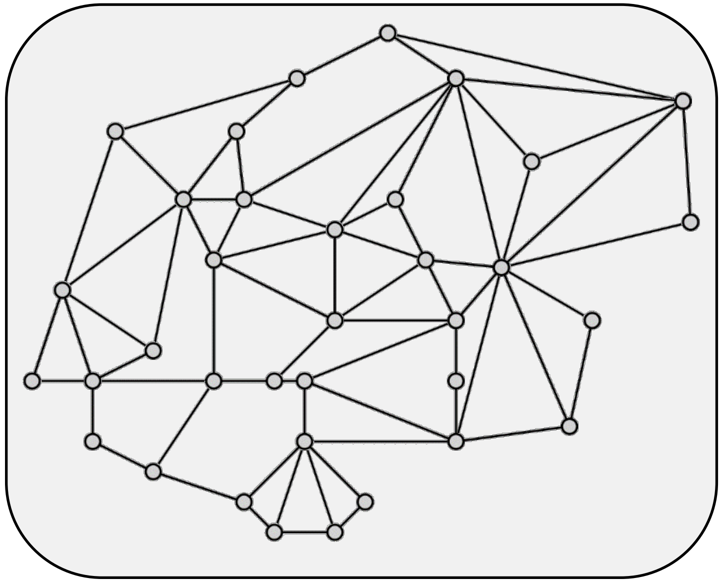
M machines with S words local memory and $M \cdot S = \Theta(m+n)$

Linear Memory $S = \tilde{O}(n)$

usual assumption for traditional MPC algorithms
single machine can see all the nodes

unrealistic for large-scale data!

- $\tilde{O}(n)$ might be prohibitively large
- sparse graphs admit trivial solution



Algorithms have been stuck at this linear-memory barrier!
Fundamentally?

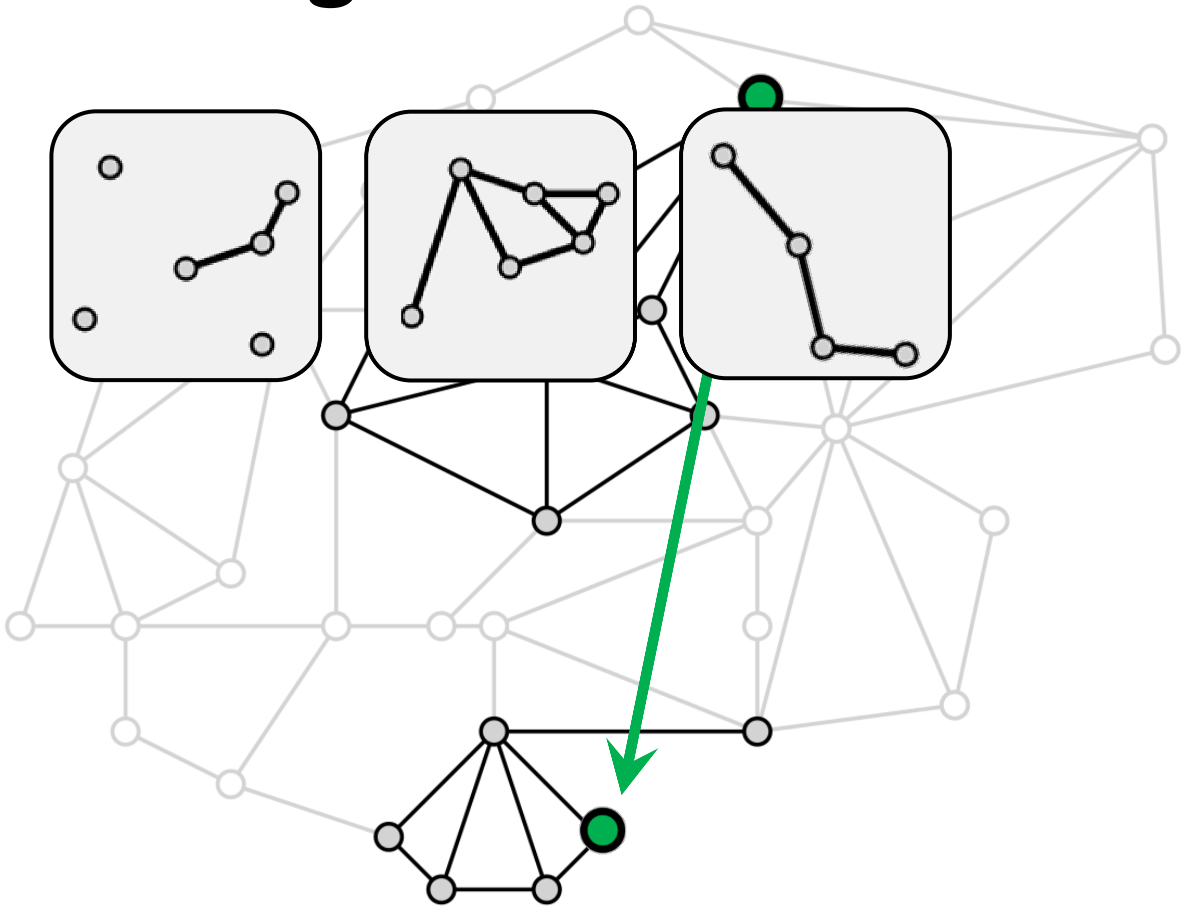
Breaking the Linear-Memory Barrier:

