Decentralized Submodular Maximization: Bridging Discrete and Continuous Settings

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Joint work with: Aryan Mokhtari Amin Karbasi

Workshop on Local Algorithms (WOLA)
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Penn
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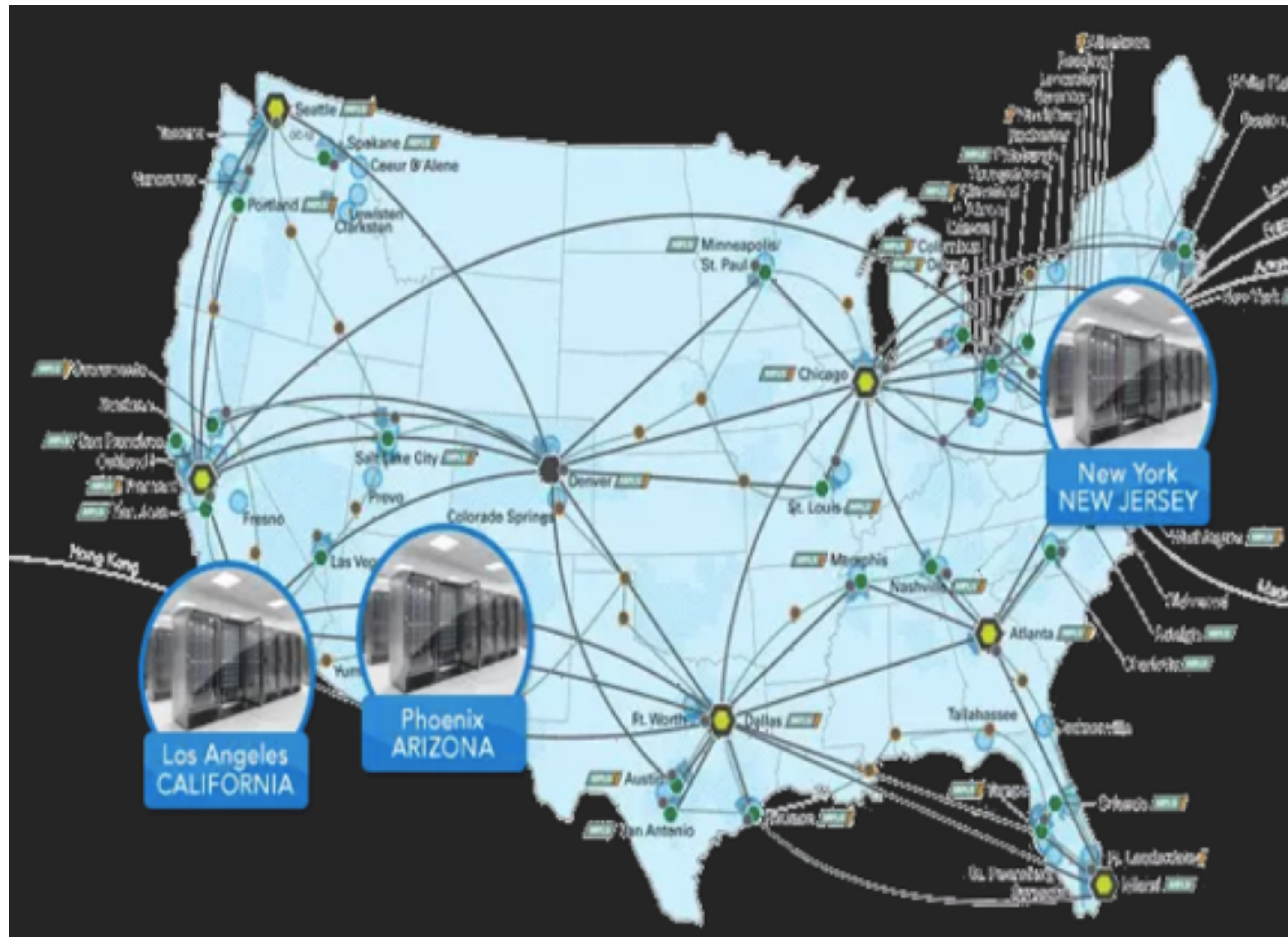
Yale

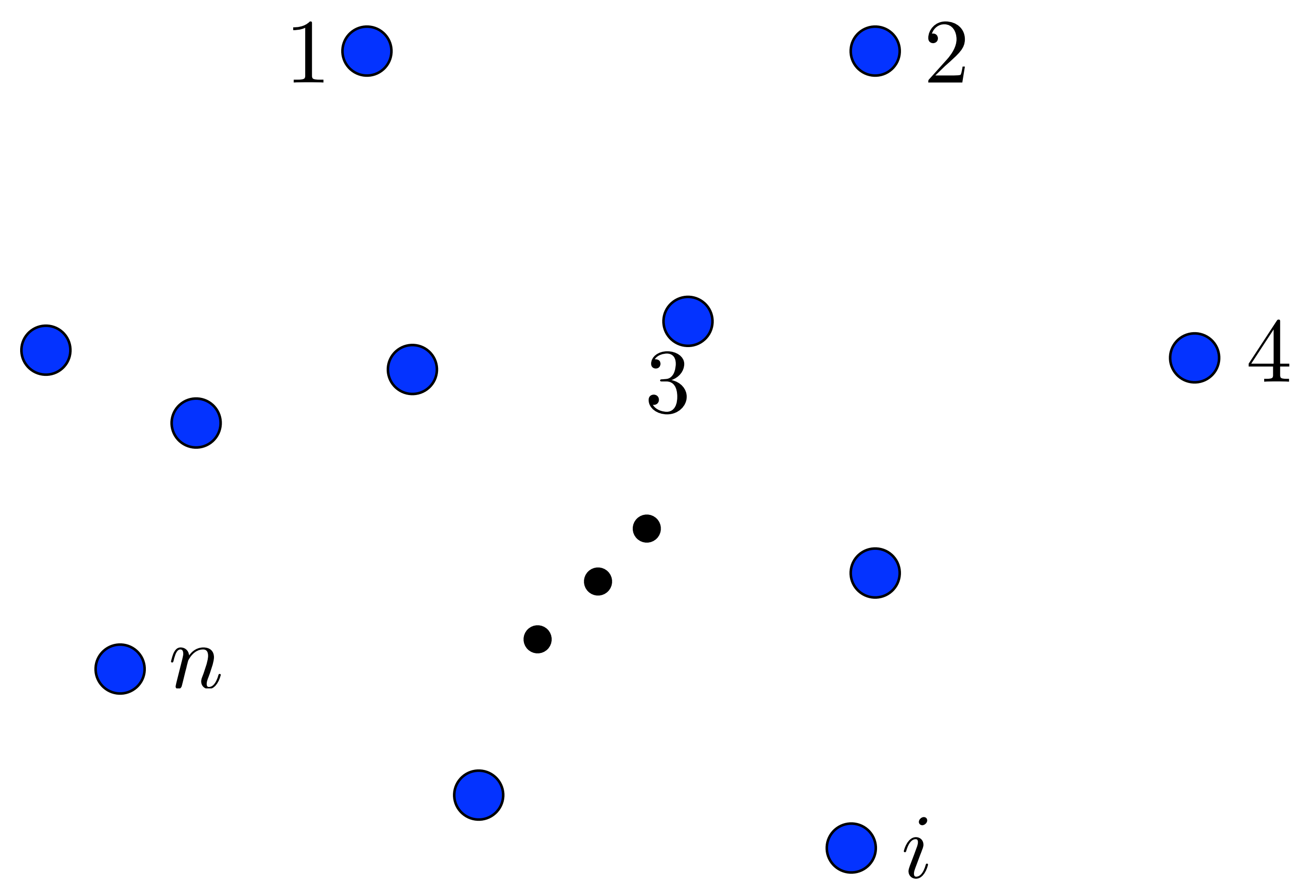
The Decentralized Setting

- Computing units distributed geographically
- Perform a global task (e.g. optimize a global function)
- Each unit has only access to a small portion of the problem (data)
- Data can’t be shared due to communication, storage, privacy constraints
- So the units have to cooperate/communicate with each other …

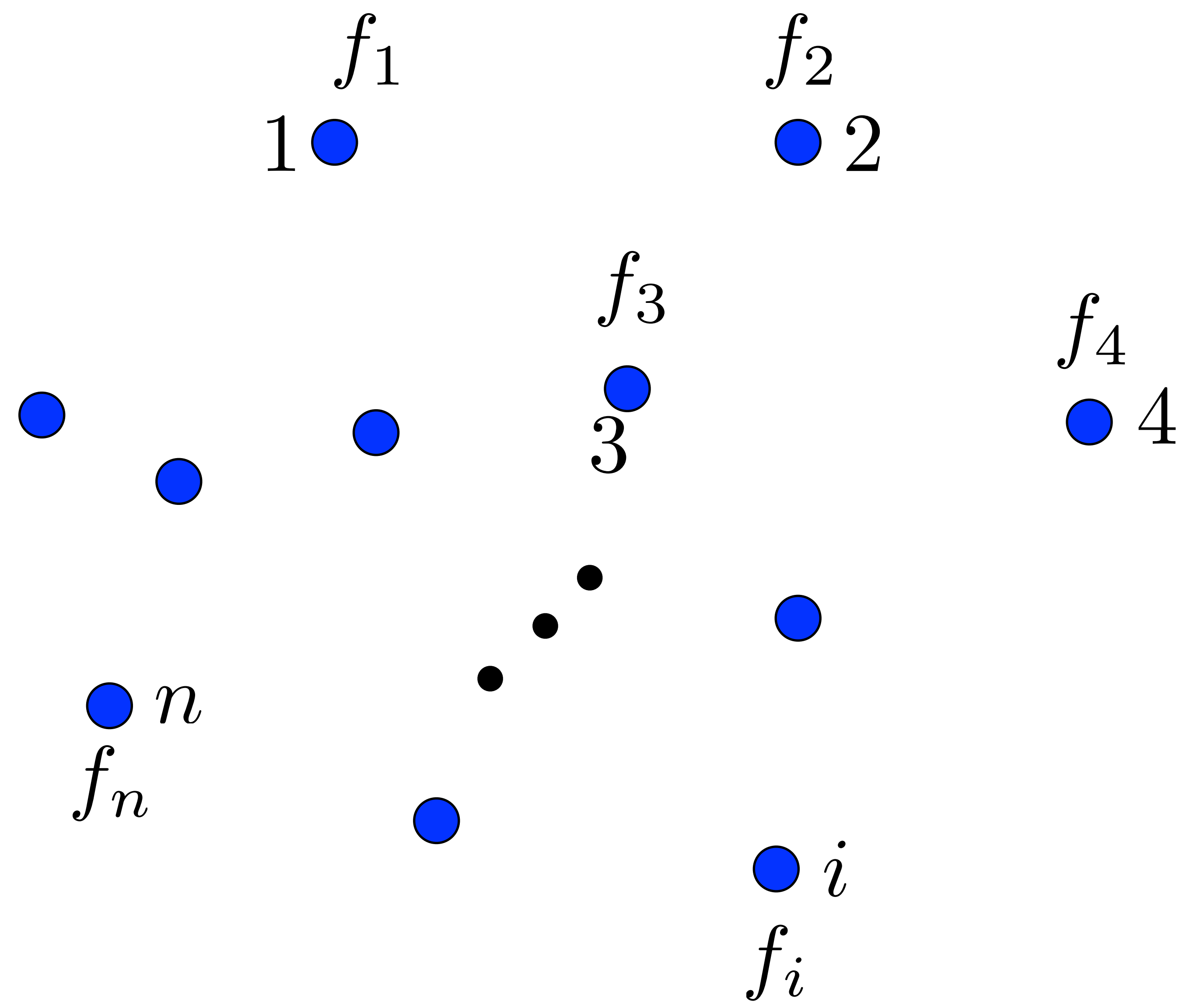


Decentralized Submodular Maximization

- n nodes (agents, computing units, etc)



Decentralized Submodular Maximization



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- Each node has access to a local function f_i

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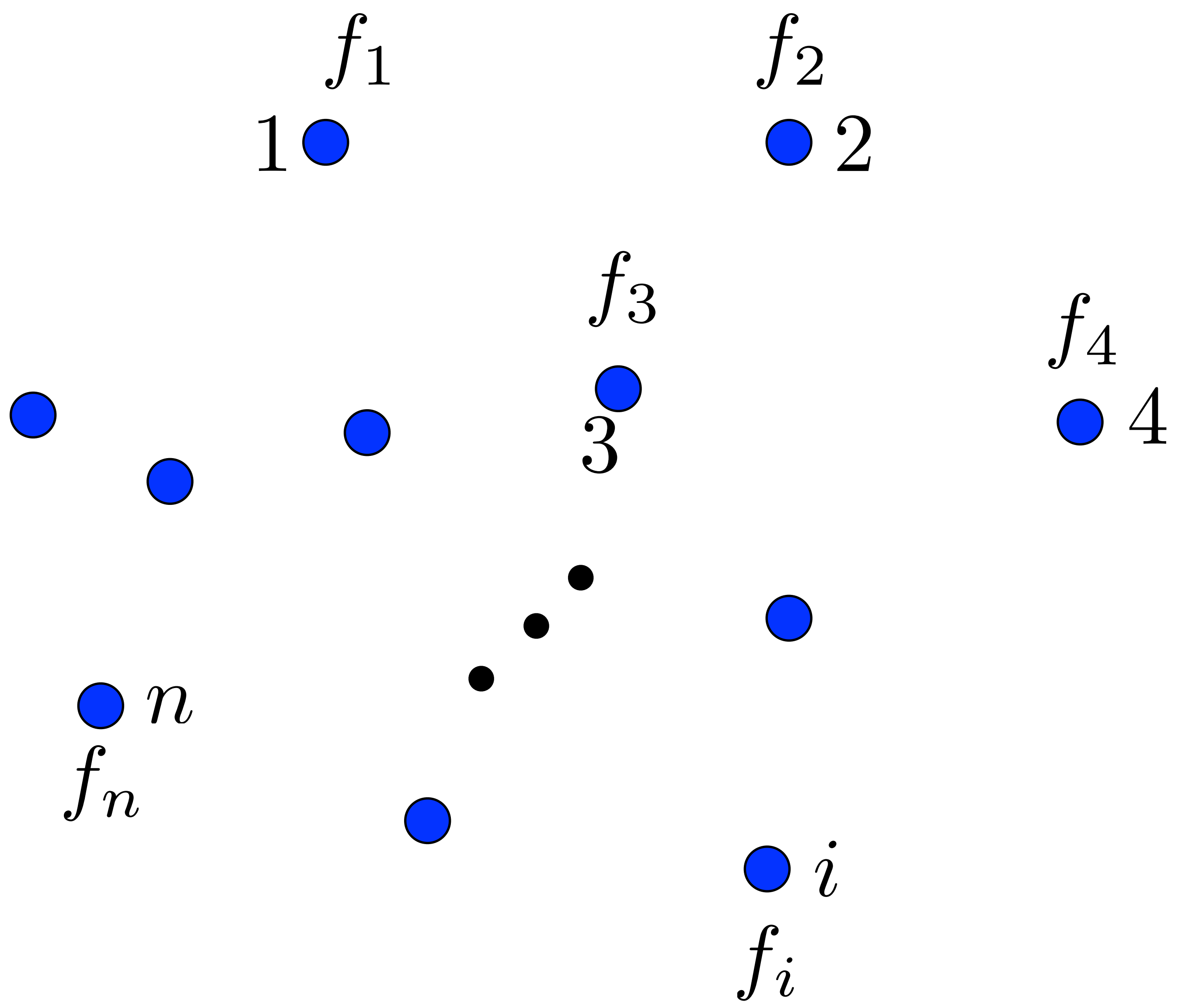

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- $f_i : 2^V \to \mathbb{R}$ is monotone and submodular

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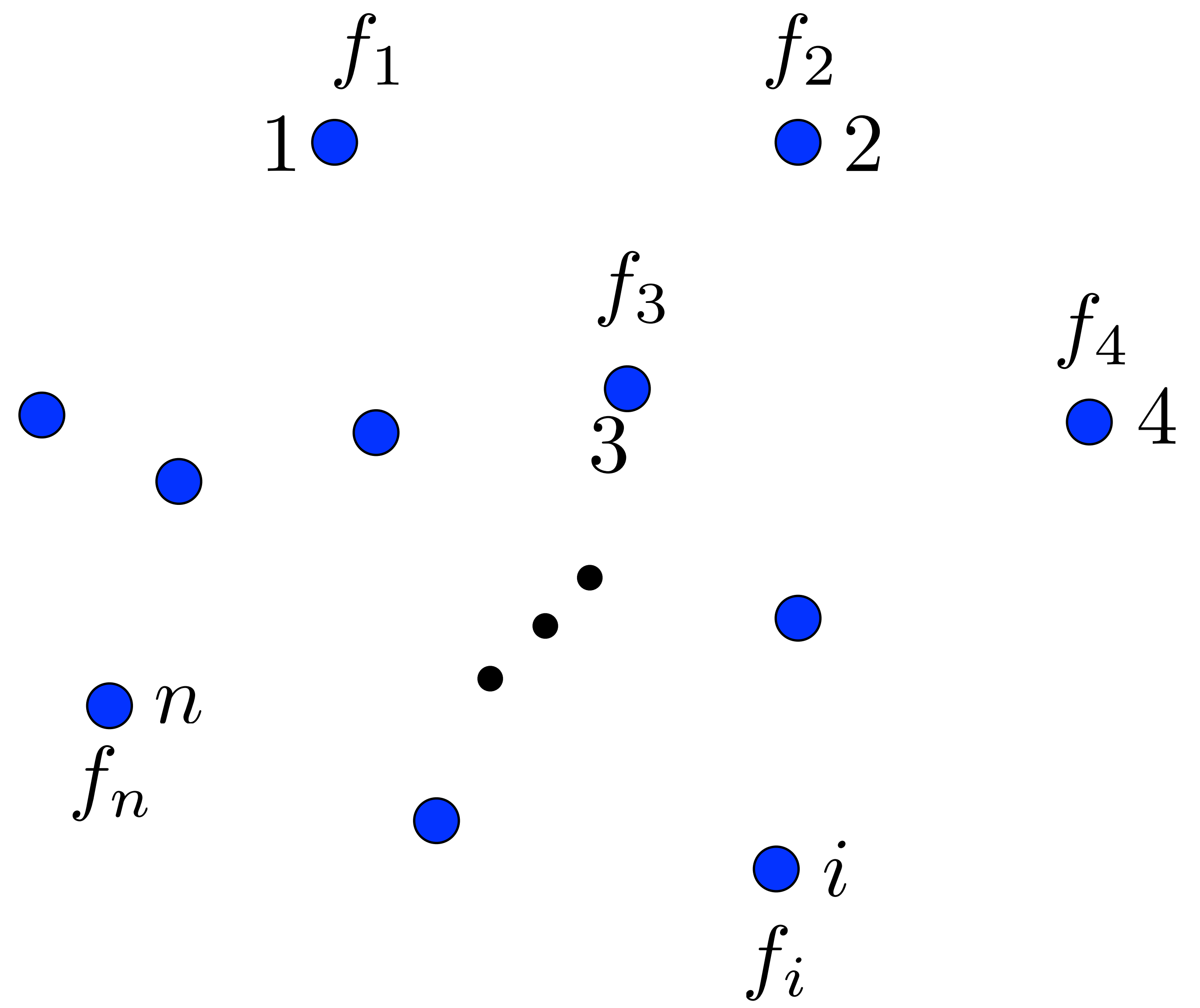

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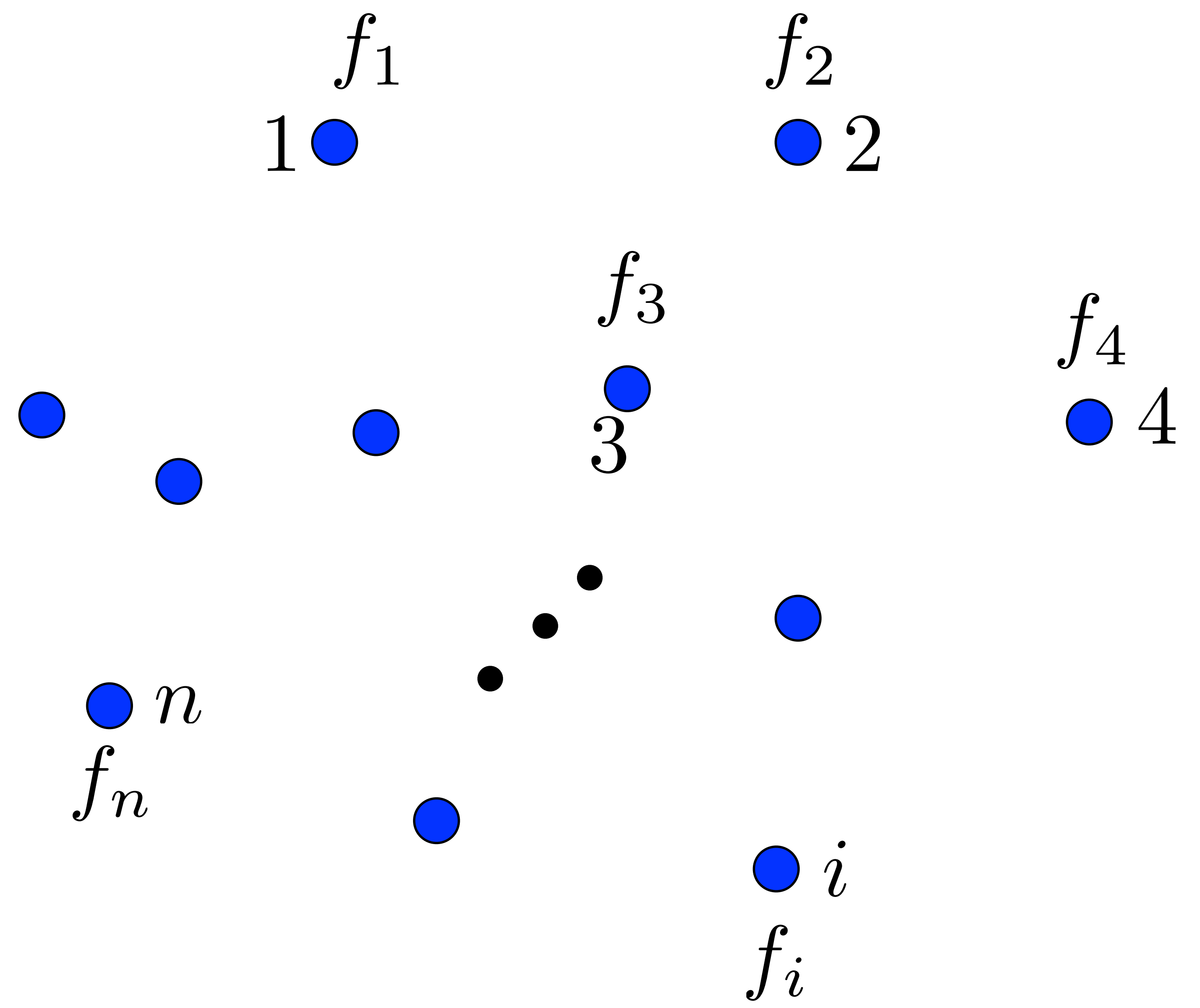