Efficient Sublinear-Memory MPC Algorithms

$\boldsymbol{S = O(n^{\varepsilon})}$ local memory

$\boldsymbol{M = O(m/n^{\varepsilon})}$ machines

poly log log $\boldsymbol{n}$ rounds



imposed locality:
machines see only subset of nodes, regardless of sparsity of graph

our approach to cope with locality:
enhance **LOCAL algorithms** with **global communication**

- exponentially faster than LOCAL algorithms due to shortcuts
- polynomially less memory than traditional MPC algorithms

$\tilde{O}(\sqrt{\log n})$ rounds

$S = O(n^{\varepsilon})$ memory

Lenzen, Wattenhofer
[PODC'11]

$O(\log \log n)$ rounds

$S = \tilde{O}(n)$ memory

Ghaffari, Gouleakis, Konrad, Mitrovic, Rubinfeld
[PODC'18]

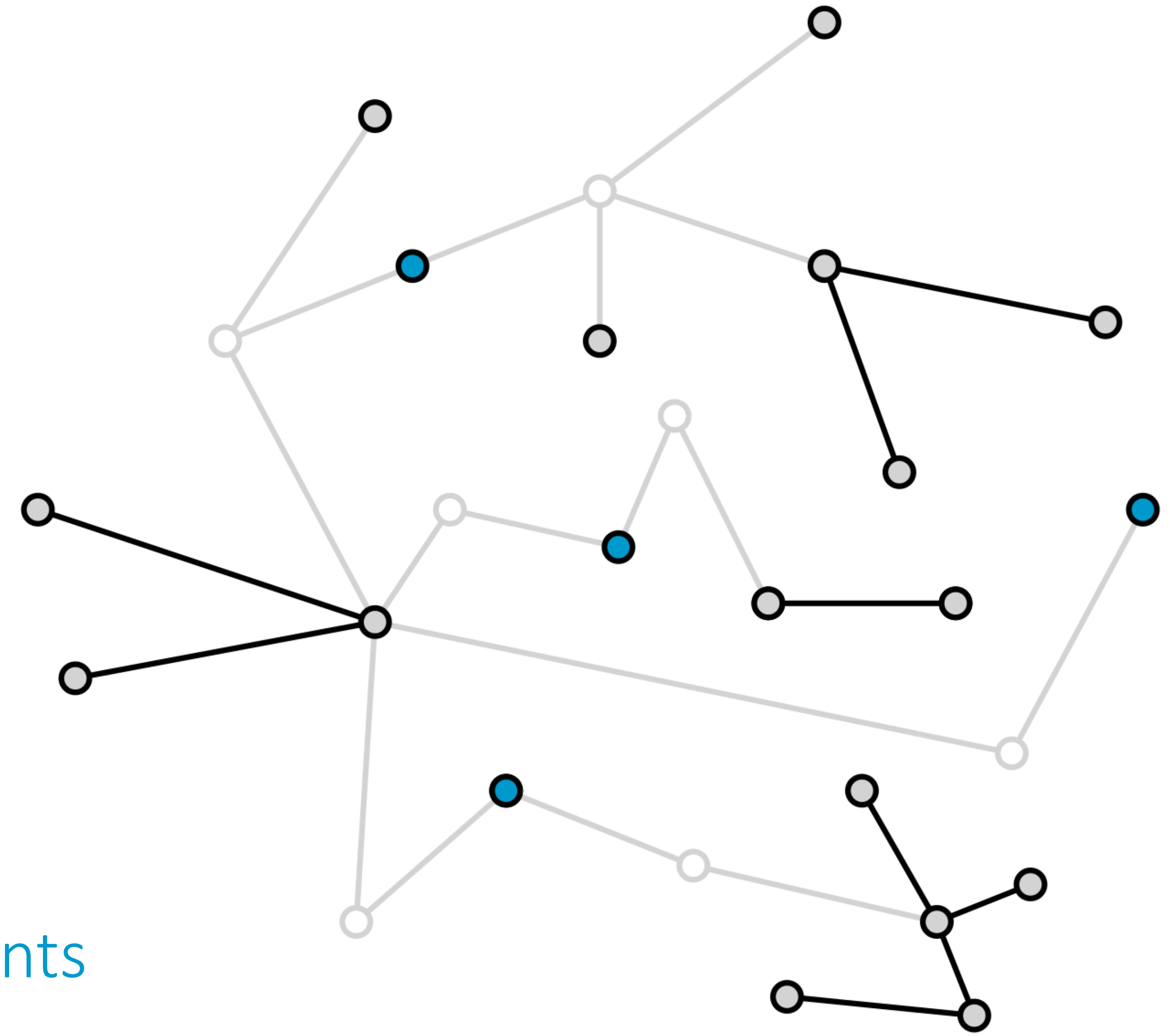
Our Result:

MPC Algorithm for MIS on Trees

randomized $O(\log^3 \log n)$-round MPC algorithm with $S = O(n^{\varepsilon})$ memory that w.h.p. computes MIS on trees.