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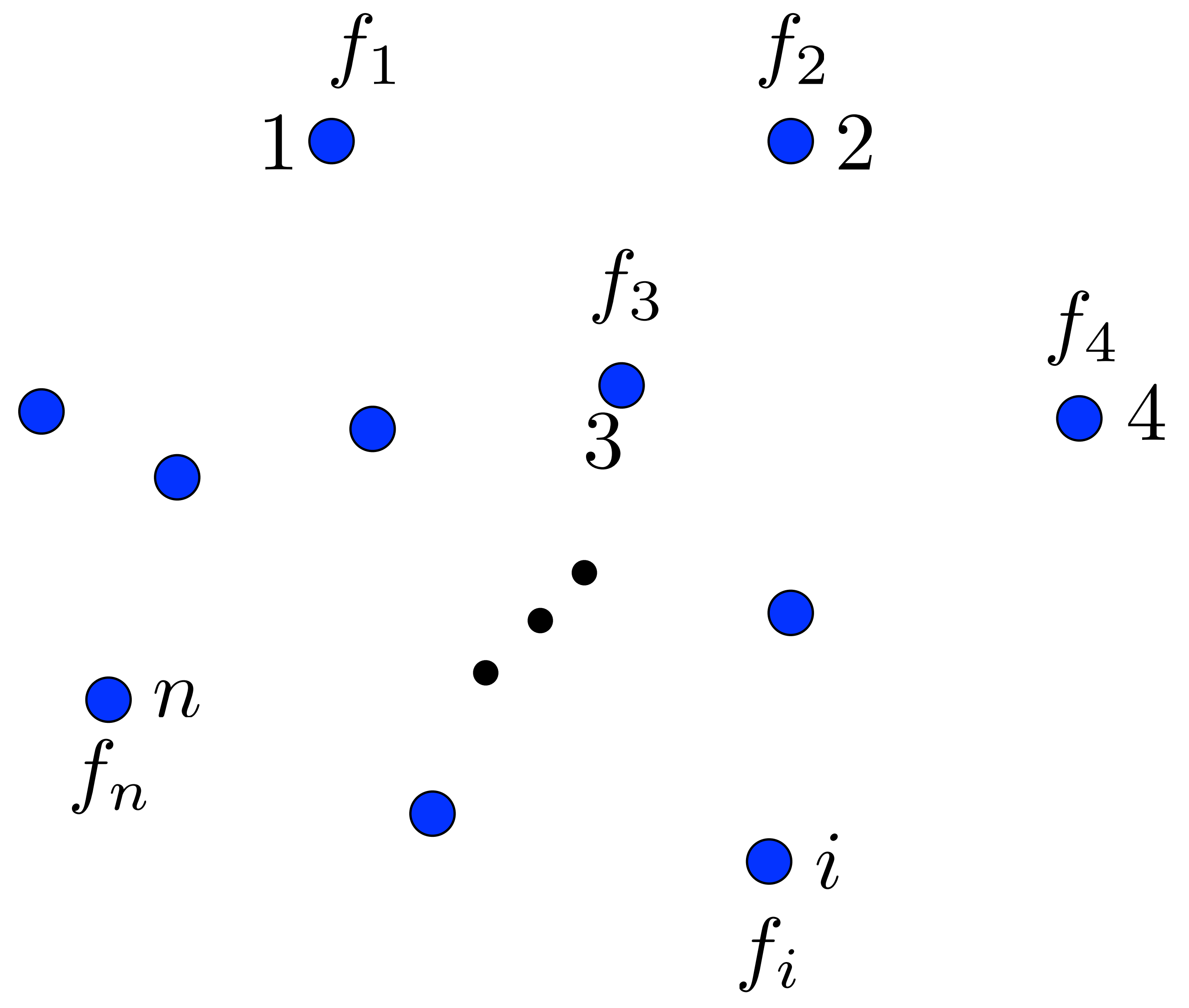

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global objective

k-cardinality constraint

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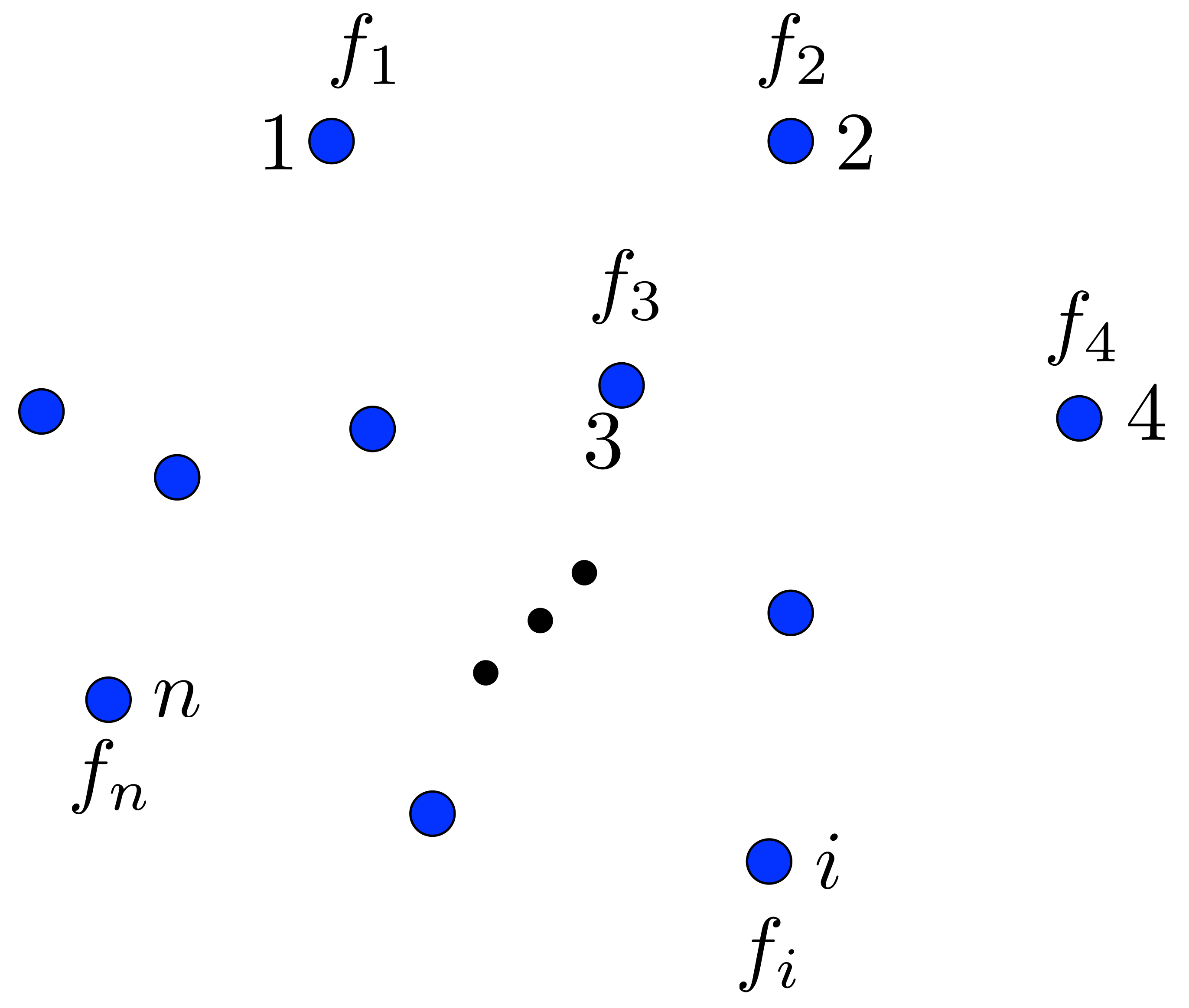

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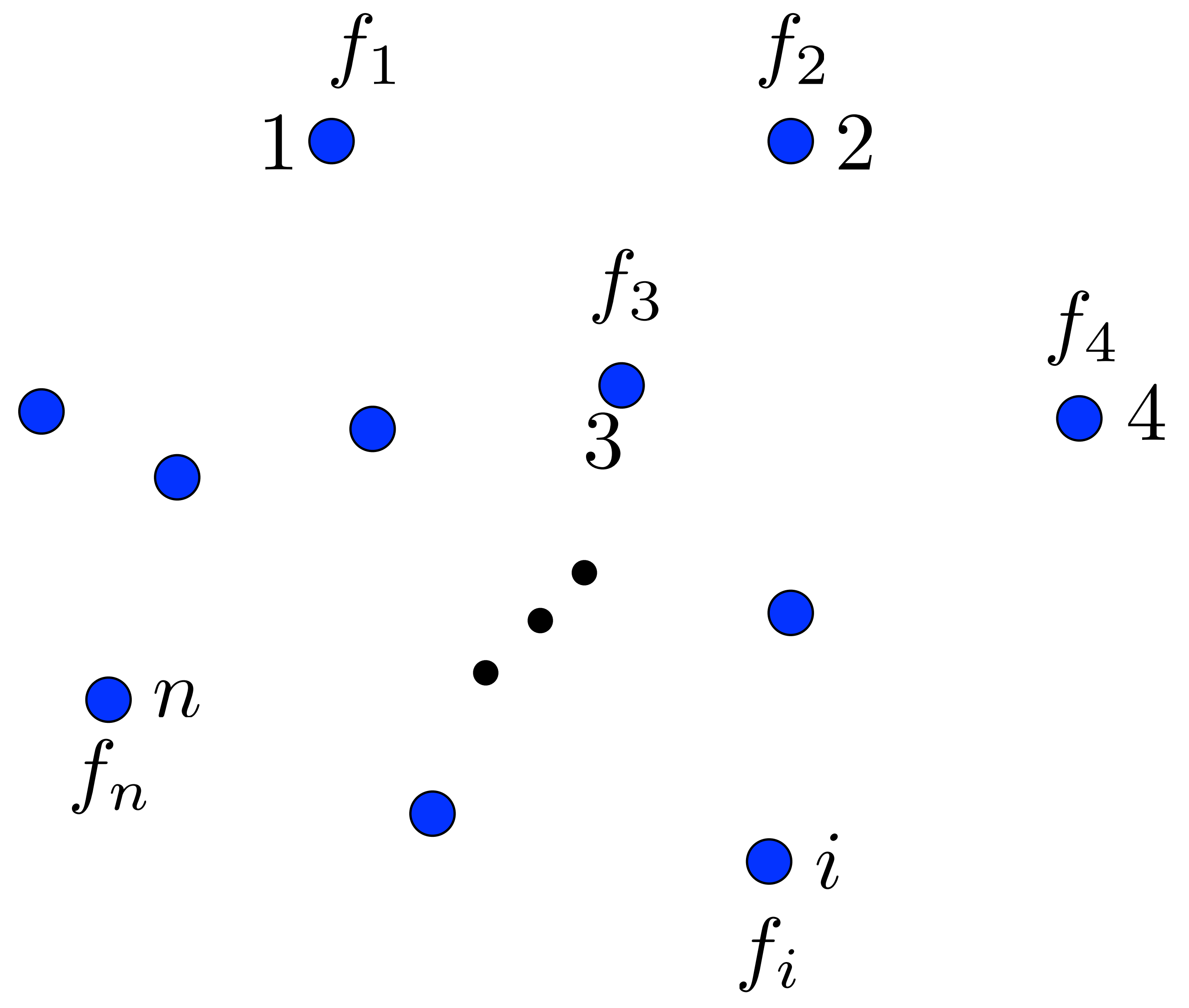
  - $\frac{1}{n}\sum_{i=1}^{n} f_i(S)$ → global objective
  - $|S| \leq k$ → k-cardinality constraint
- Challenge: Each node only knows its local f_i

Decentralized Submodular Maximization



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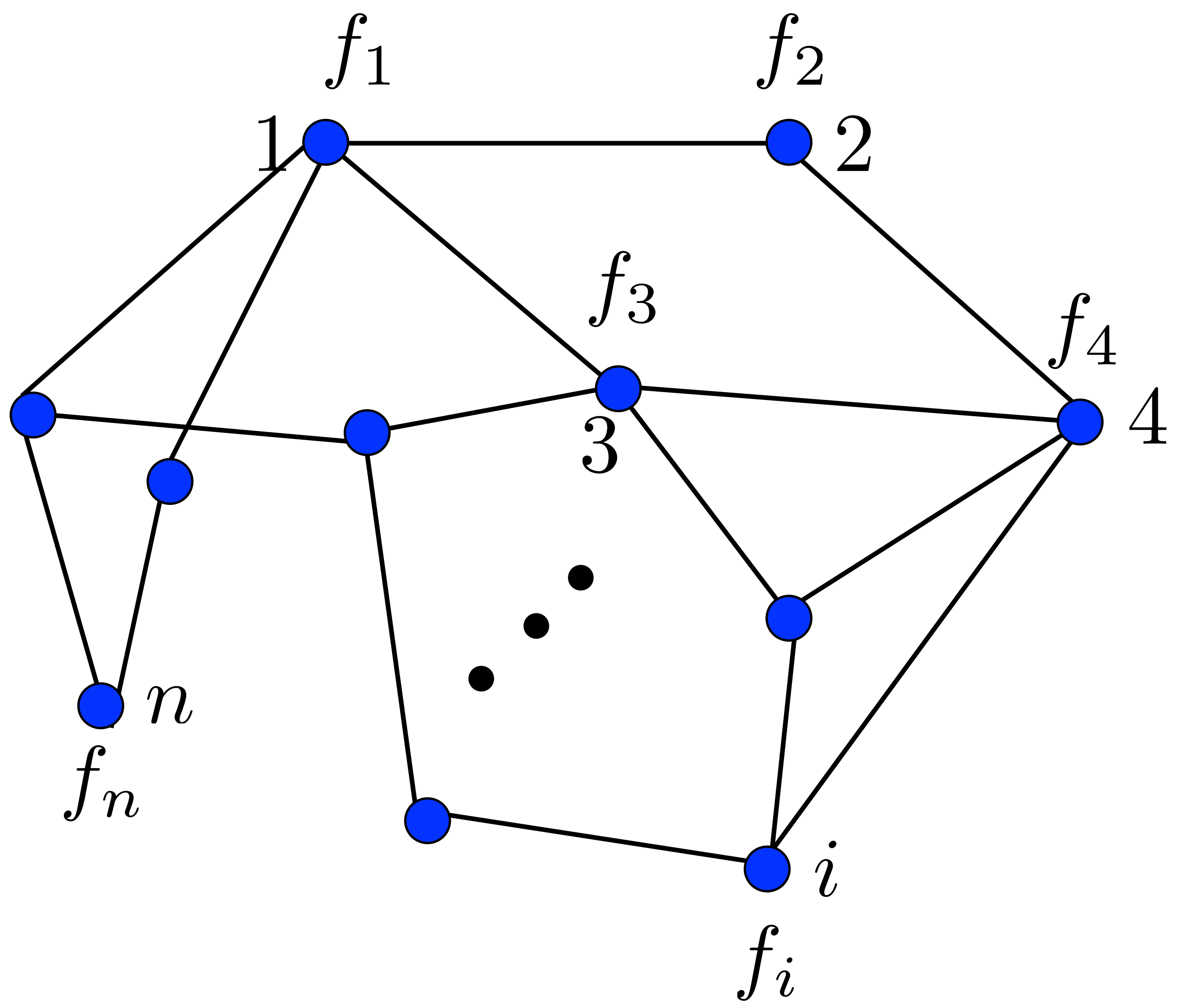
Decentralized Submodular Maximization



$G = (N, E)$

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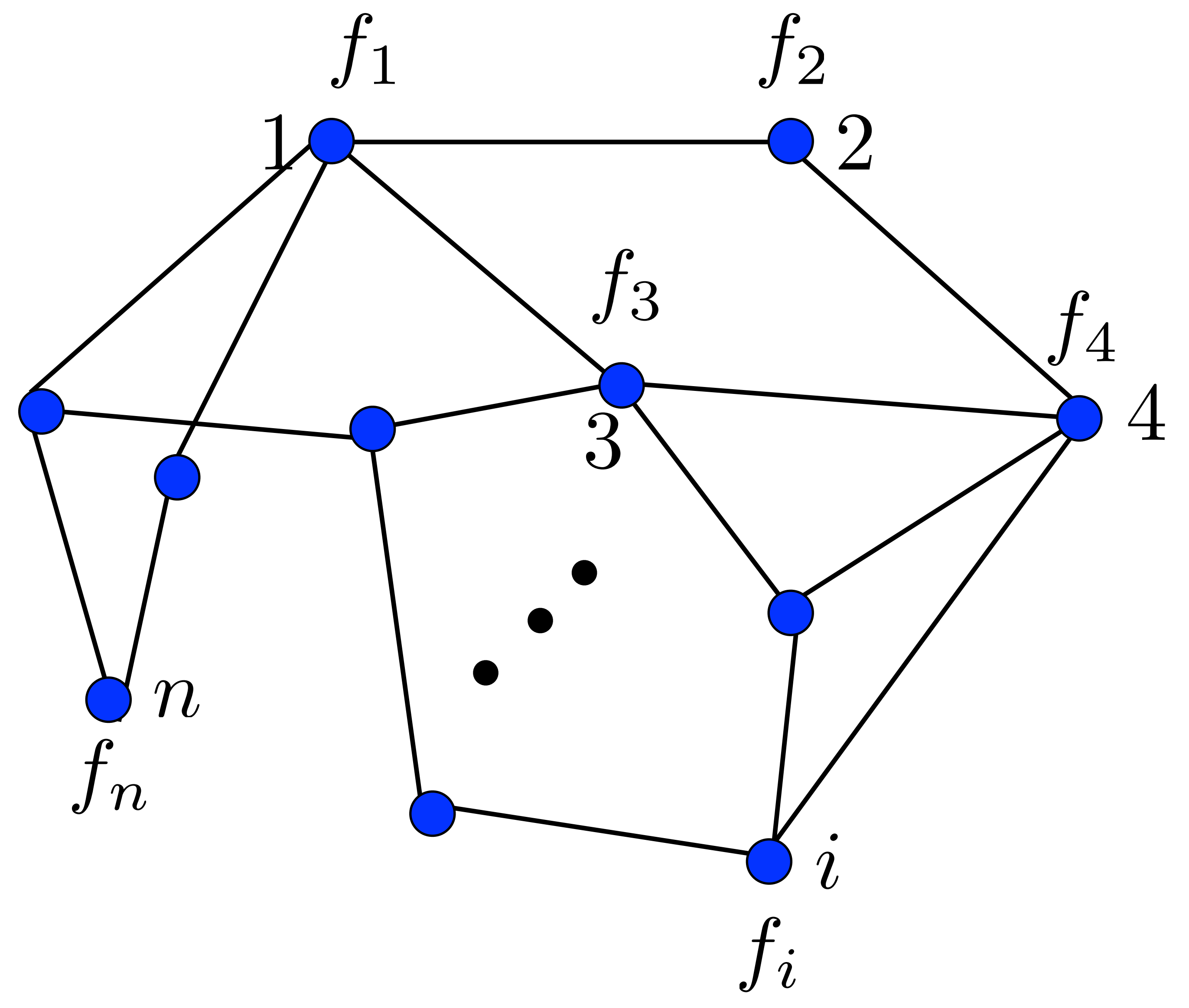

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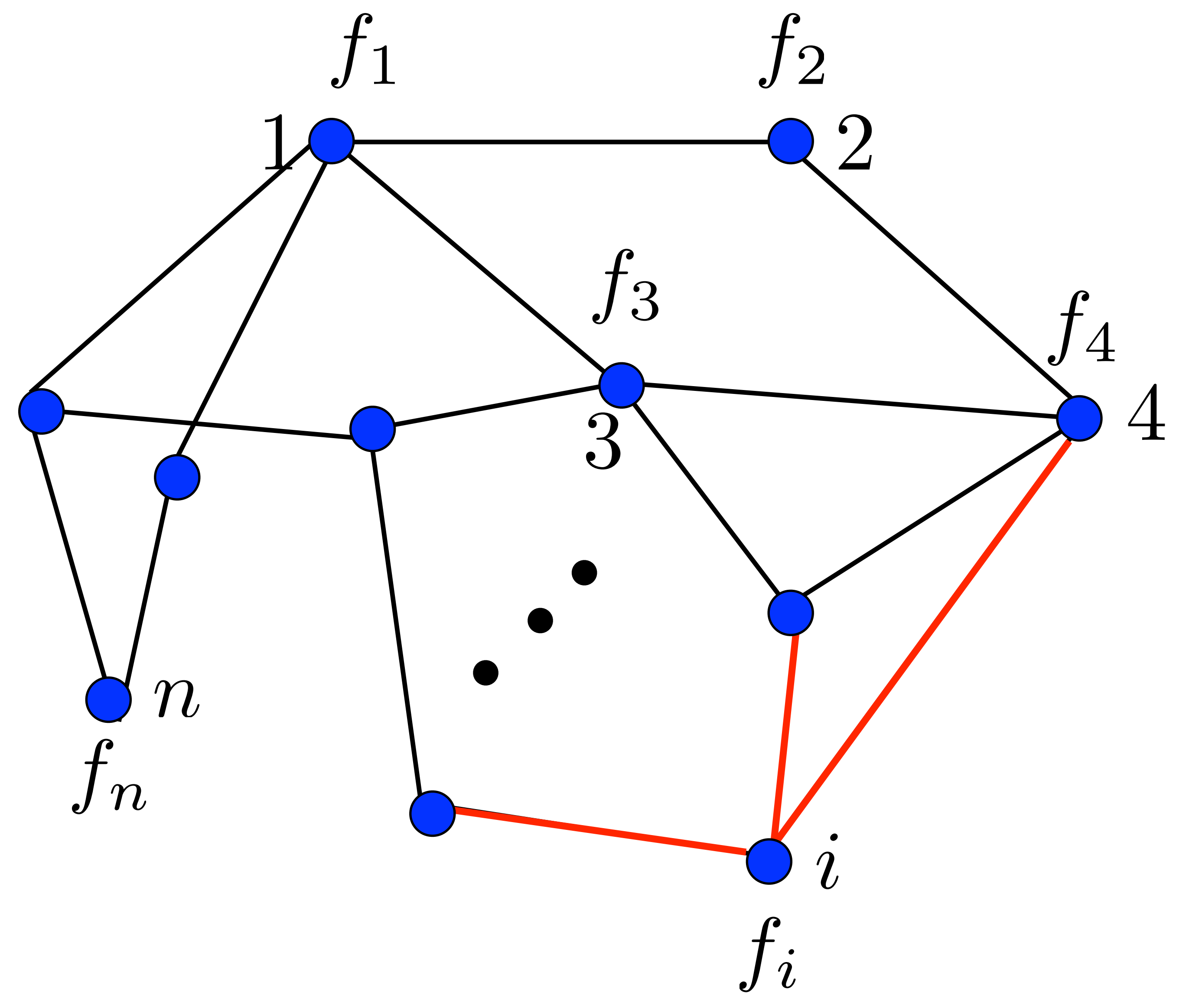

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$A \subseteq B$ ➡ $f(A \cup \{e\}) - f(A) \geq f(B \cup \{e\}) - f(B)$

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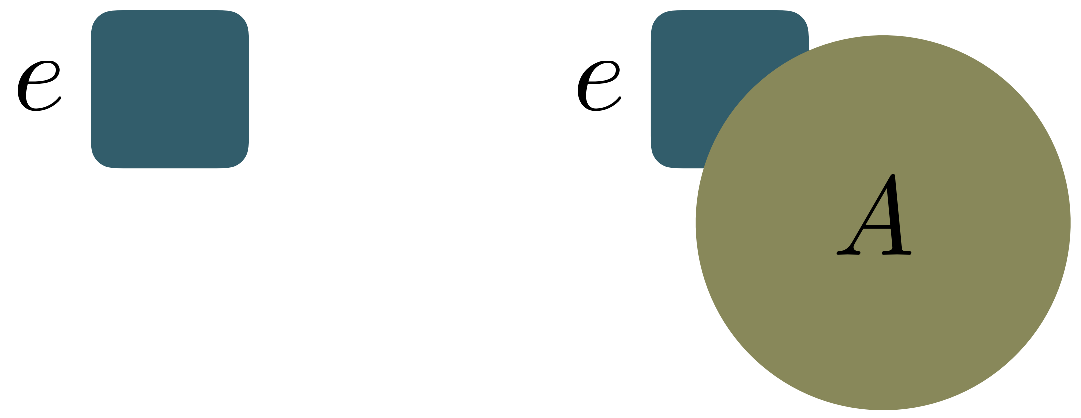

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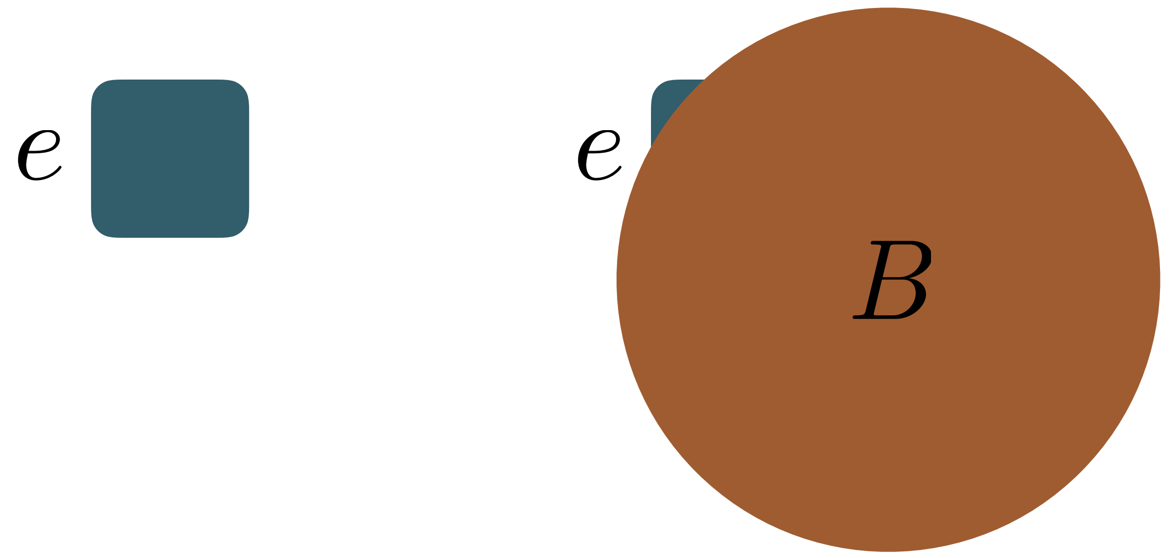

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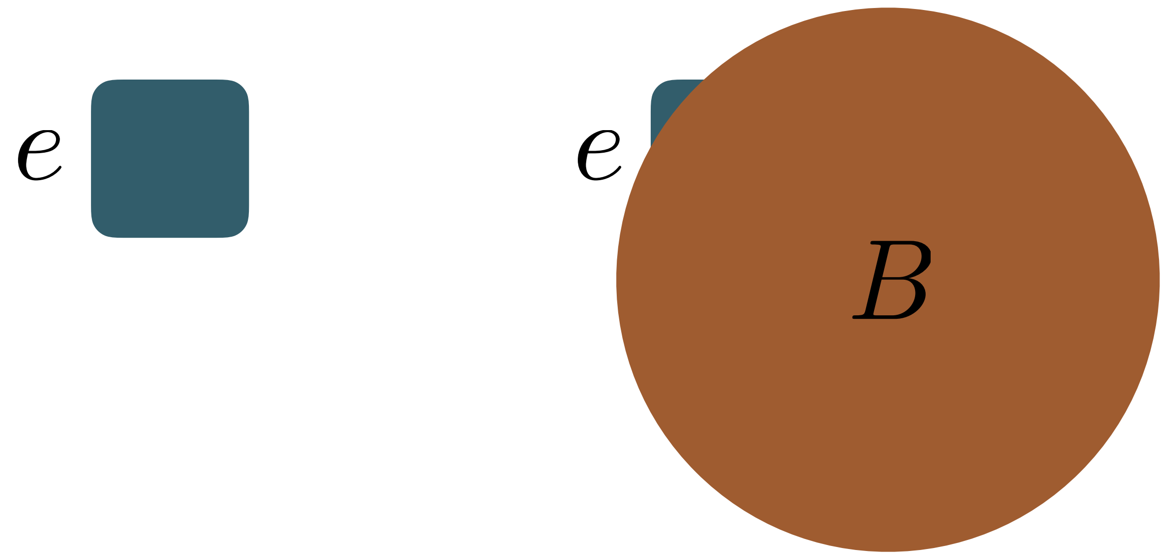

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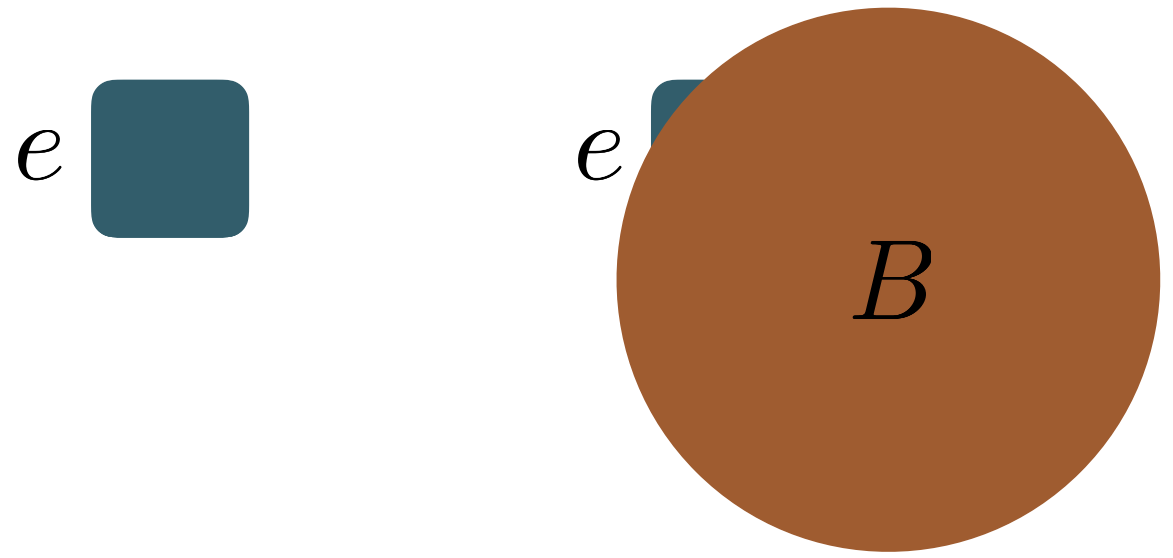

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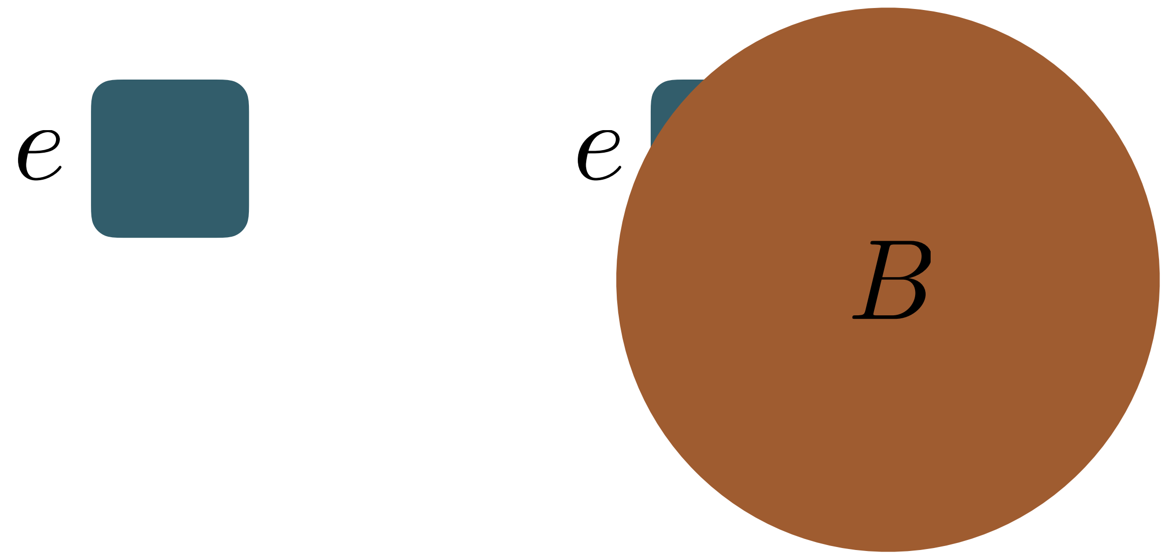

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- Greedy algorithm: $(1 - 1/e)$-approximation

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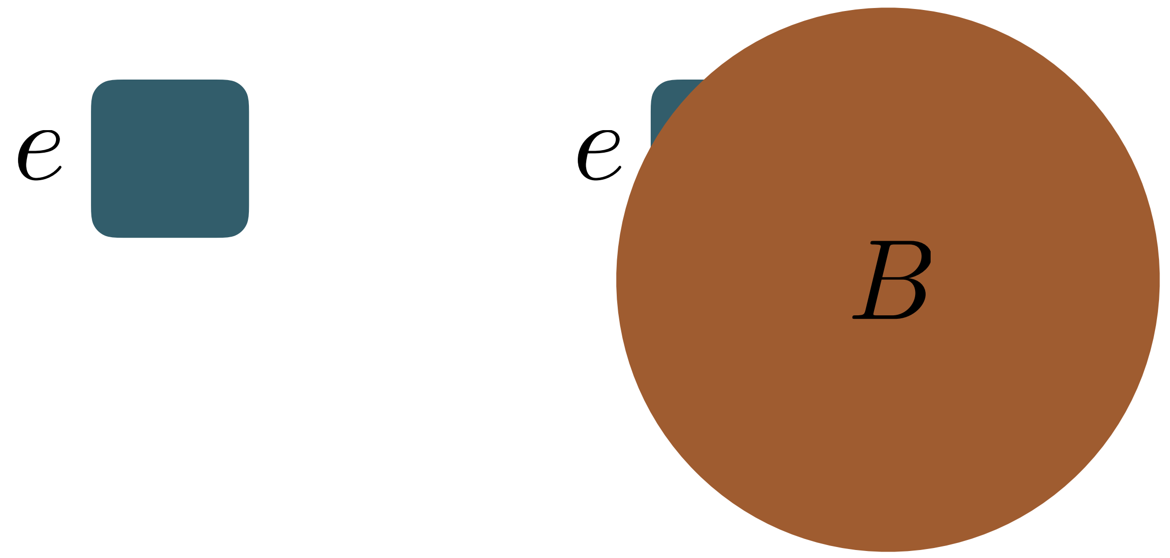

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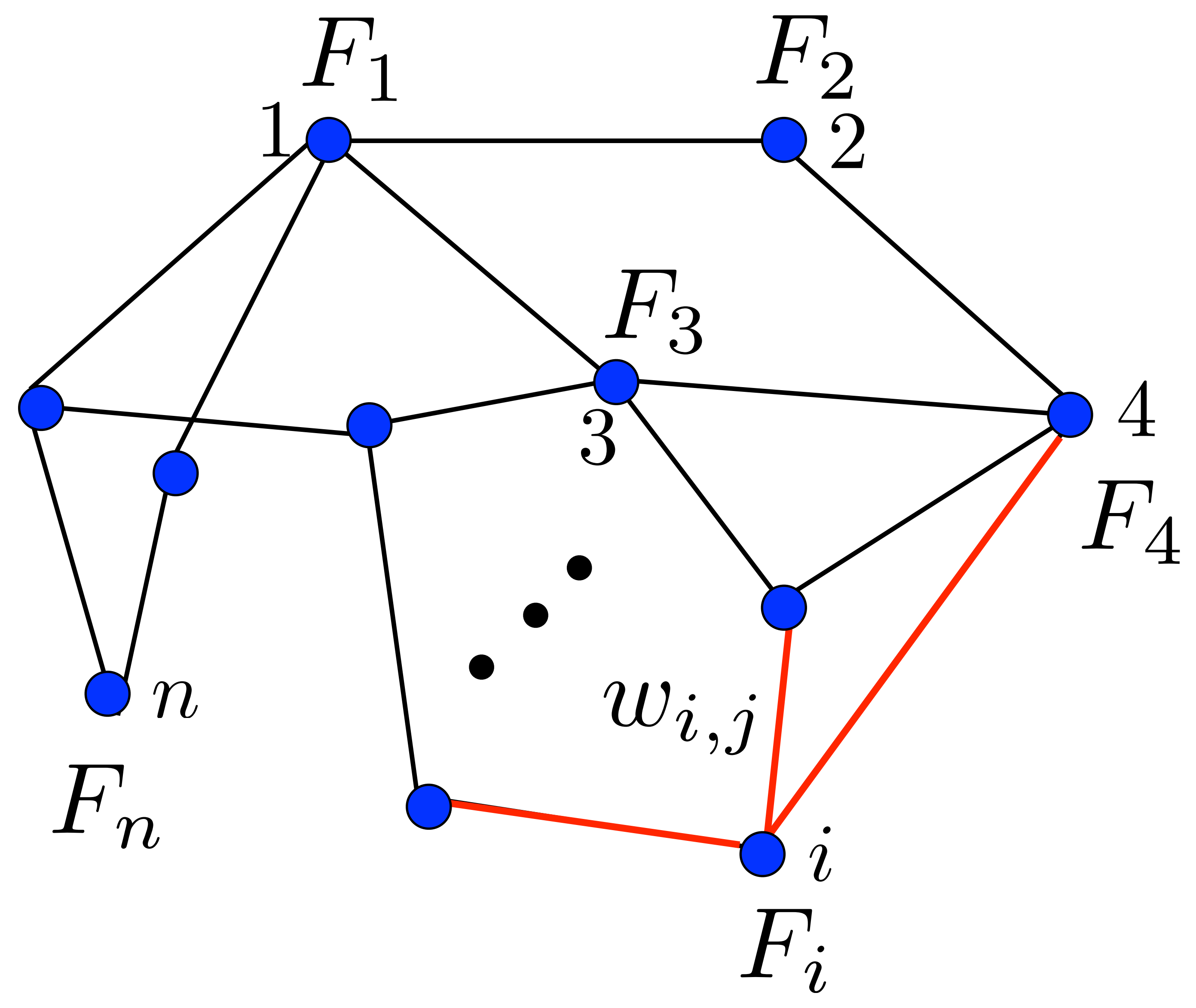
- Graph cut
- Coverage (vertex or set cover)
- Entropy
- Feature selection
- Experimental design
- Social Influence Maximization

Decentralized Submodular Maximization

- Outline for the rest of the talk:
    - Decentralized convex optimization
    - Continuous extensions of submodular functions
    - Centralized maximization of the continuous extension
    - The Decentralized Continuous Greedy (DCG) algorithm
    - Experiments

Decentralized Convex Minimization



$G = (N, E)$

- **Goal**: $\underset{x \in \mathcal{C}}{\text{minimize}} \; \frac{1}{n}\sum_{i=1}^{n} F_i(x)$ (F_i's are convex)

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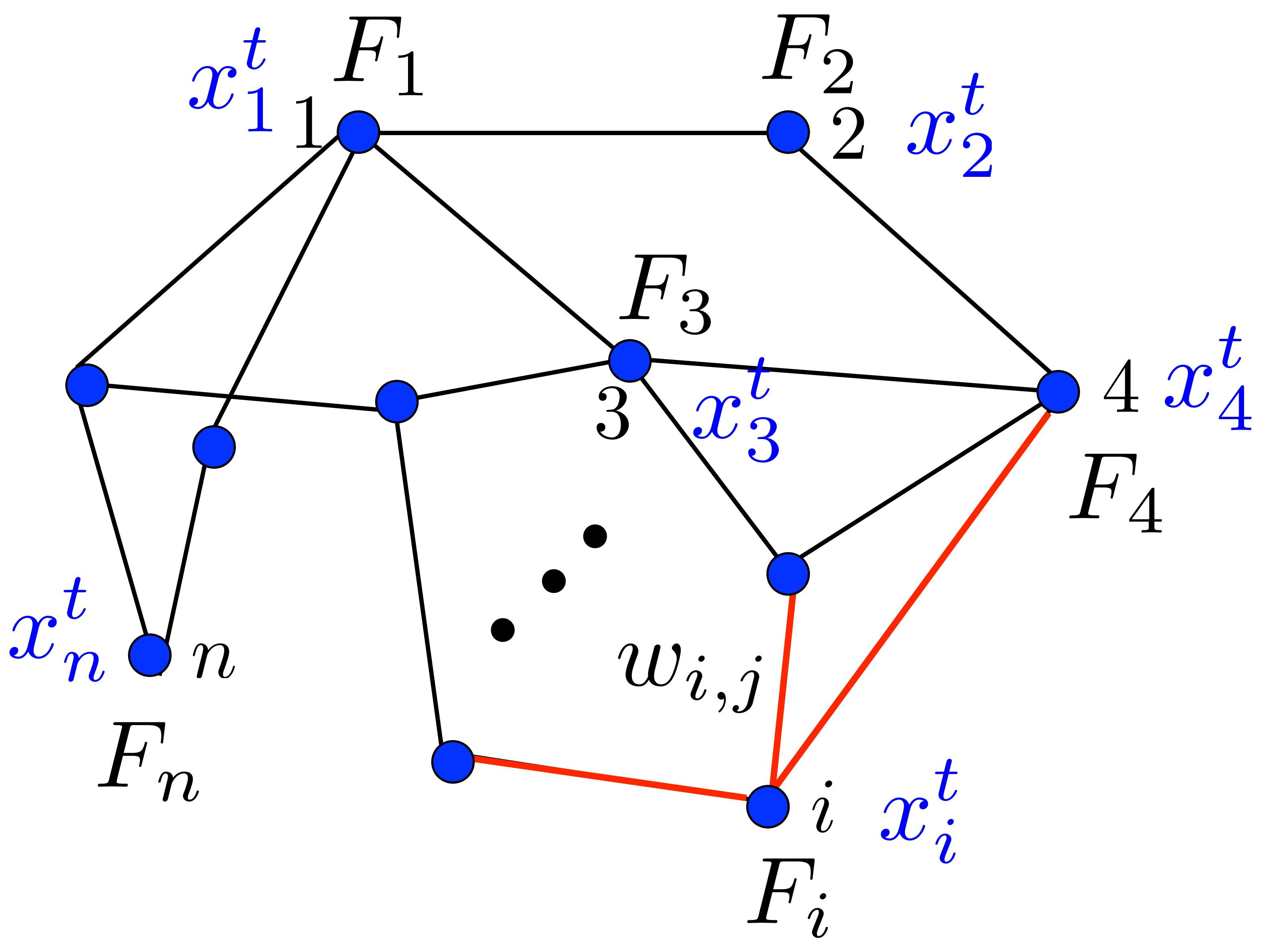

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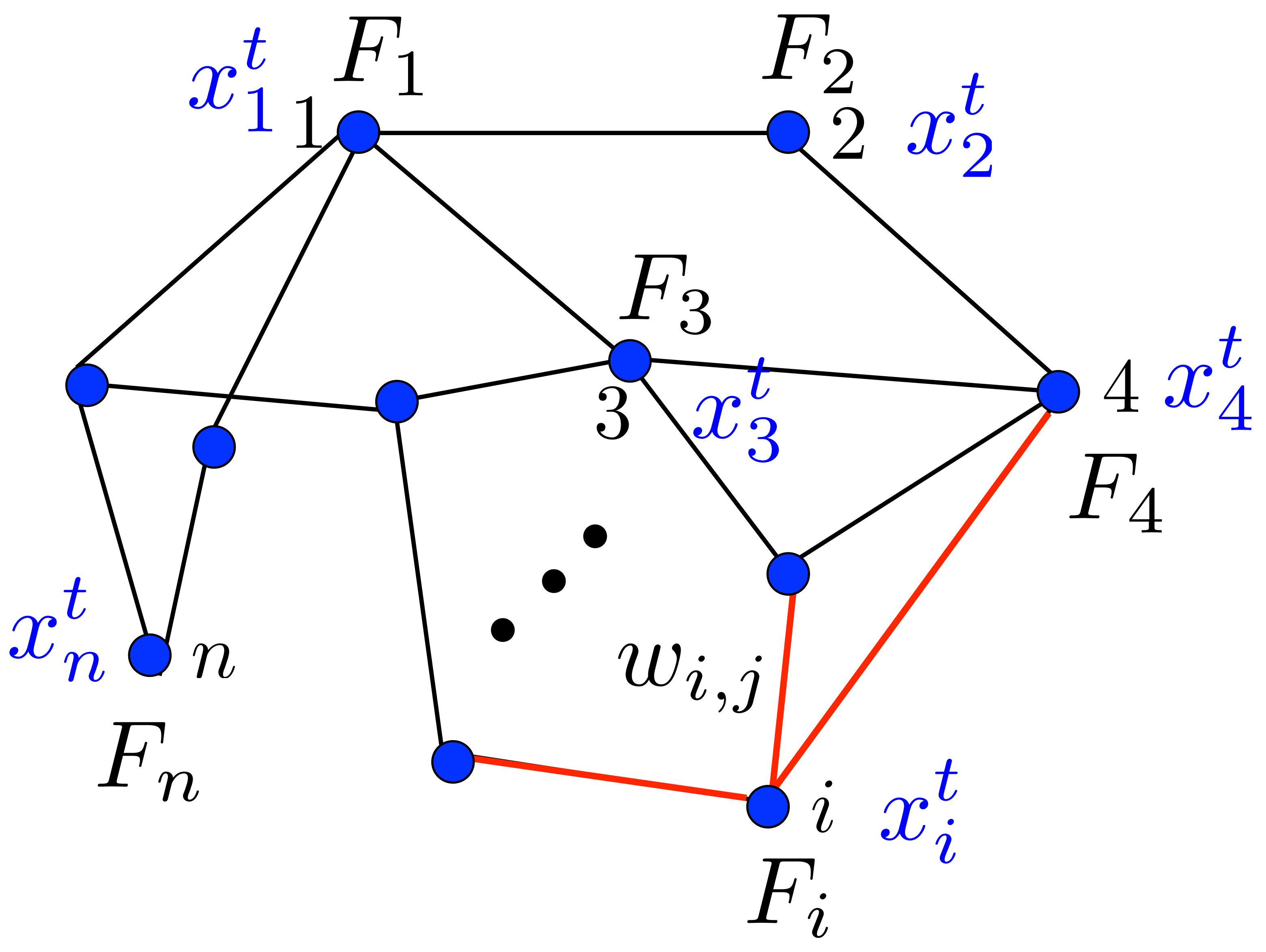

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[Nedic, Ozdaglar '09]

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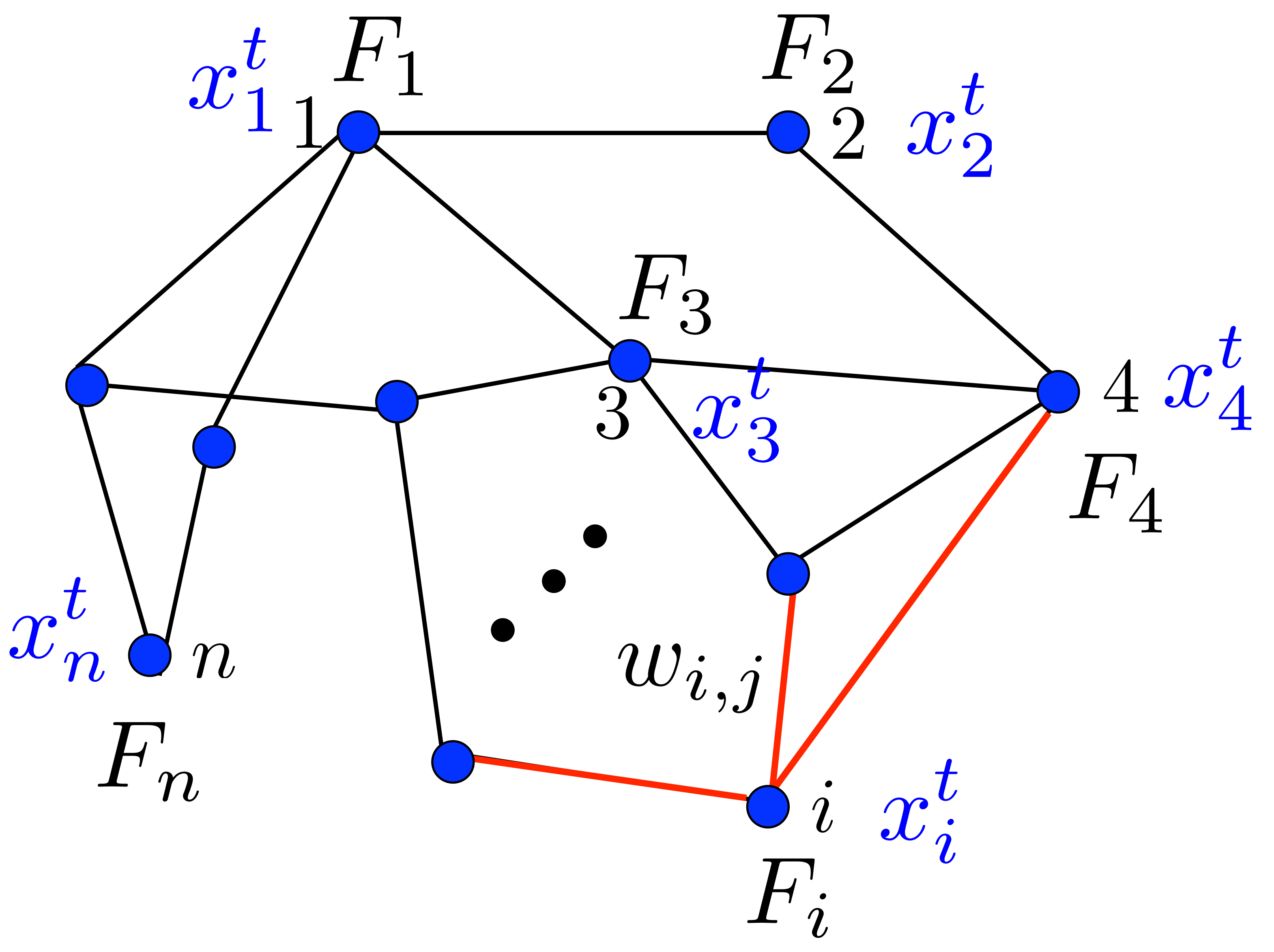