Brandt, F., Uitto 2018

Algorithm Outline



1) Shattering

break graph into small components

i) Degree Reduction

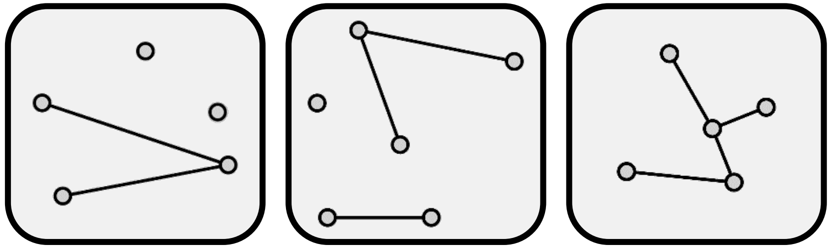
ii) LOCAL Shattering

2) Post-Shattering

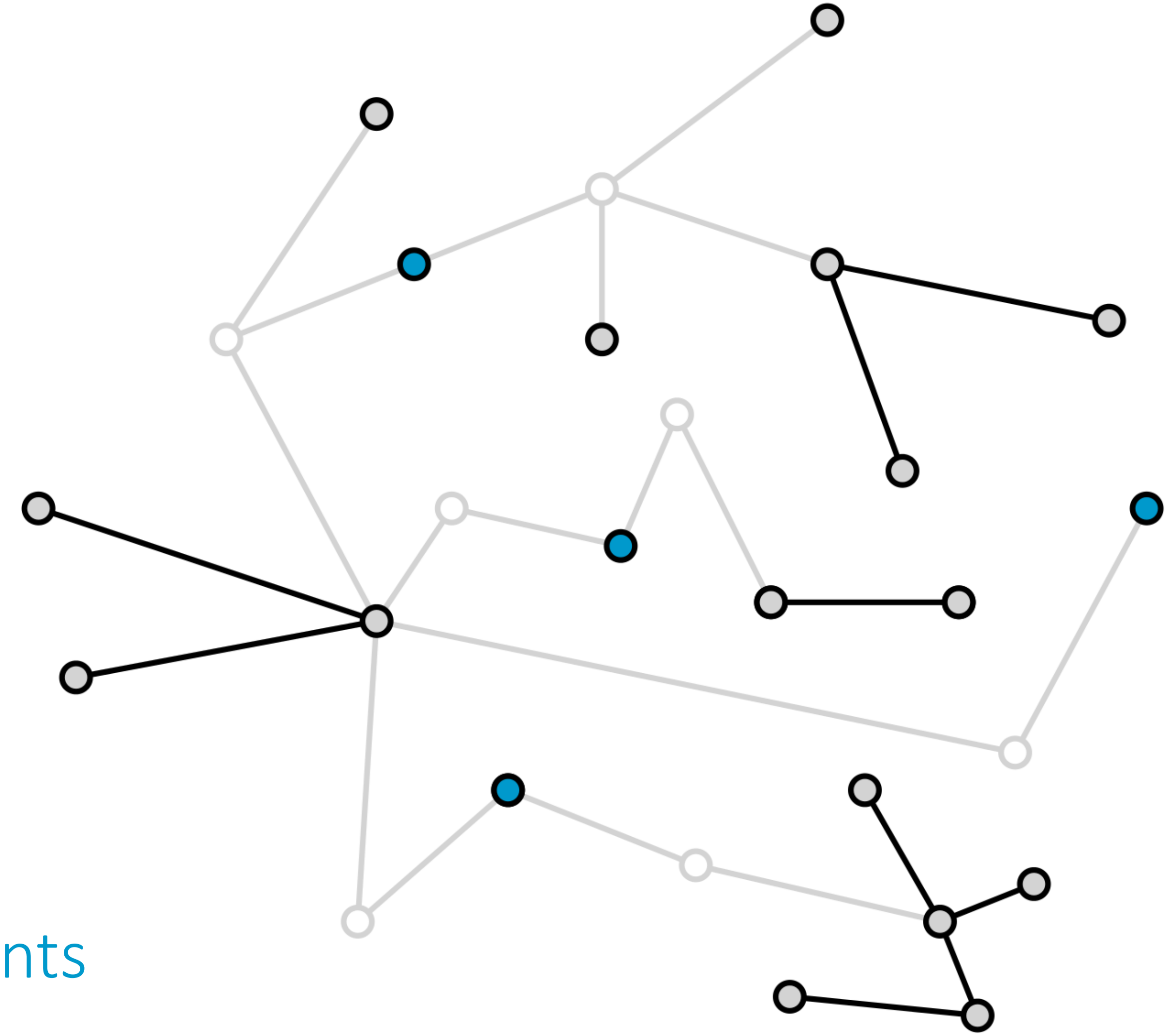
solve problem on remaining components

i) Gathering of Components *Distributed Union-Find*

ii) Local Computation



Algorithm Outline



1) Shattering

break graph into small components

i) **Degree Reduction**

ii) **LOCAL Shattering**

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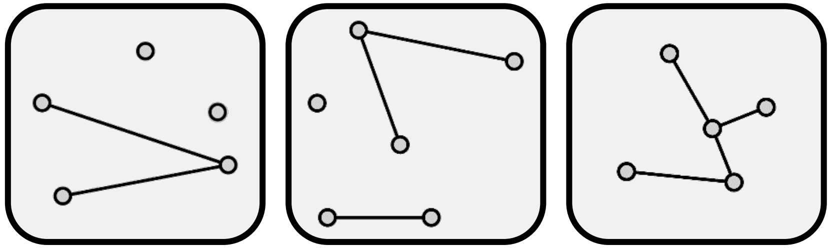
solve problem on remaining components