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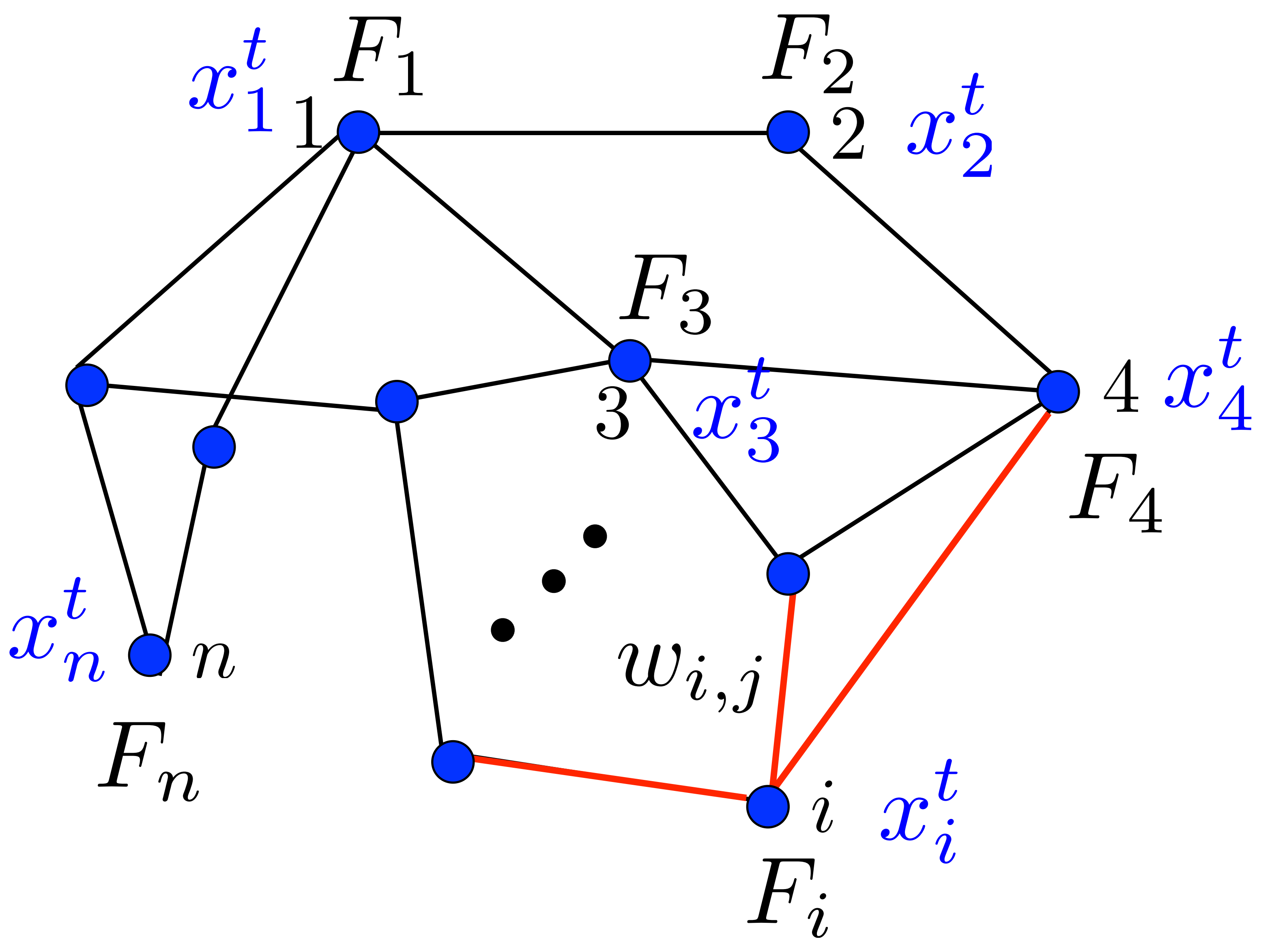
$$w_{i,j} = w_{j,i} \geq 0$$

forcing consensus

Decentralized Convex Minimization



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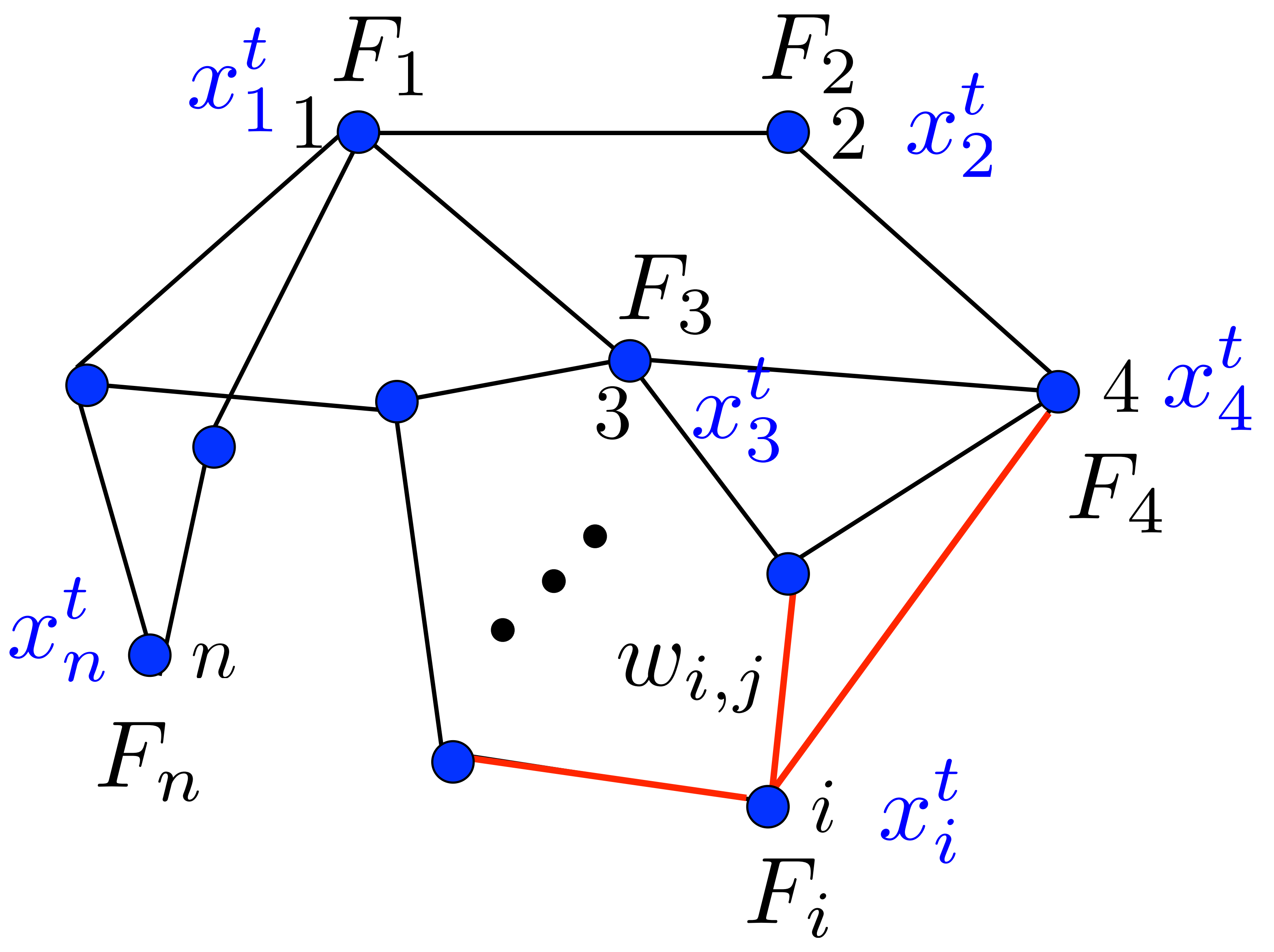

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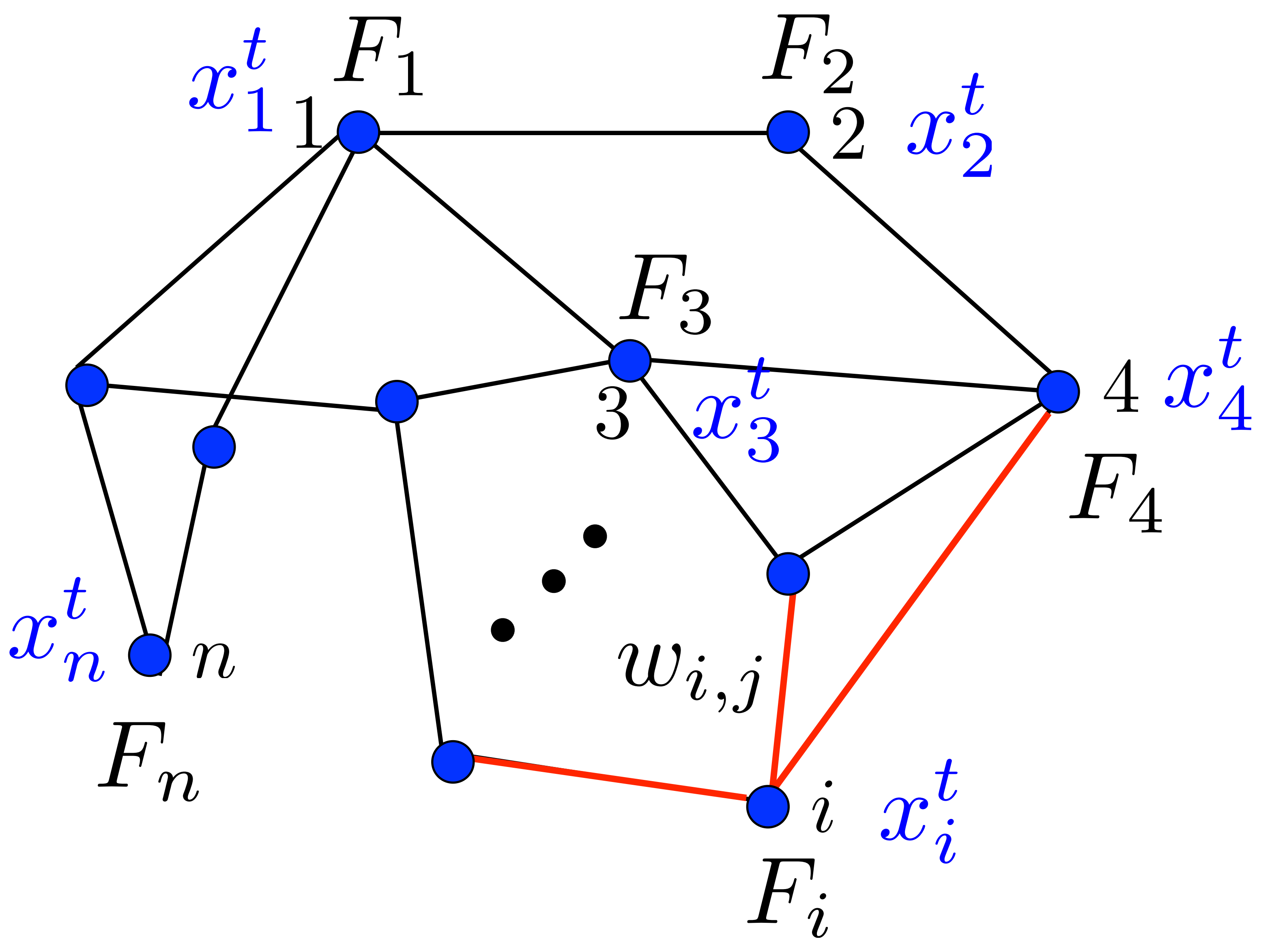

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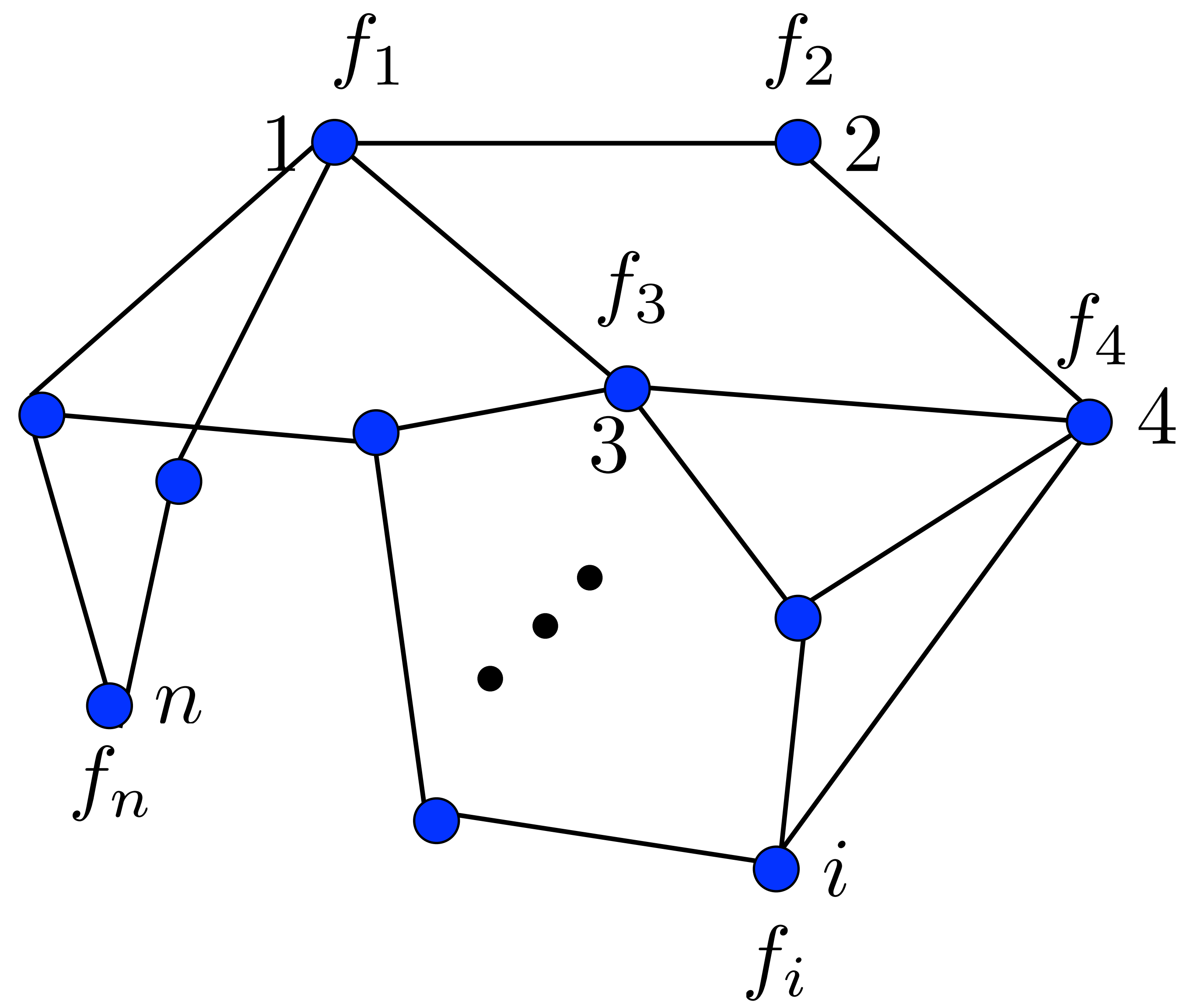

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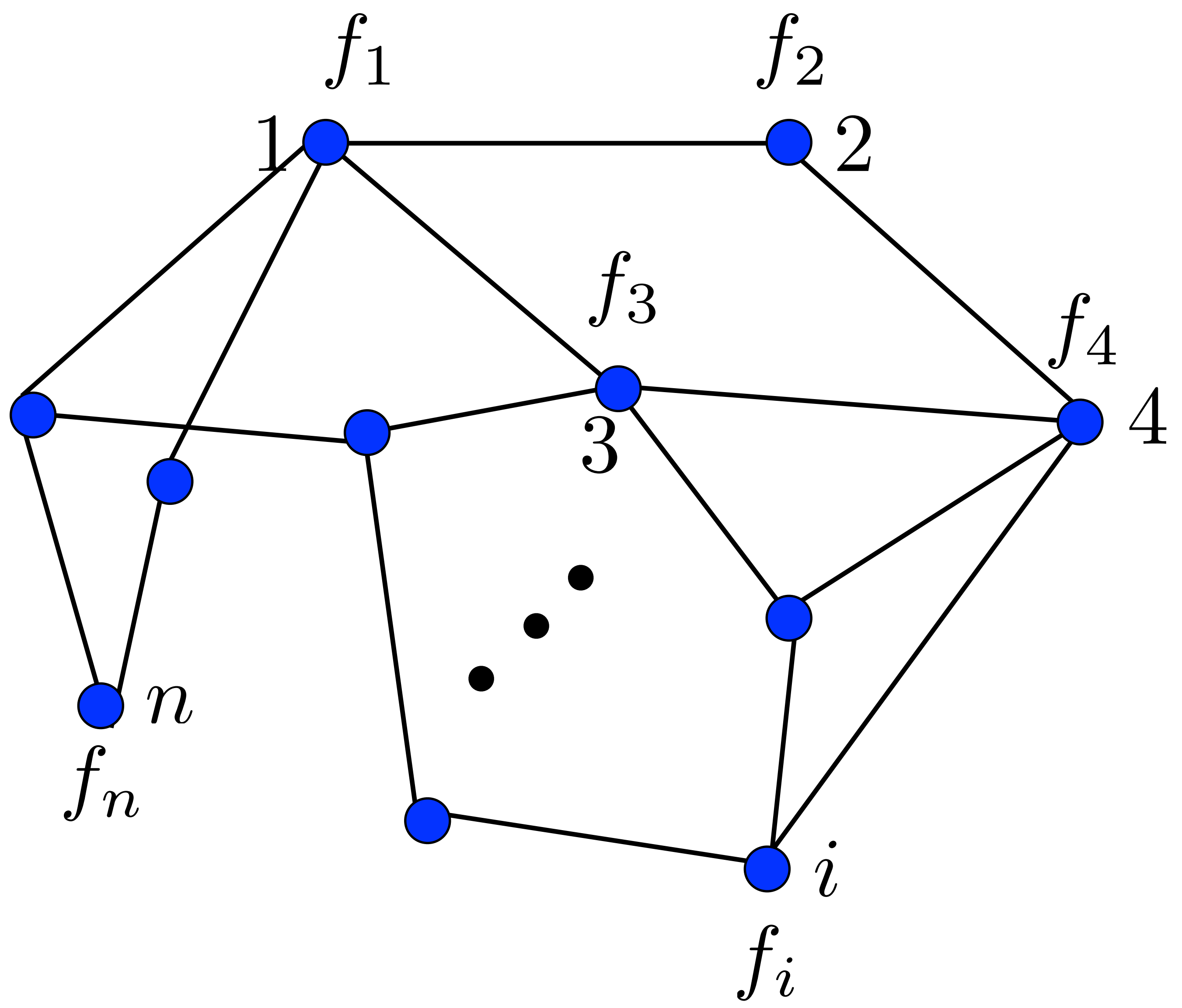

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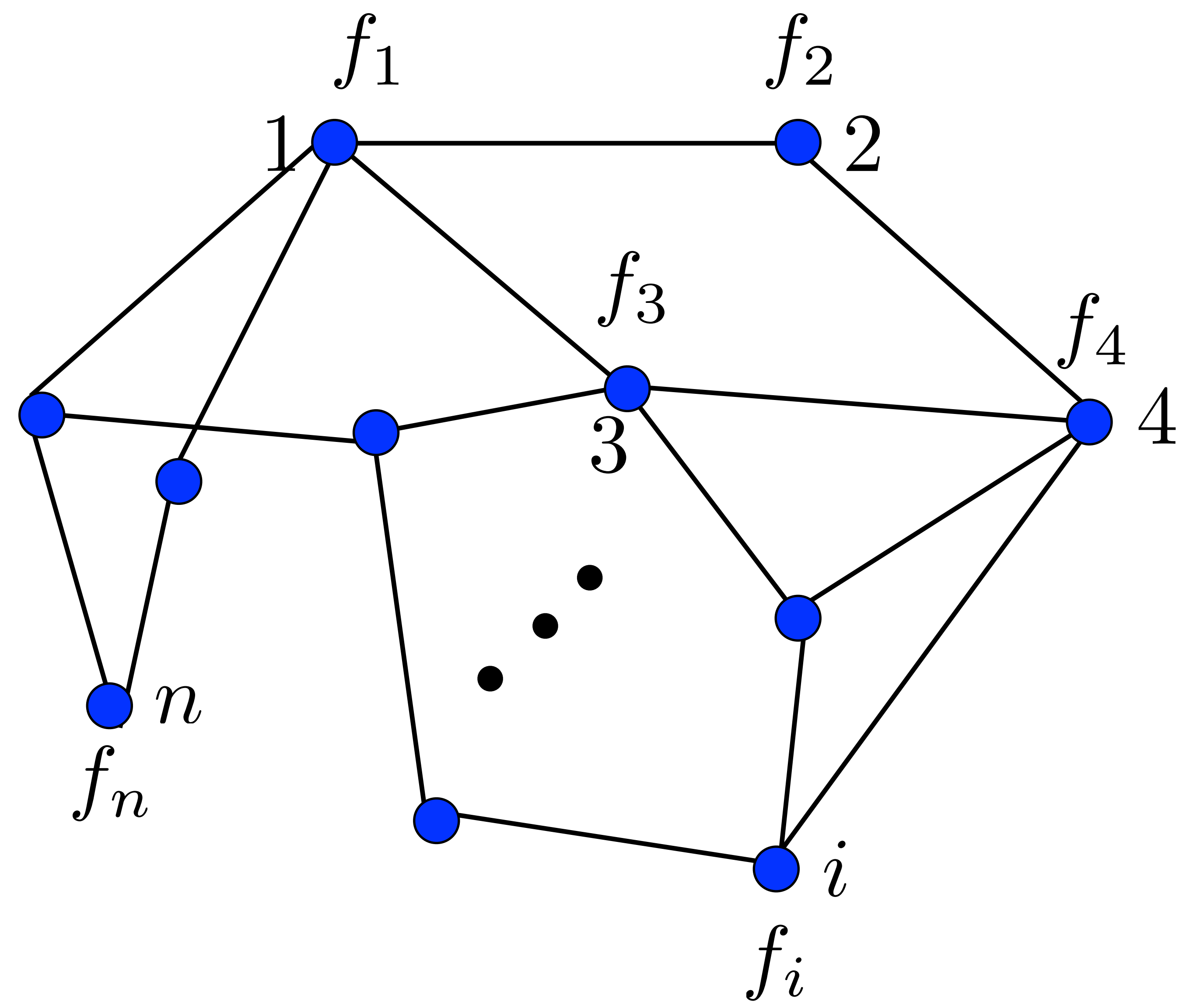

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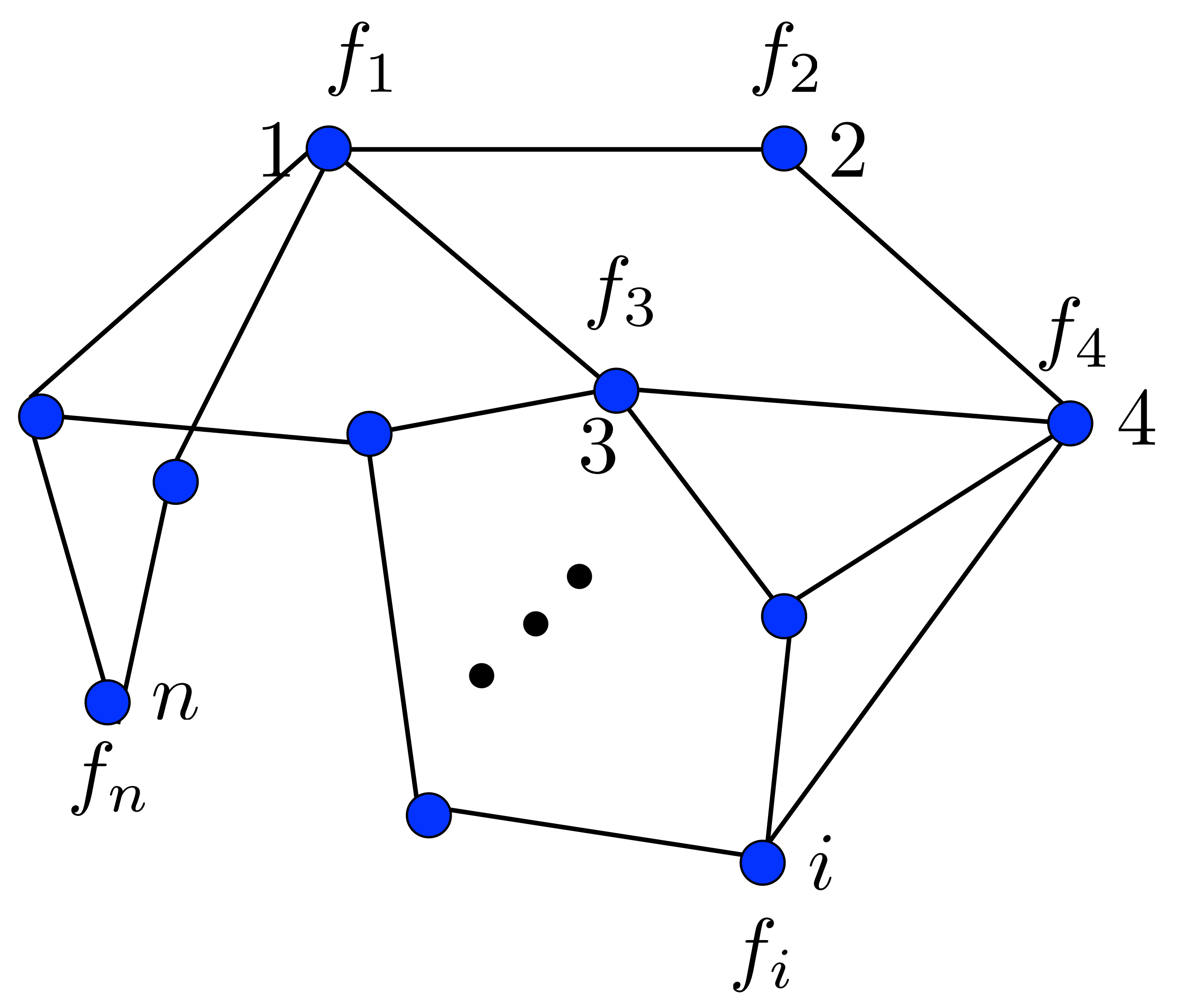

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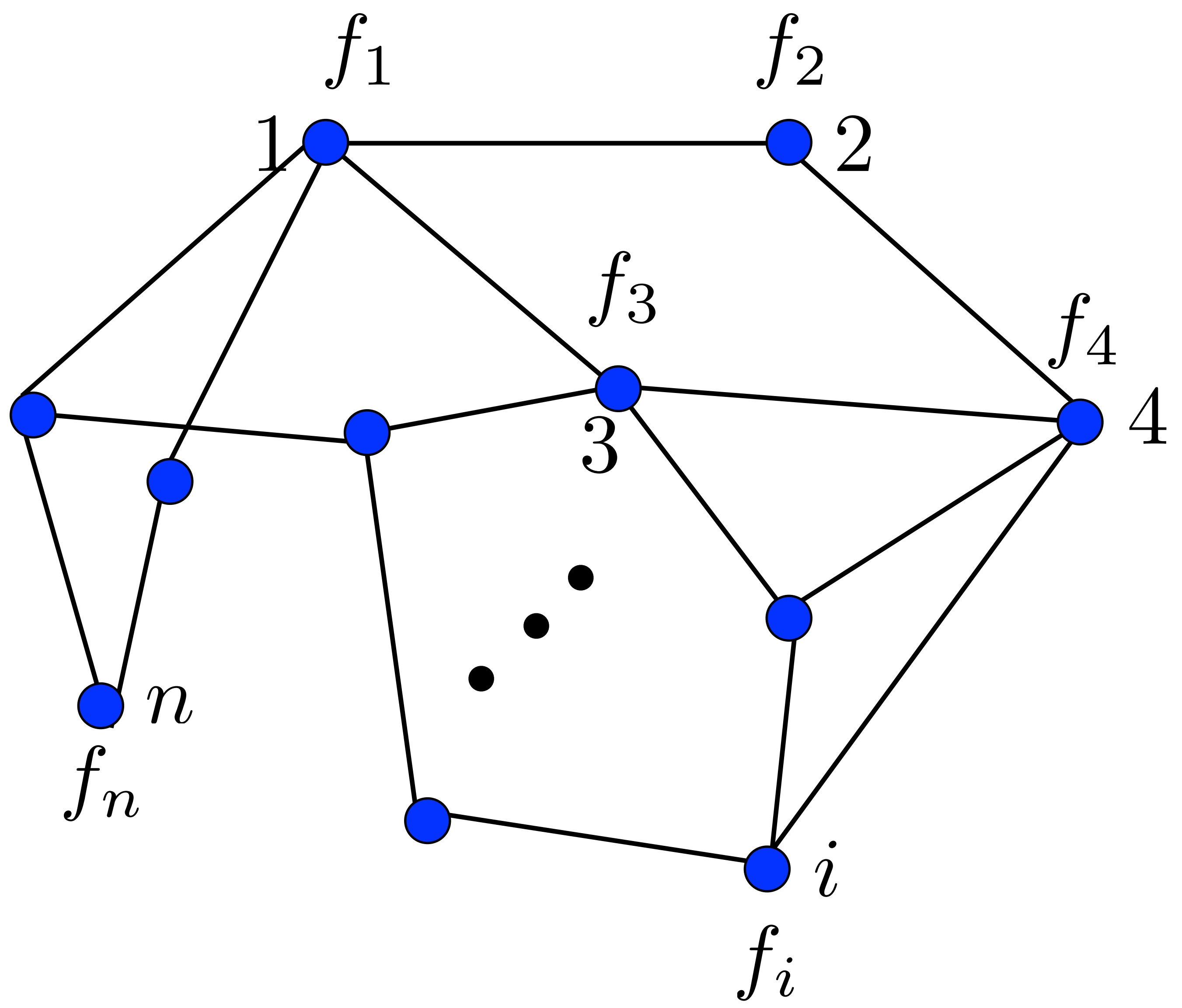

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Multilinear Extension

$f : 2^V \to \mathbb{R}$ ➡ $F : [0,1]^m \to \mathbb{R}$

$V = \{1, 2, \cdots, m\}$

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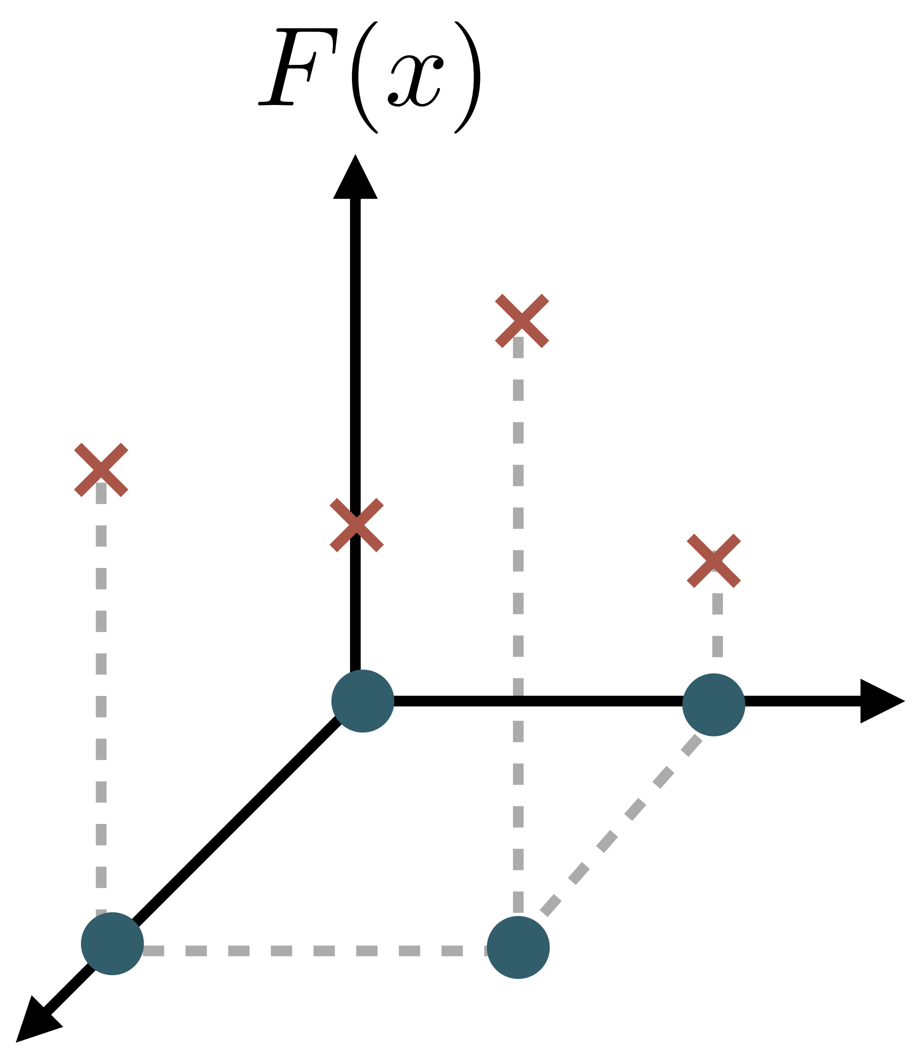

$f(S)$ →

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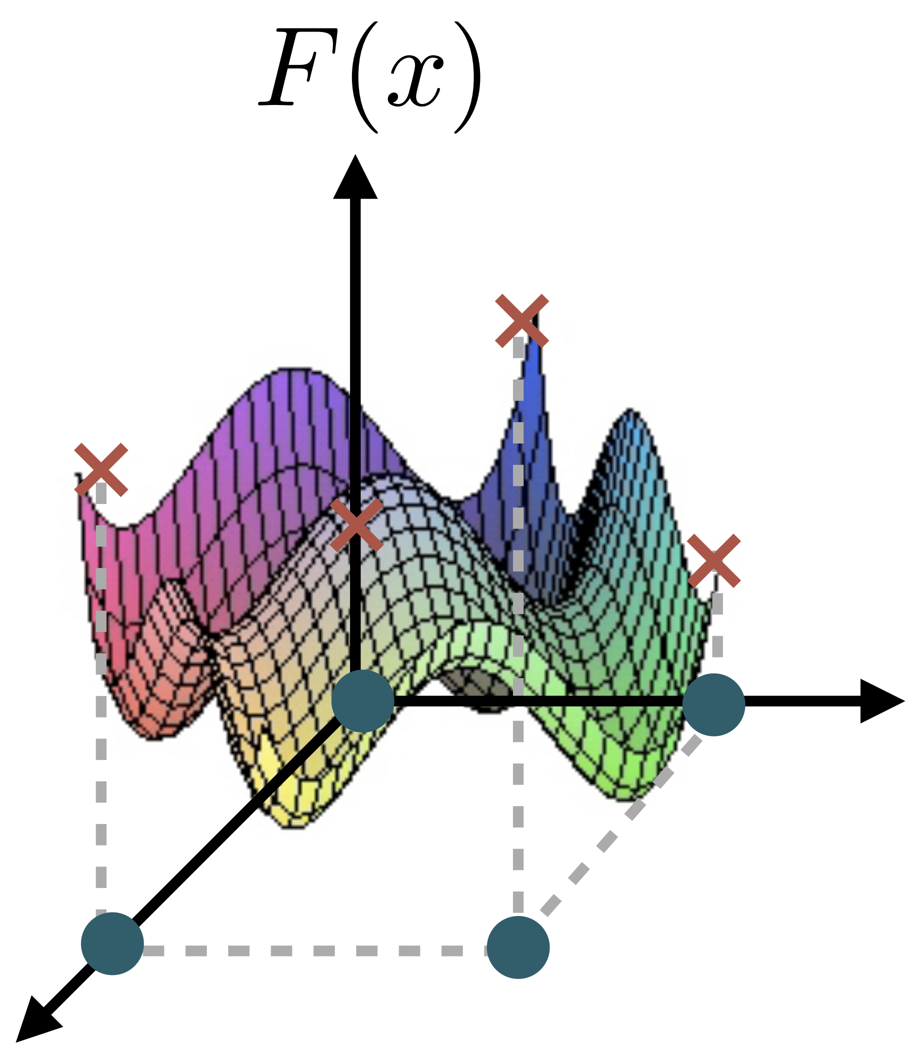

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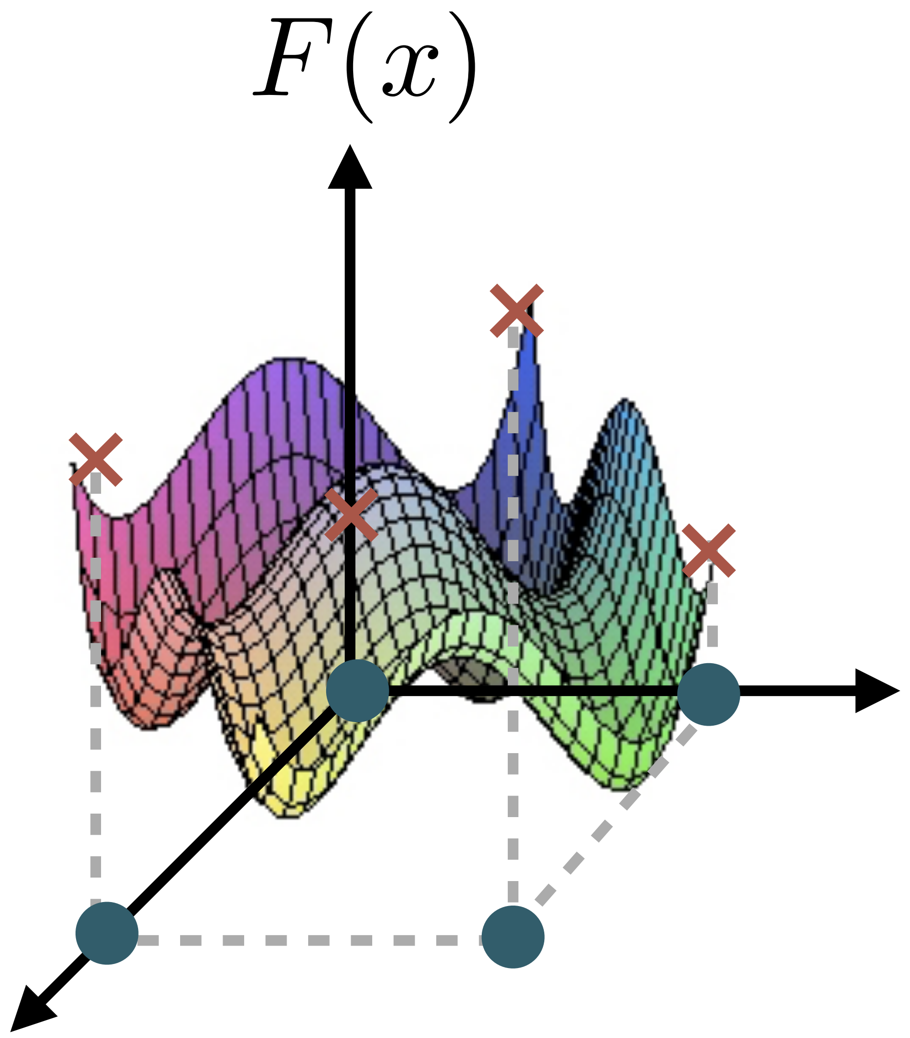

$f(S)$ ➡

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$x = (x_1, x_2 \cdots, x_m) \in [0,1]^m$

$$F(x) = \sum_{S \in V} f(S) \prod_{a \in S} x_a \prod_{b \notin S} (1 - x_b) = \mathbb{E}_{S \sim (x_1, \cdots, x_m)}[f(S)]$$

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$f(S)$

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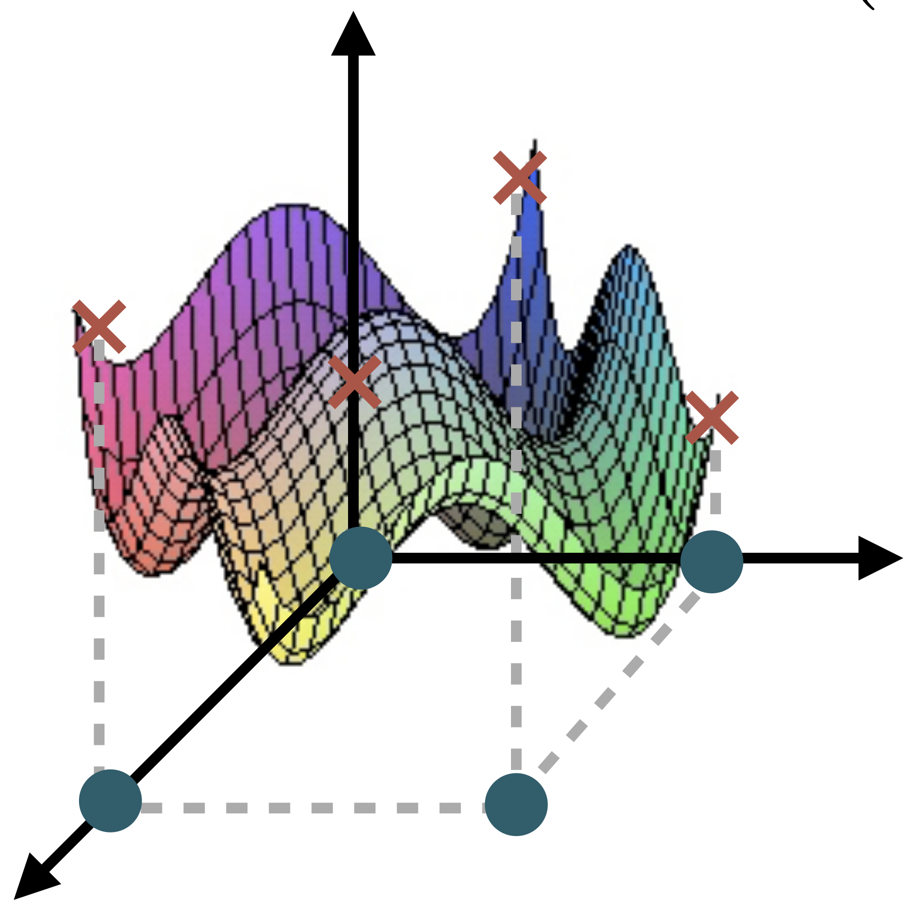

$$\max_{|S| \leq k} f(S) \quad = \quad \max_{x \in \mathcal{C}} F(x)$$

[Calinescu, Chekuri, Vondrak 2011]

Multilinear Extension

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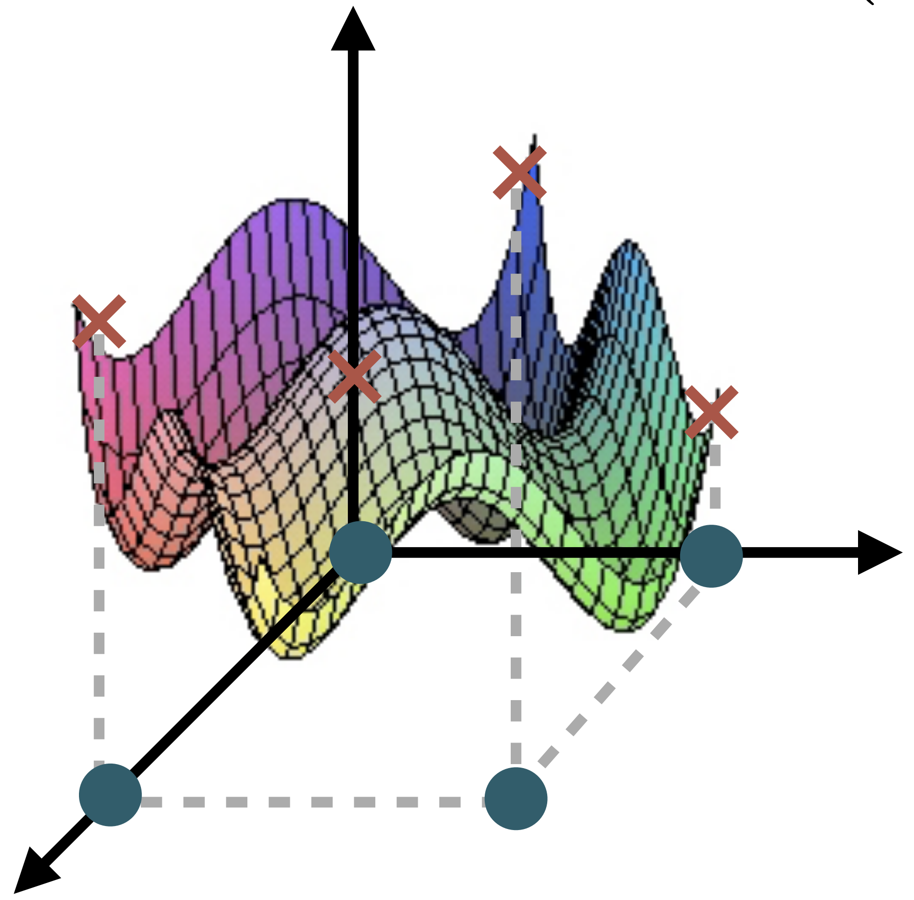
$f(S)$

$S \subseteq \{1, \cdots, n\}$



$$\max_{|S| \leq k} f(S) \quad = \quad \max_{x \in \mathcal{C}} F(x)$$

[Calinescu, Chekuri, Vondrak 2011]

Greedy algorithm provides a (1 - 1/e)-OPT solution

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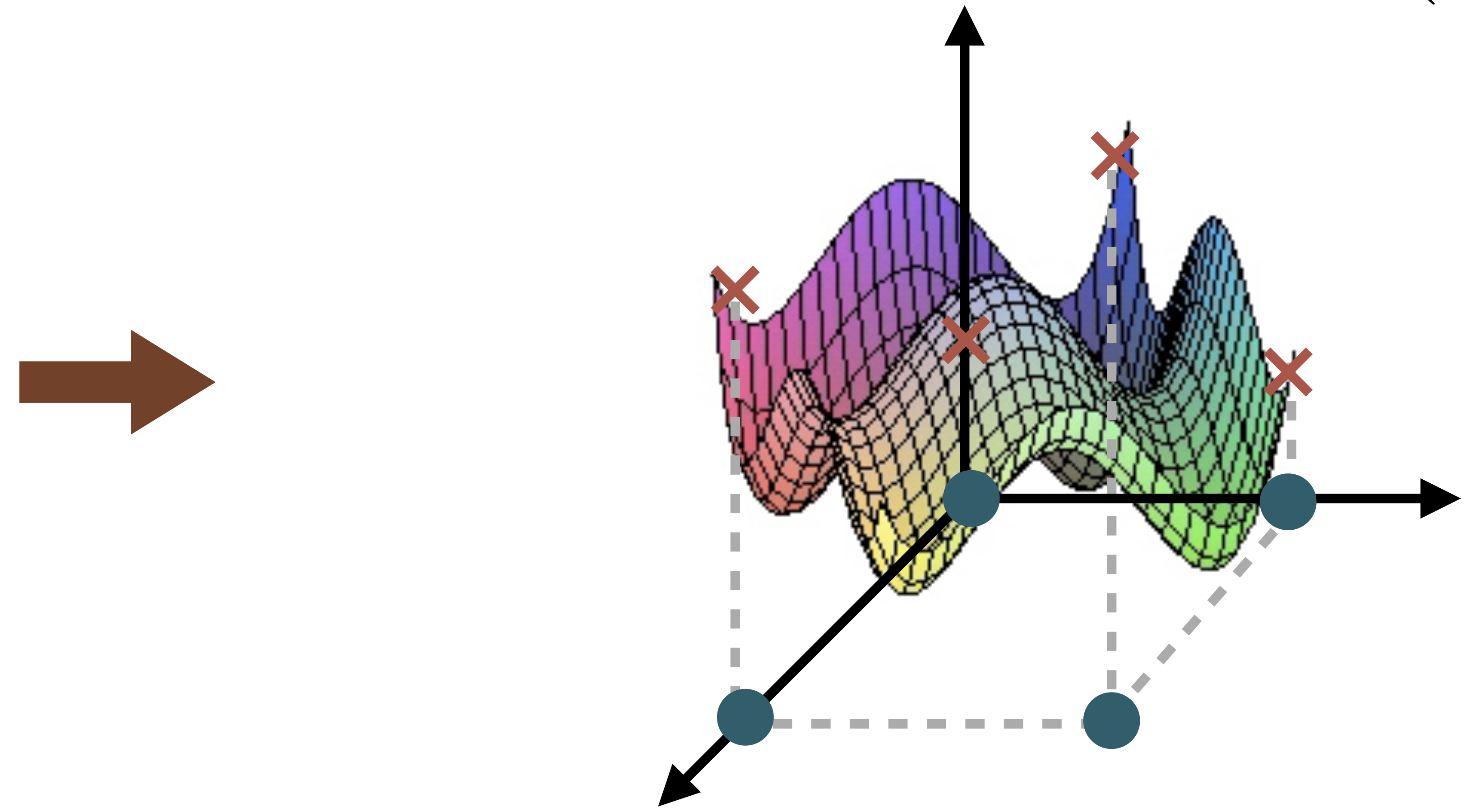