i) **Gathering of Components** *Distributed Union-Find*



ii) **Local Computation**

Polynomial Degree Reduction:
Subsample-and-Conquer

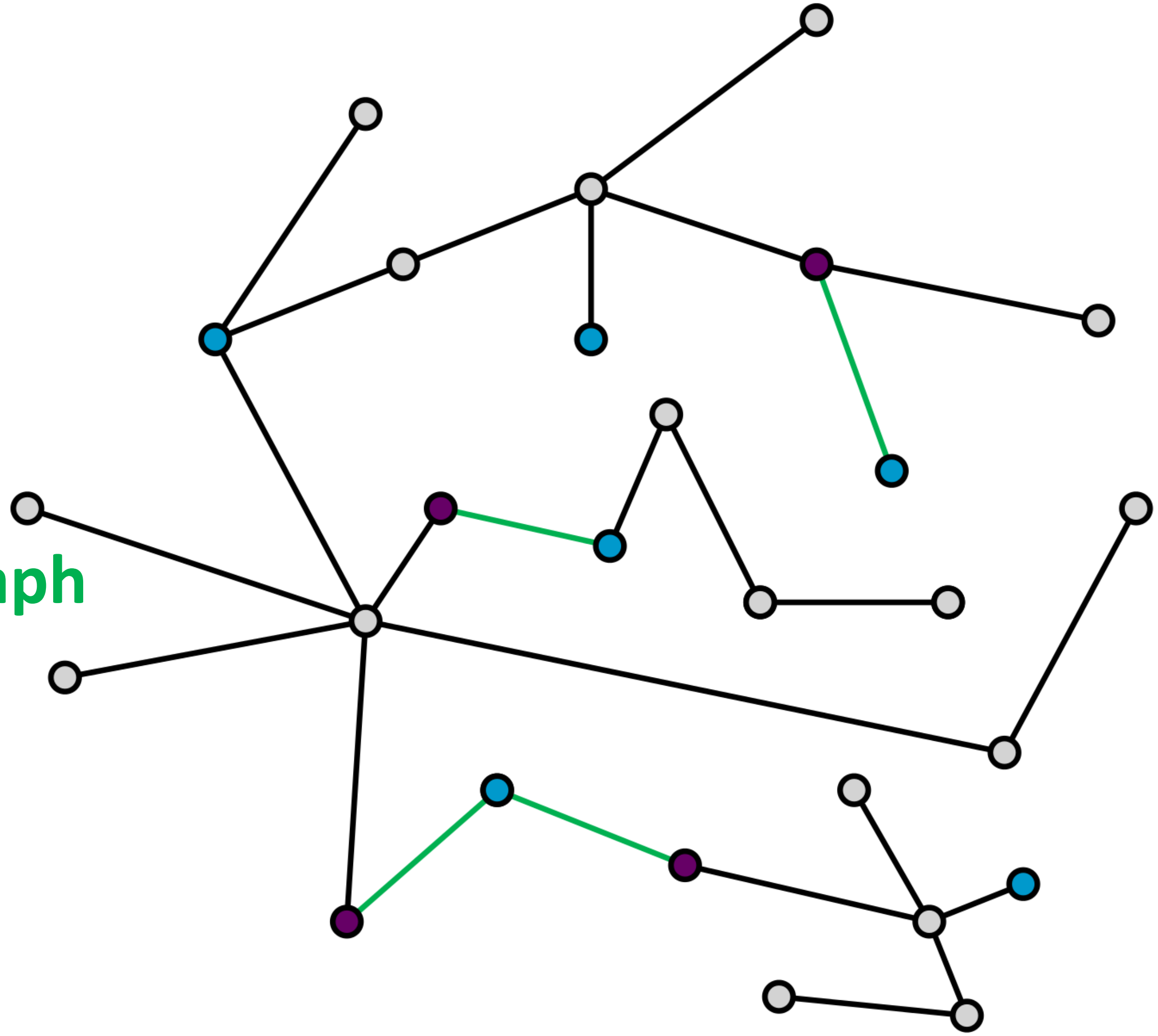
Subsample

subsample **nodes** independently

Conquer

compute **random MIS** in **subsampled graph**

- gather connected components
- locally compute random 2-coloring
- add a color class to MIS



non-subsampled **high-degree node**

- w.h.p. has many **subsampled neighbors**
- thus w.h.p. has at least one **MIS neighbor**
- hence will be removed from the graph