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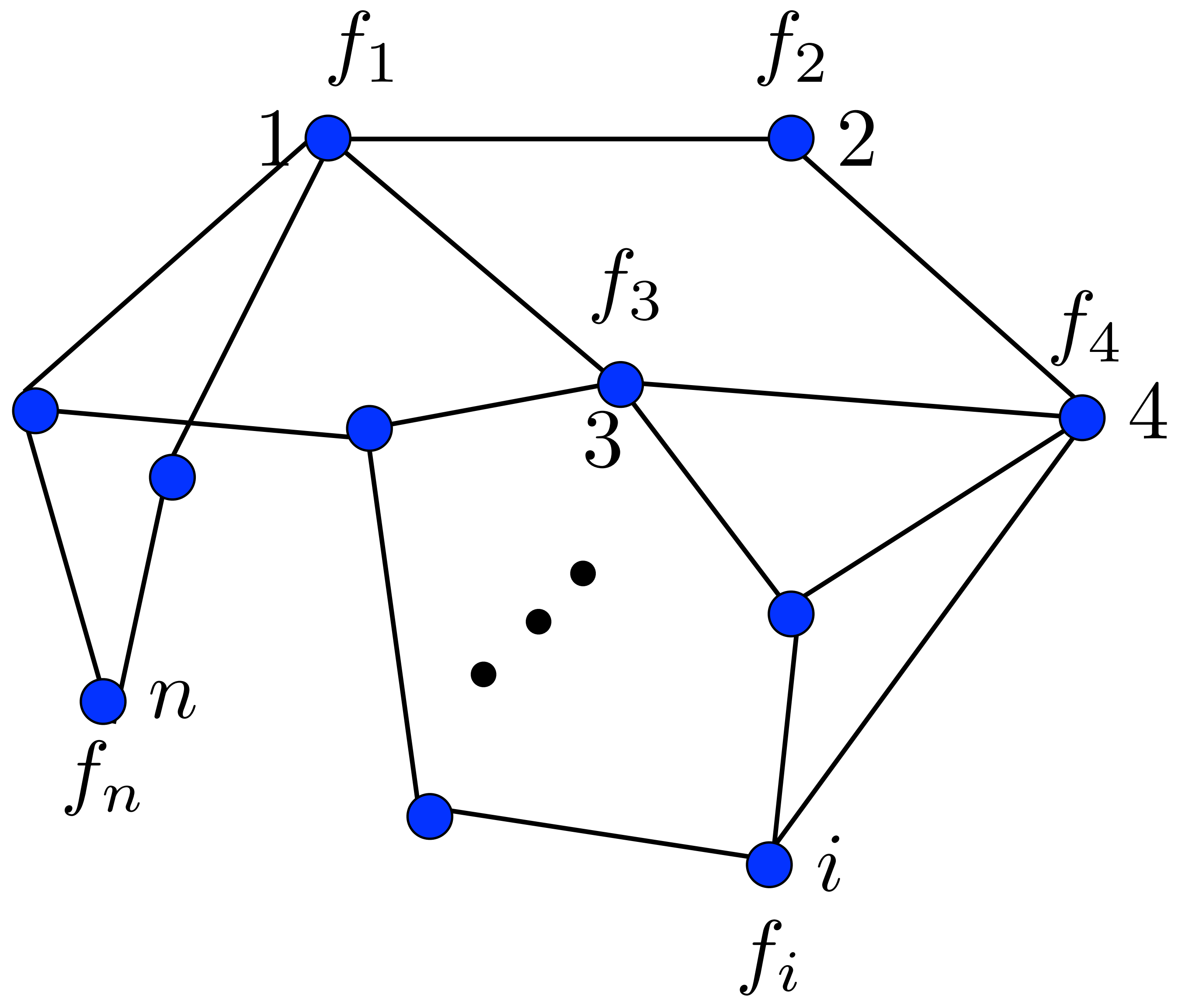
[Calinescu, Chekuri, Vondrak 2011]

Greedy algorithm provides a (1 - 1/e)-OPT solution

Continuous Greedy algorithm provides a (1 - 1/e)-OPT solution

Decentralized Submodular Maximization

Replace by the multilinear extension



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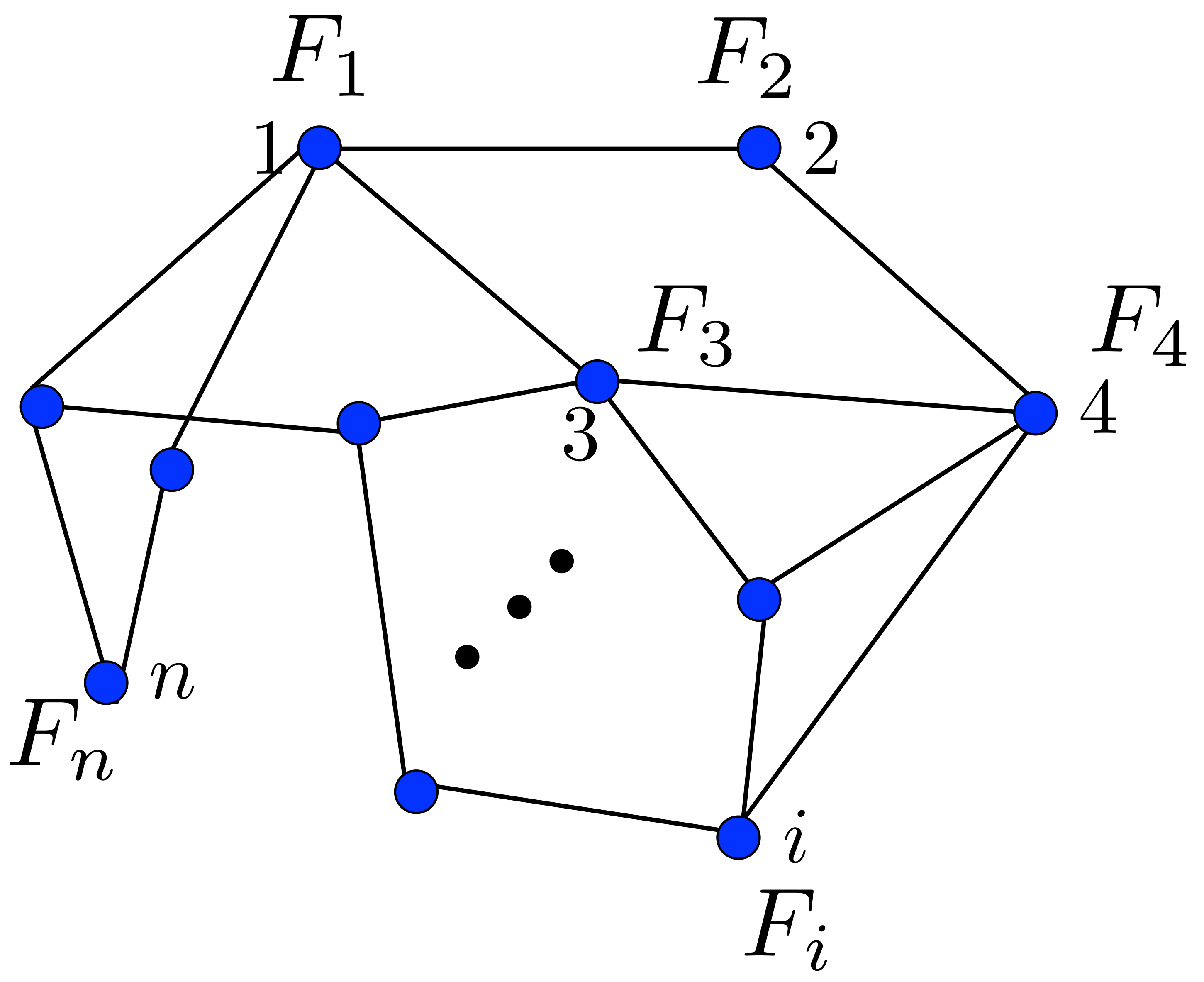

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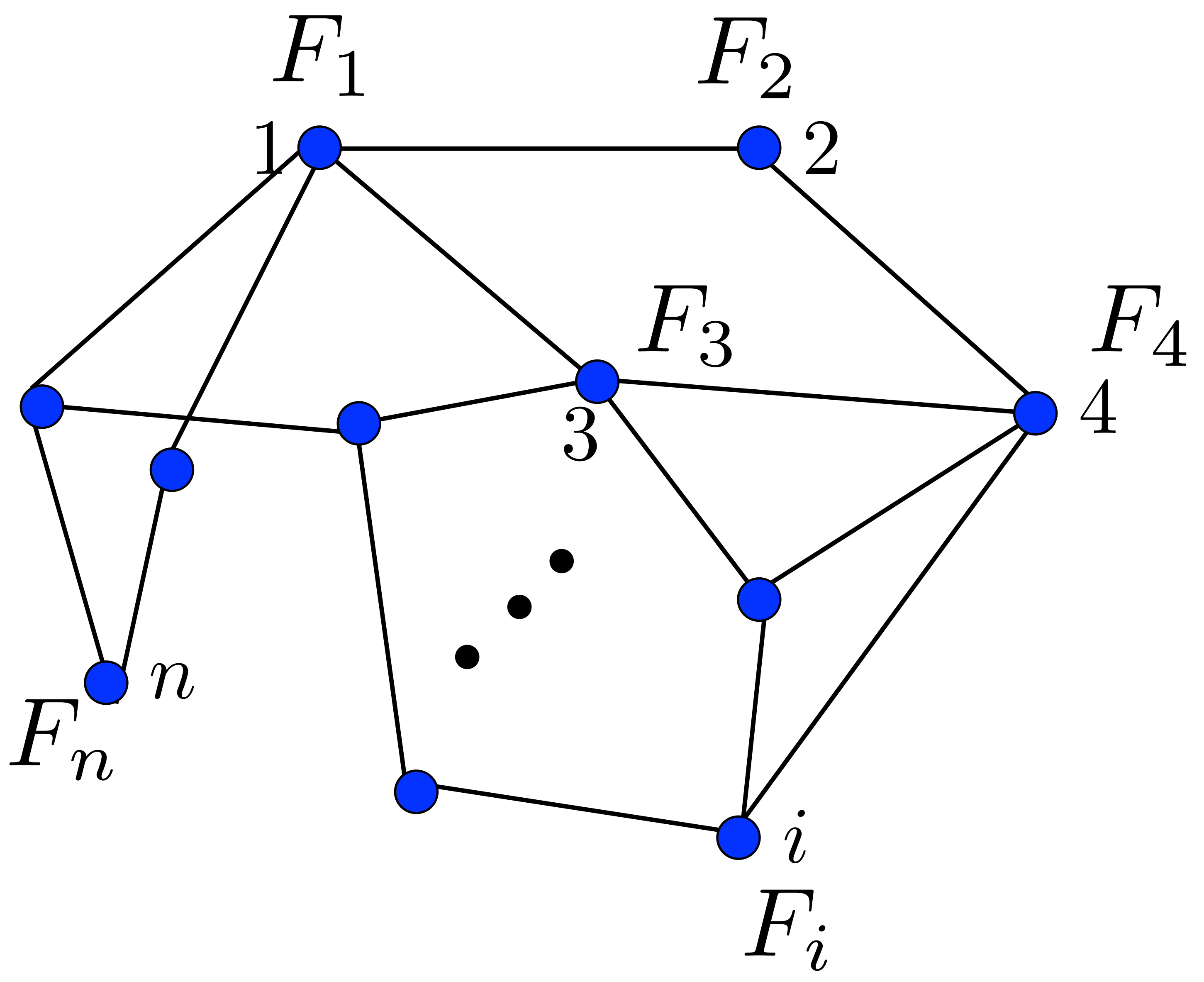

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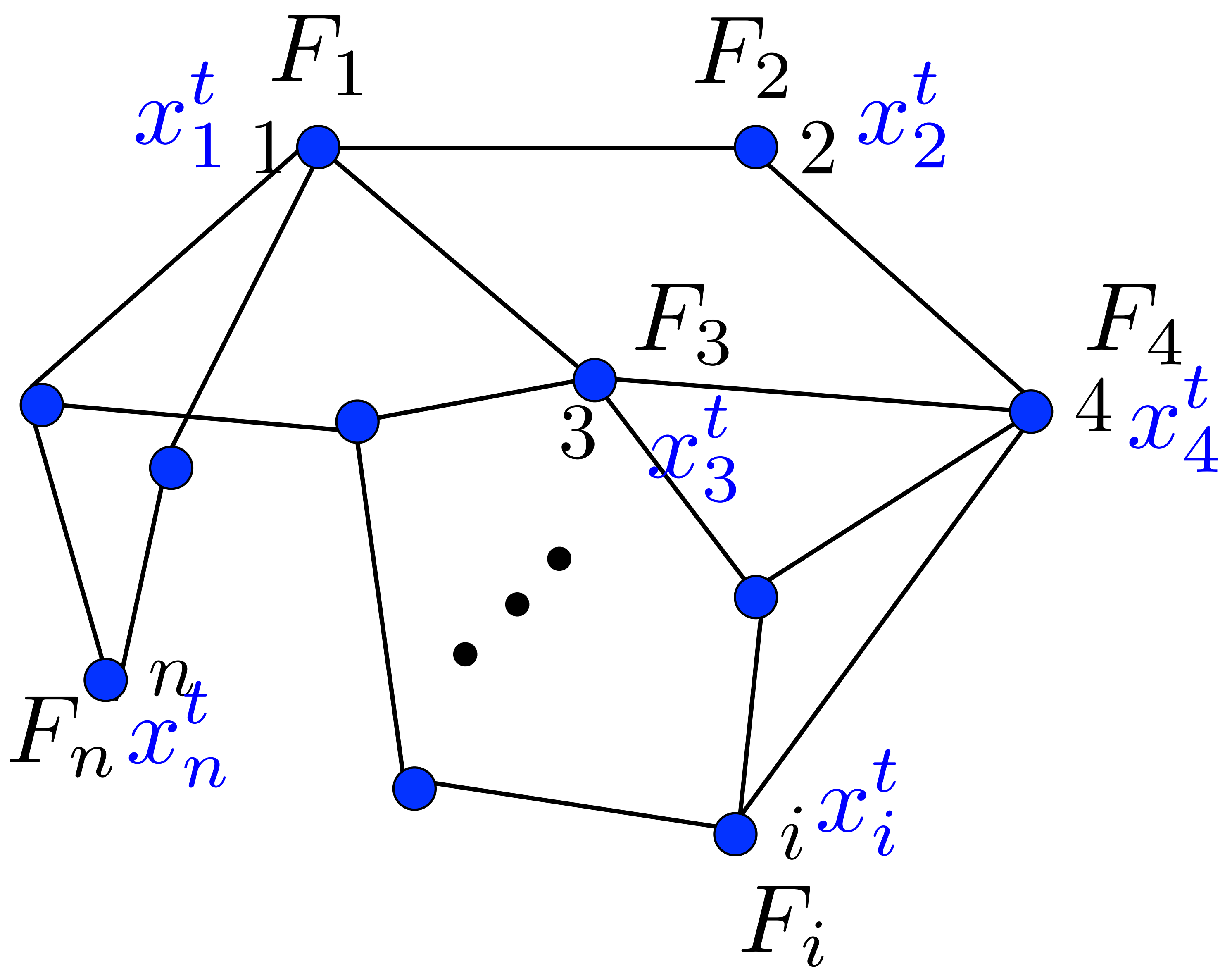

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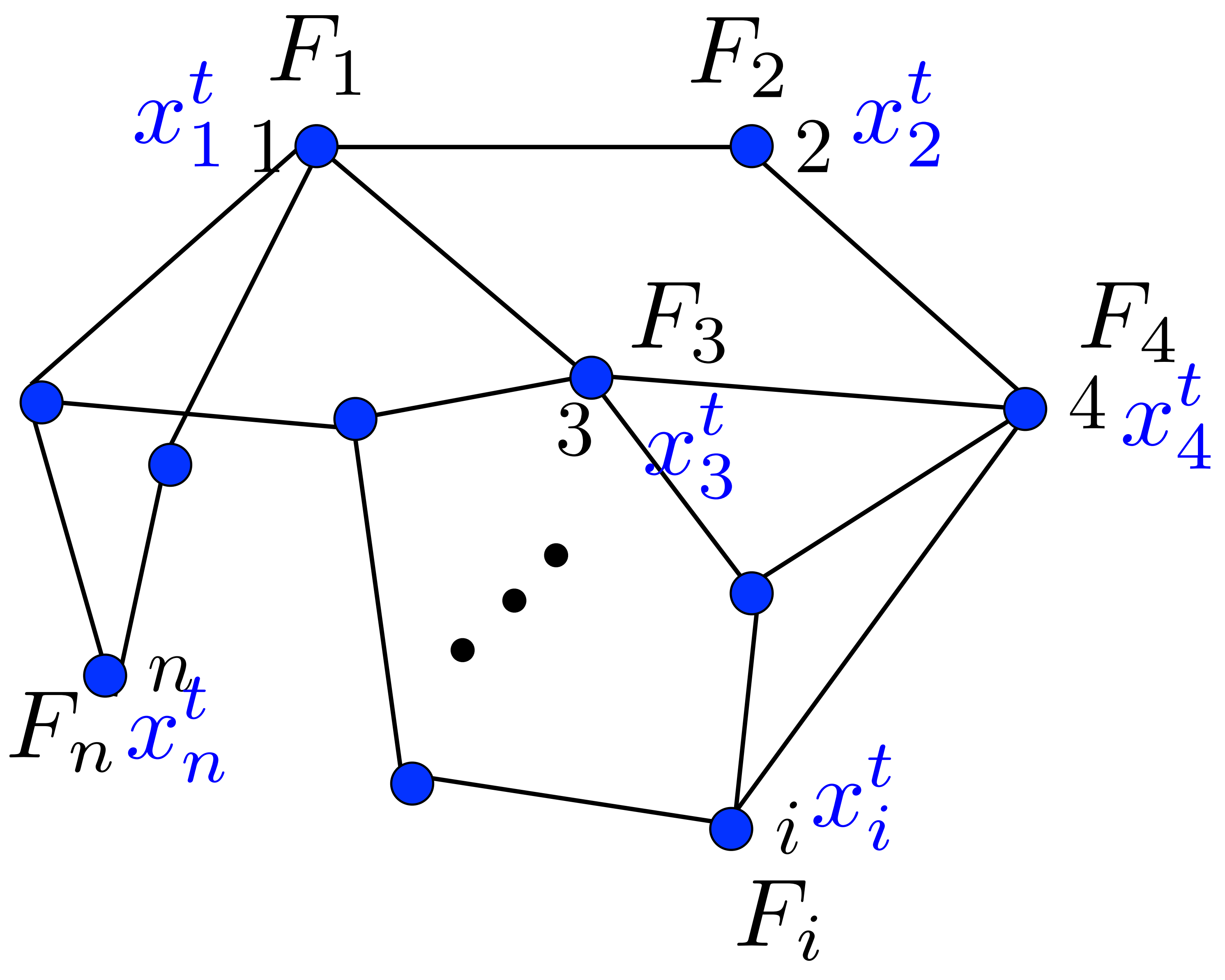

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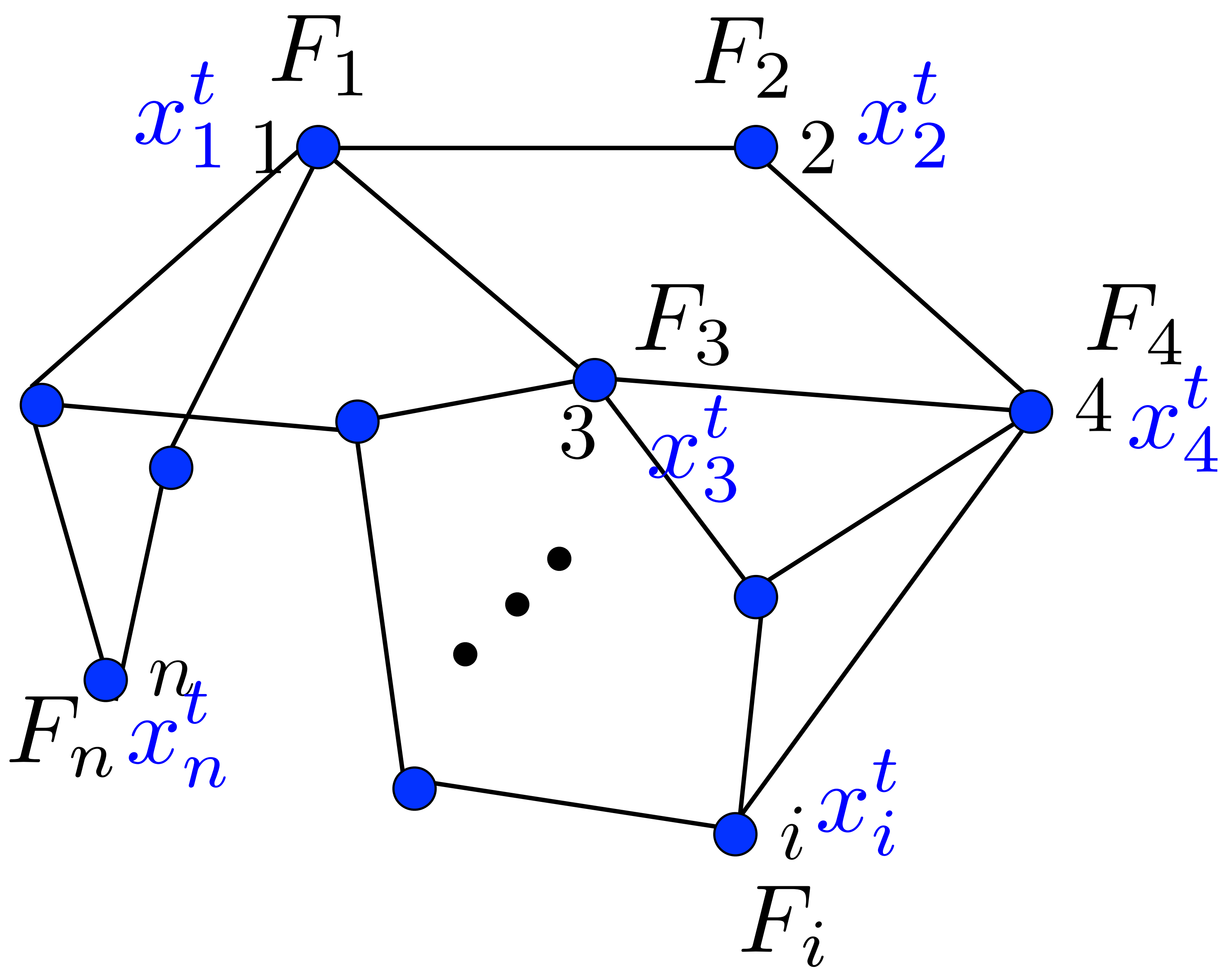

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$F : [0,1]^m \to \mathbb{R}$

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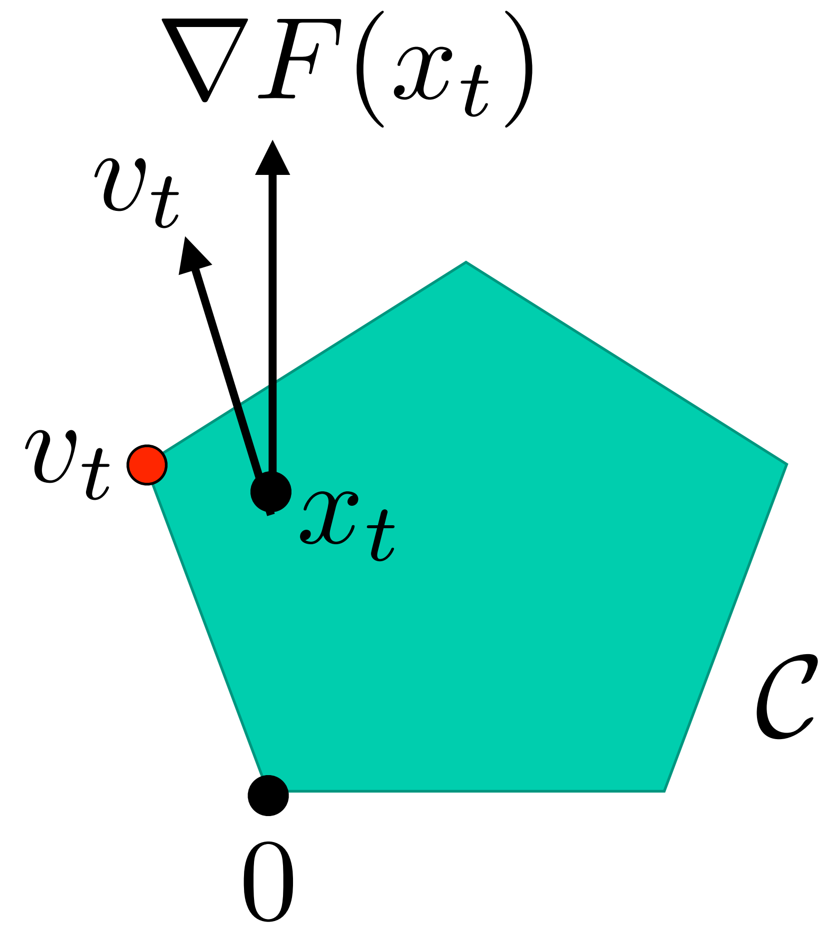

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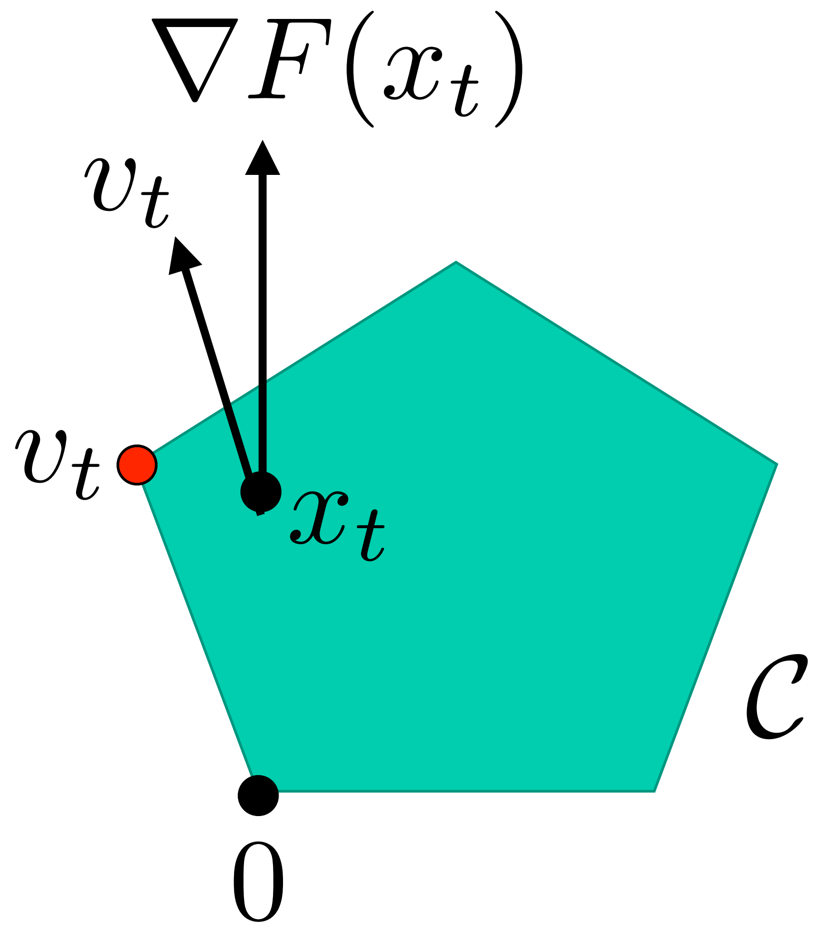
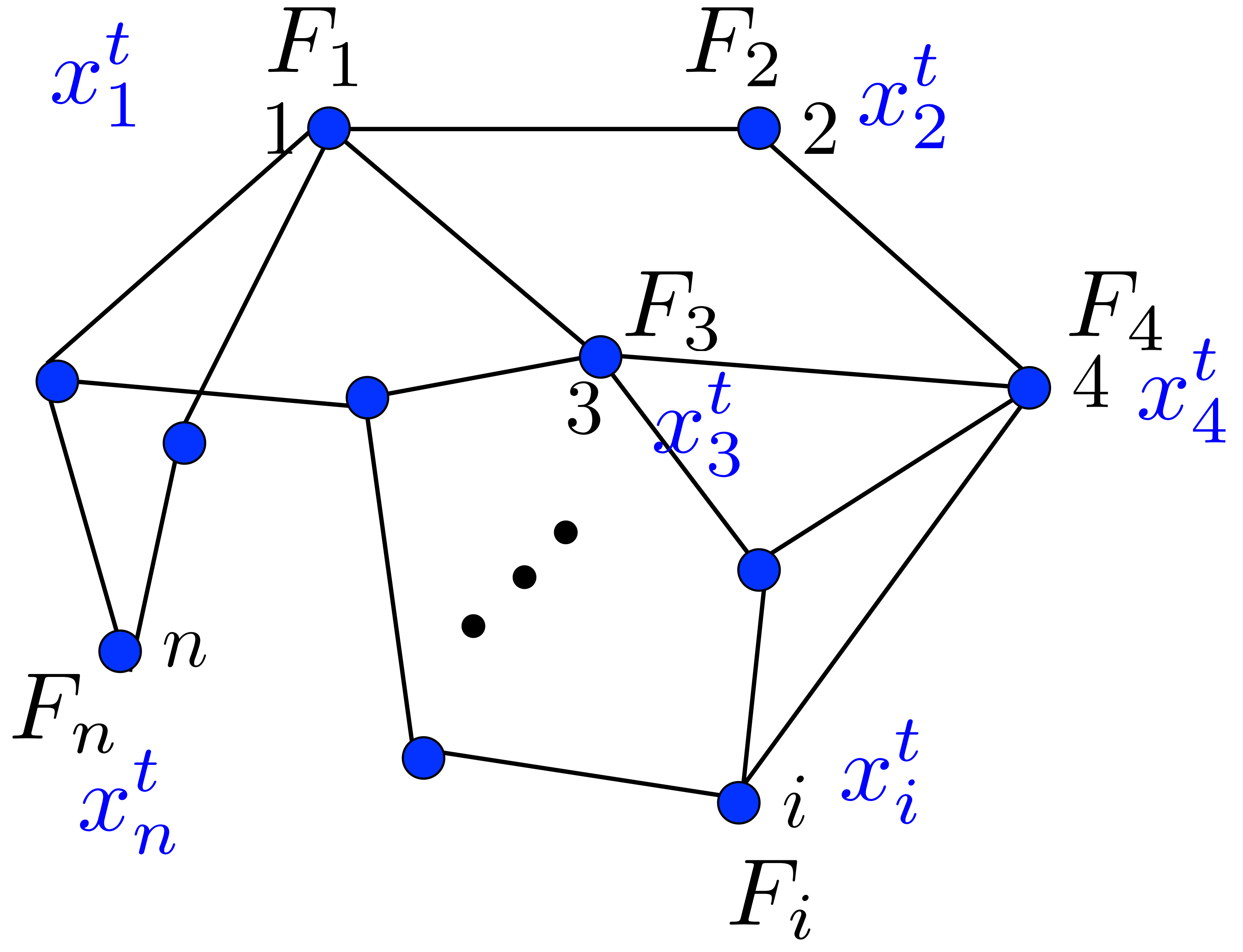


The Continuous Greedy Algorithm provides a tight (1-1/e)-optimum solution

[Calinescu, Chekuri, Vondrak 2011]

Decentralized Continuous Greedy (DCG)

Replace by the multilinear extension



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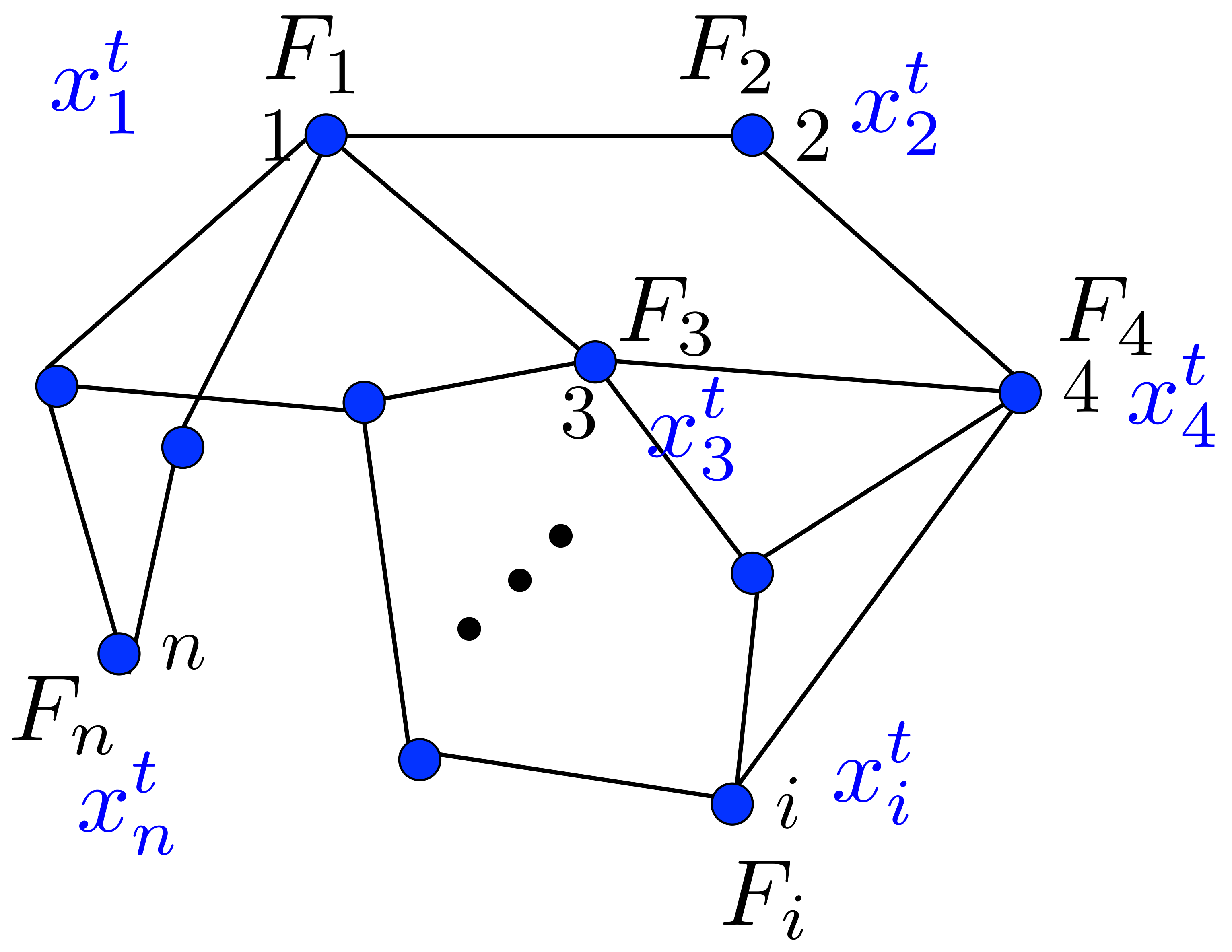

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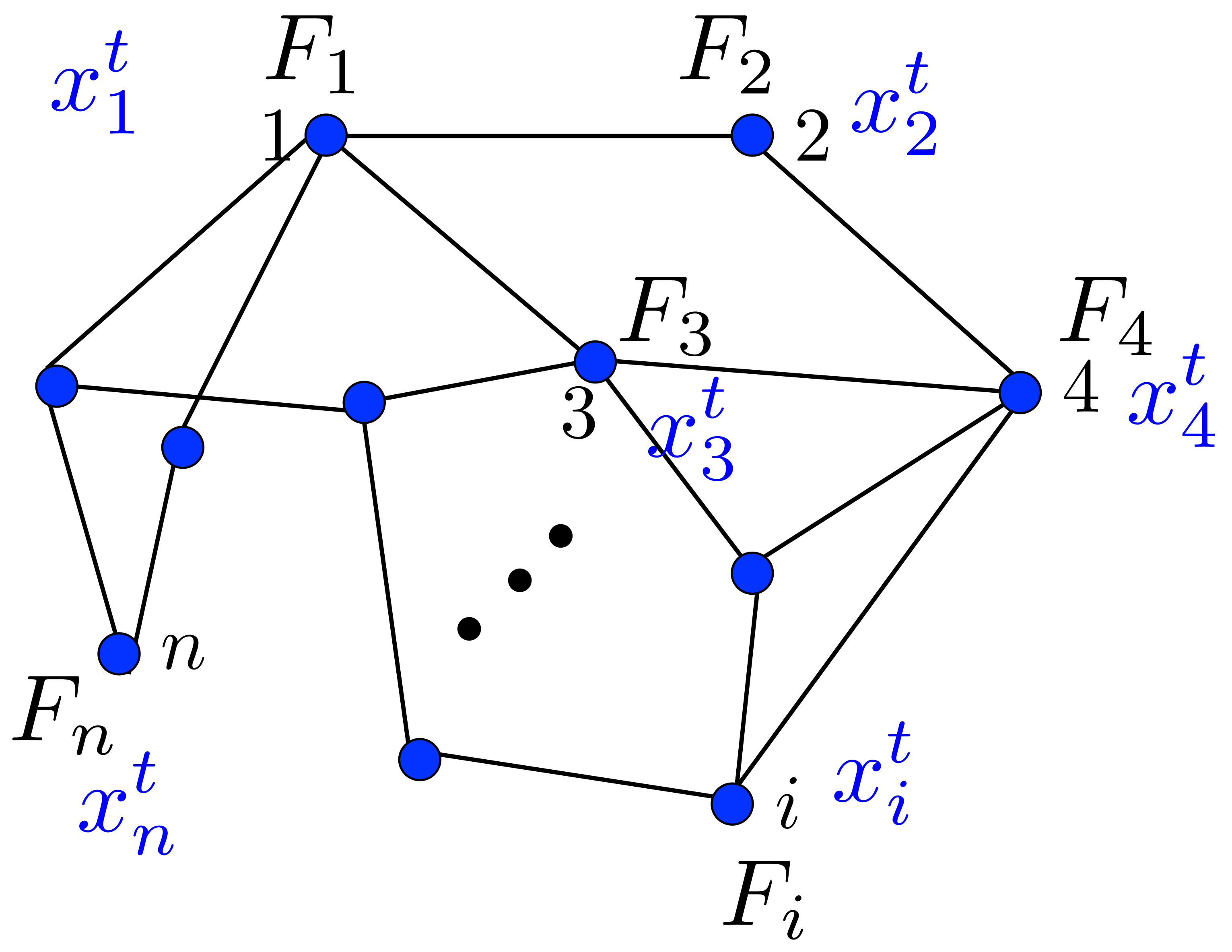

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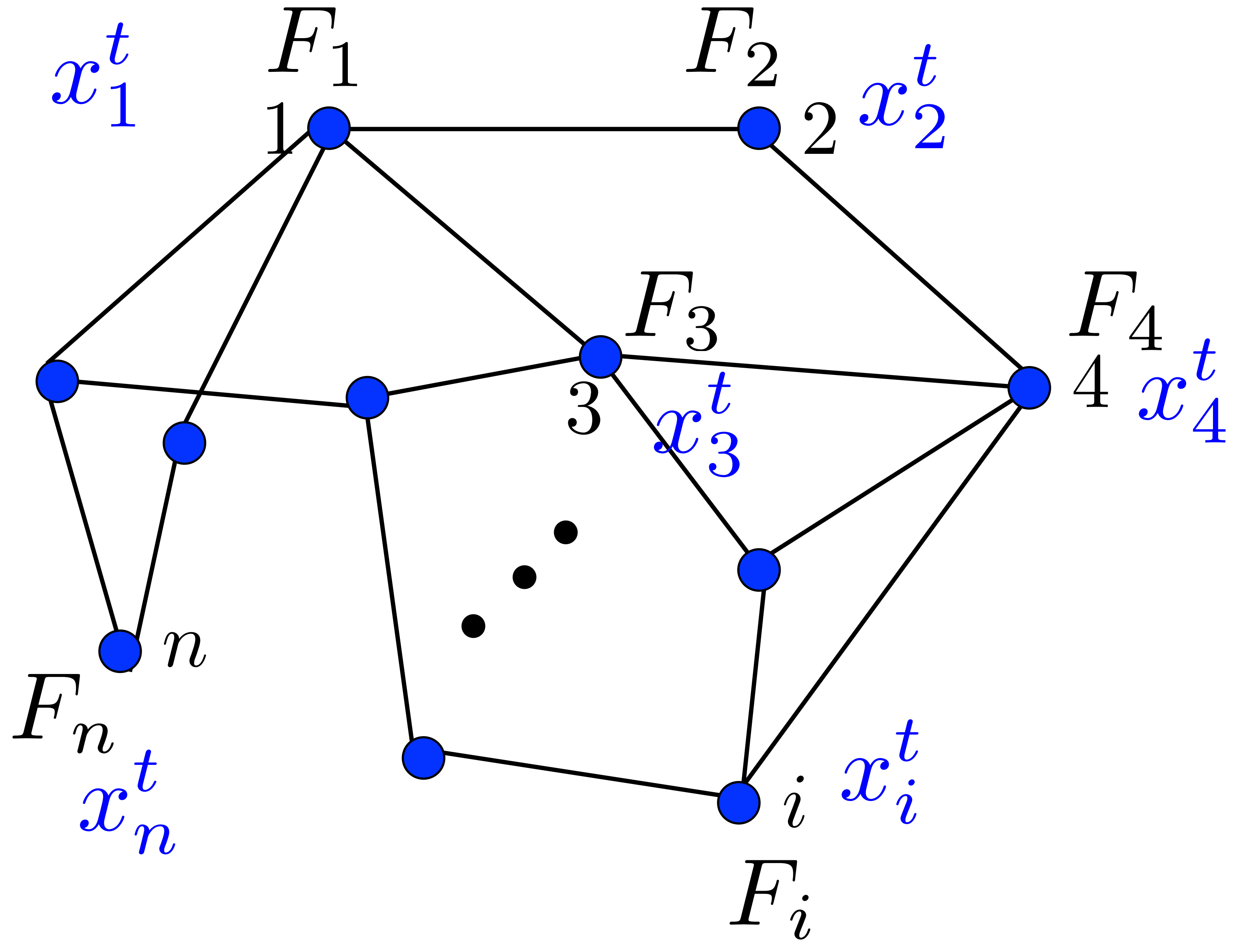

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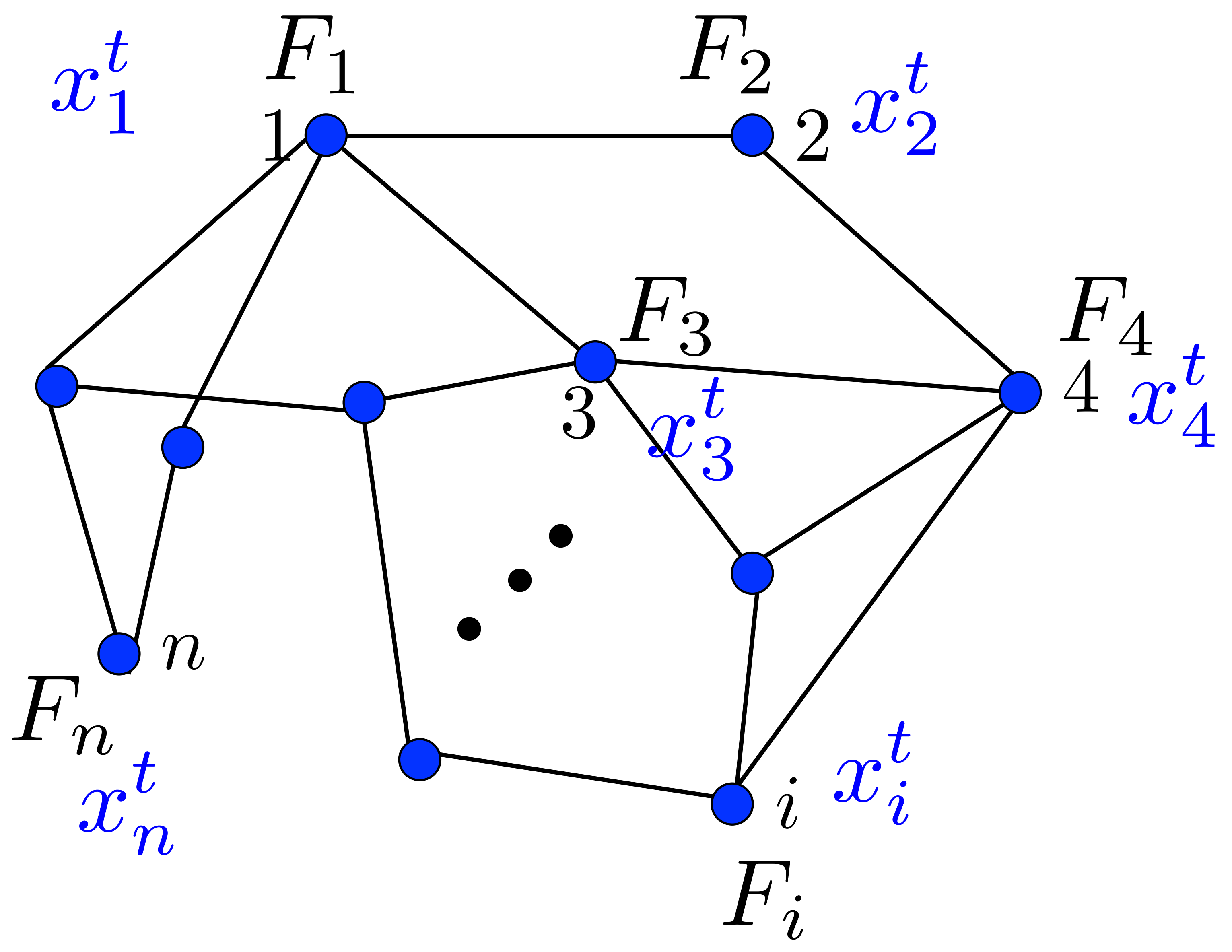

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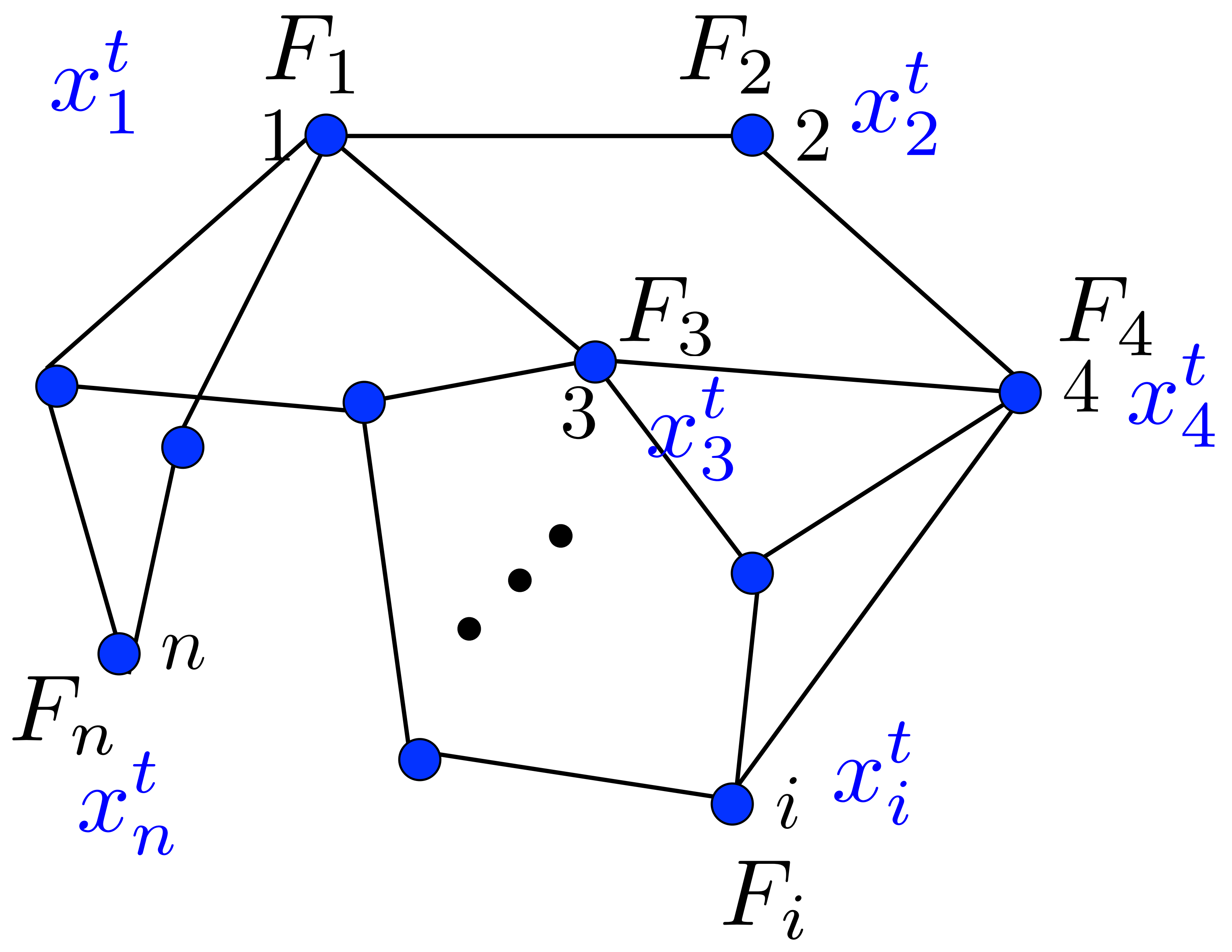

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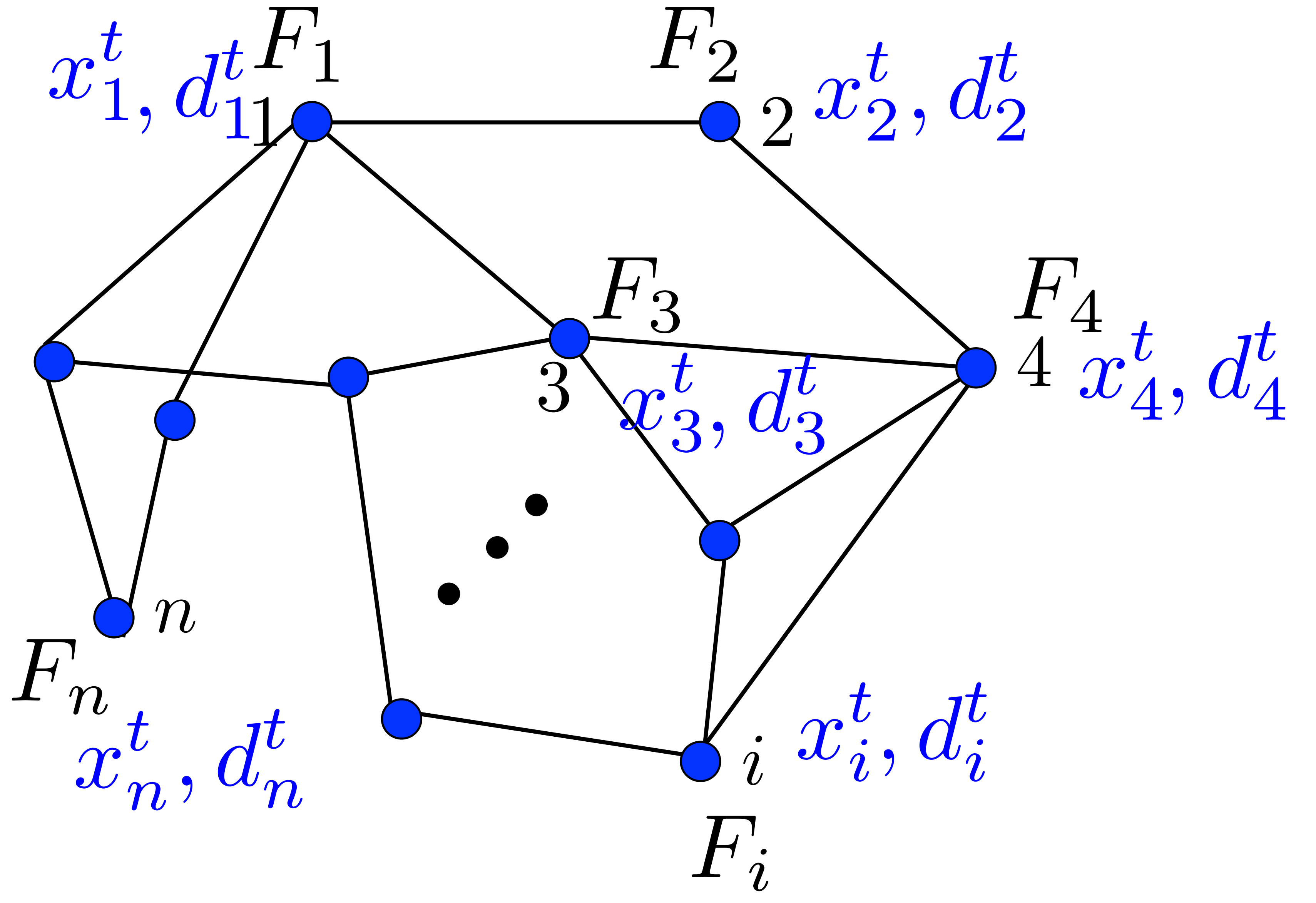

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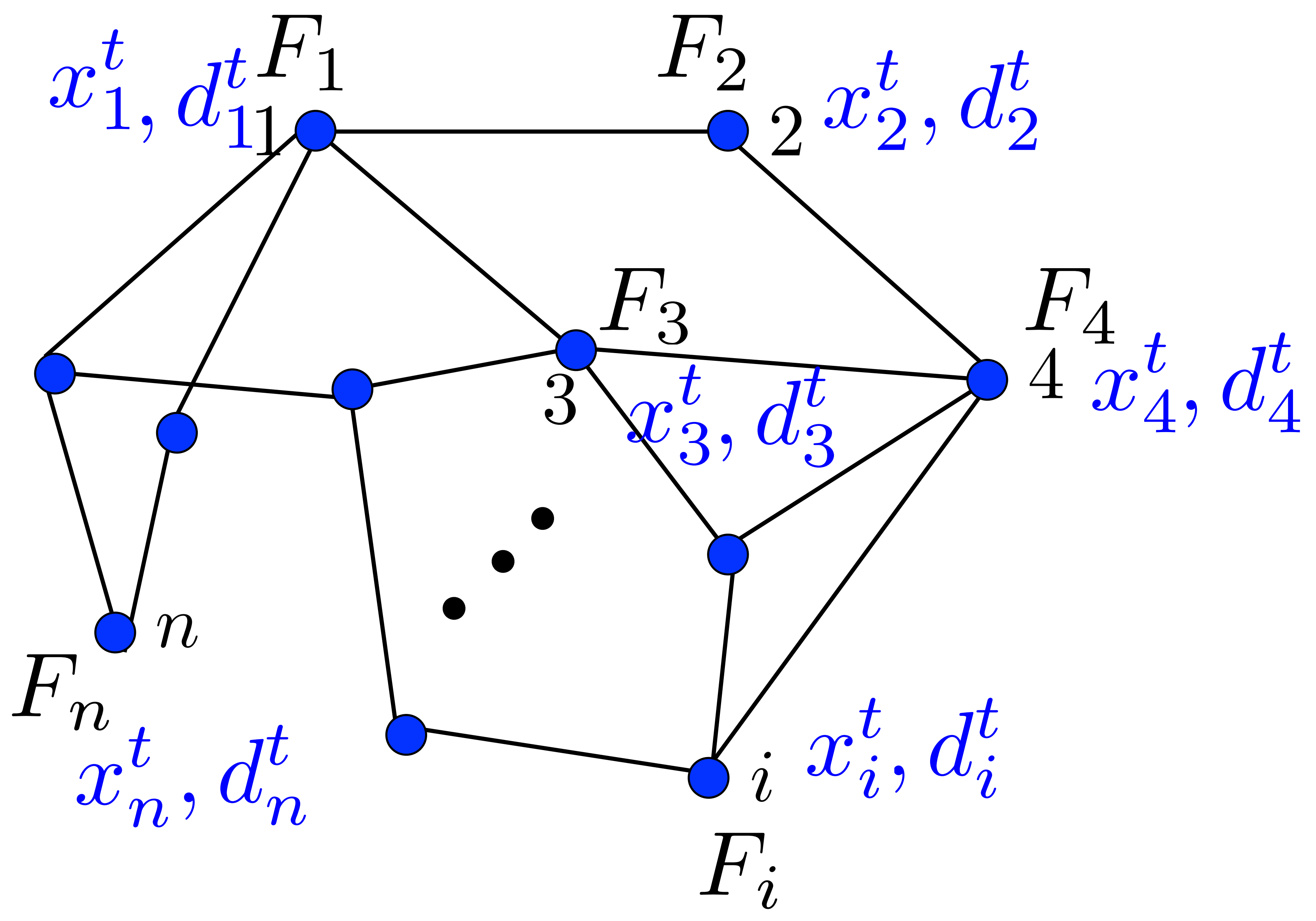

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consensus on gradients

Decentralized Continuous Greedy (DCG)

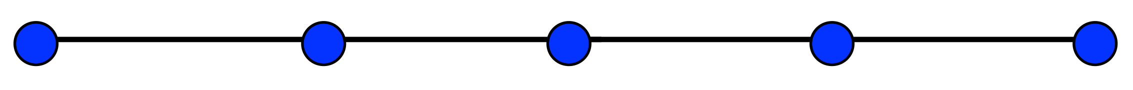
- Theorem: By choosing $\alpha = O(1/\sqrt{T})$, for any node j we have:

$$F(\mathbf{x}_j^T) \geq (1 - 1/e)F(\mathbf{x}^*) - \mathcal{O}\left(\frac{1}{(1-\beta)T^{1/2}}\right)$$

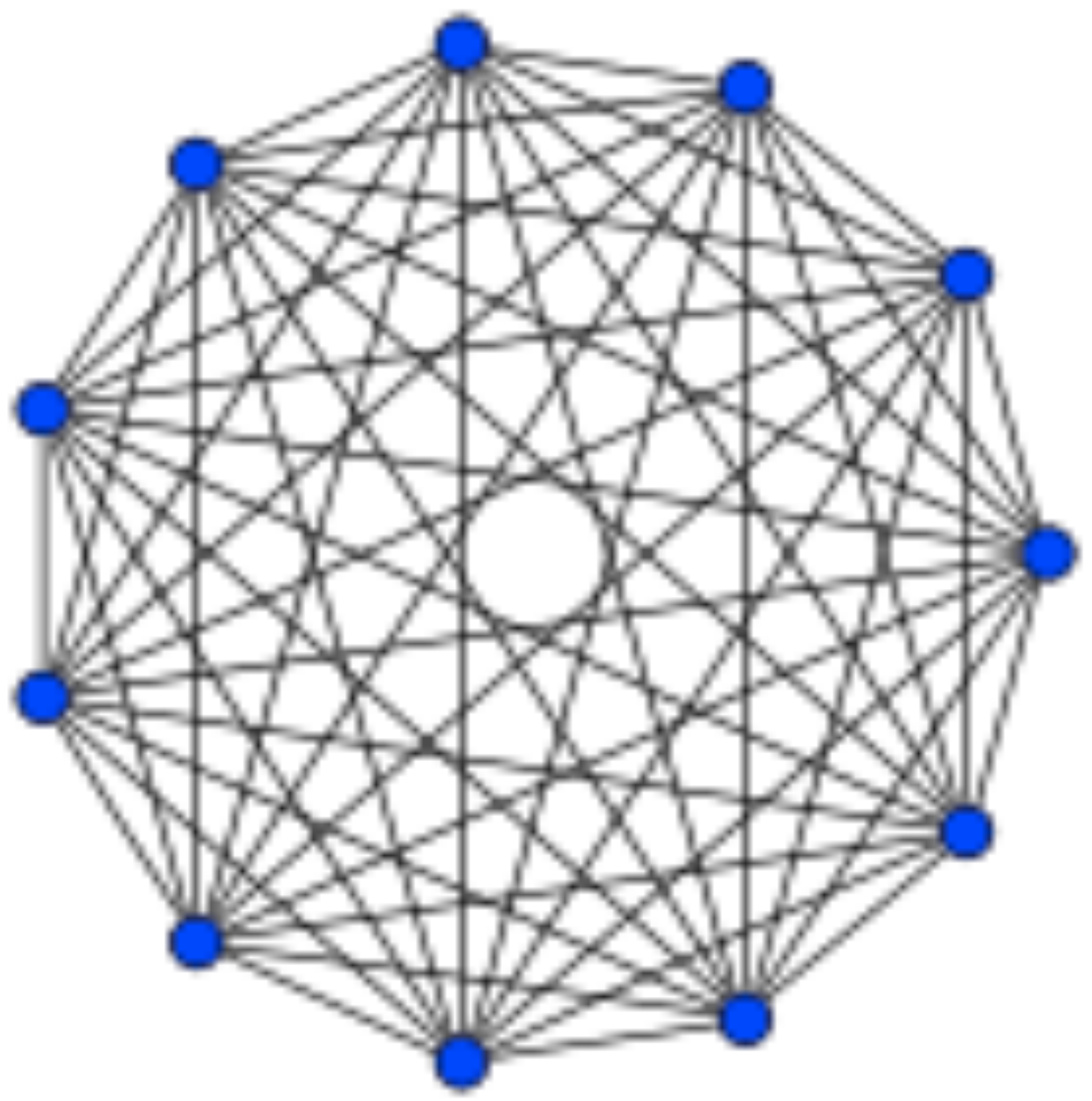
where β is the second largest magnitude of the eigenvalues of the weight matrix W.

- Note: There are many other details (computing gradients, assumptions, choice of the weights, etc)

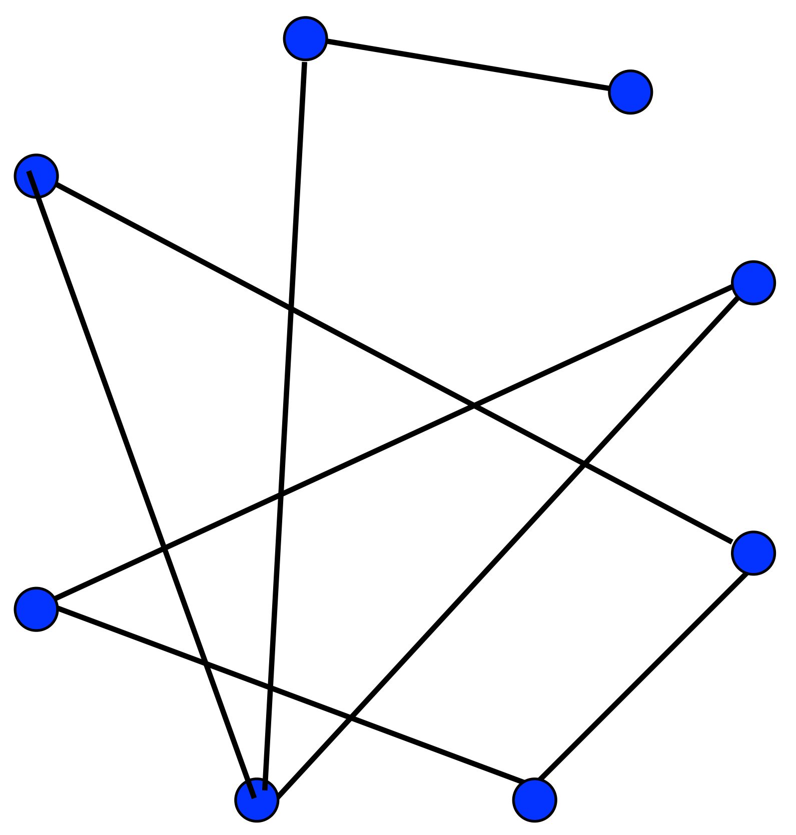
Experiments



Line graph



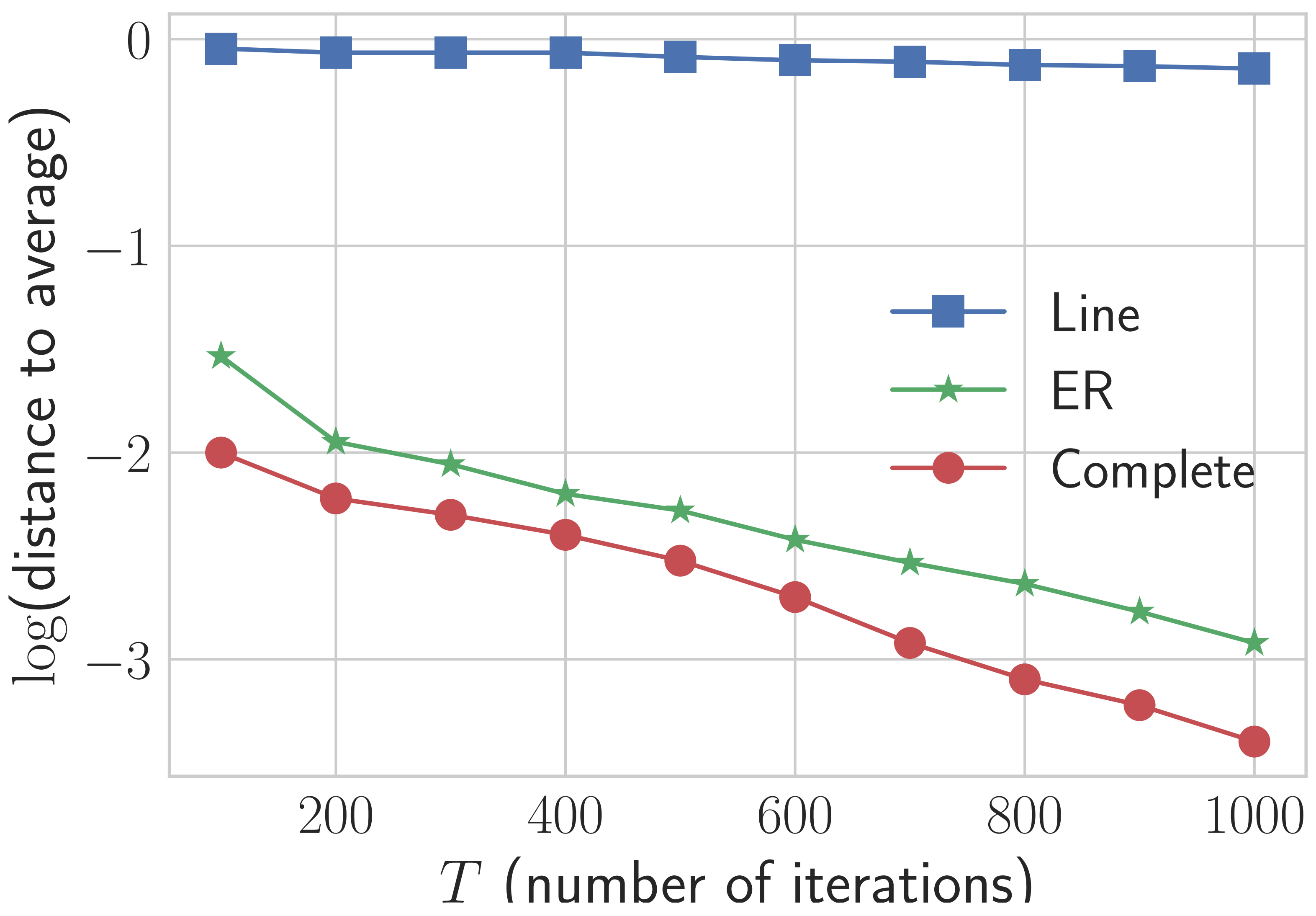
Complete graph

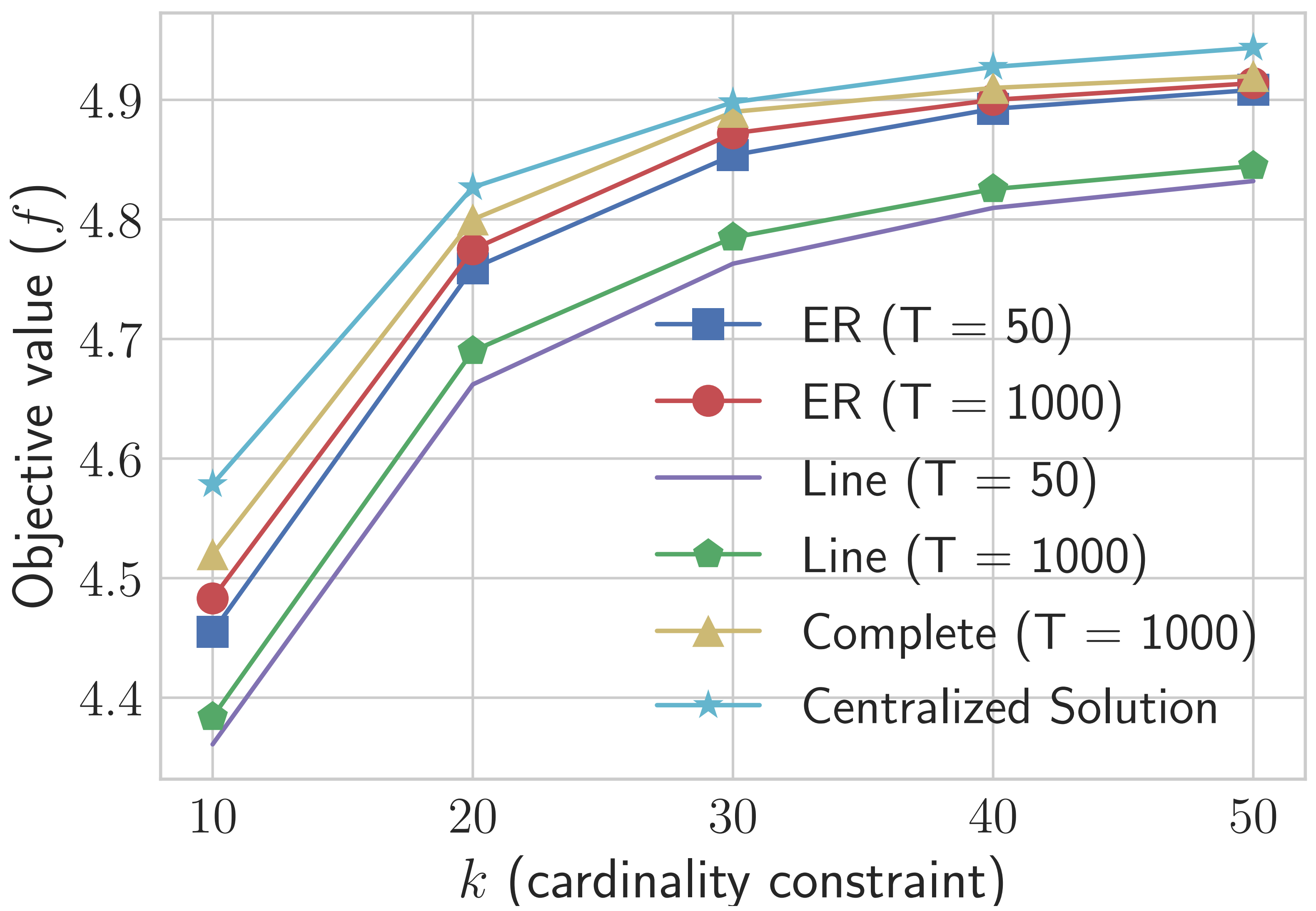


Erdos-Renyi Graph

Experiments

- MovieLens-1M data set, data distributed evenly between 100 nodes (units)
- Task: Find k movies that are most satisfactory
- Three types of networks: Line, Erdos-Renyi, Complete





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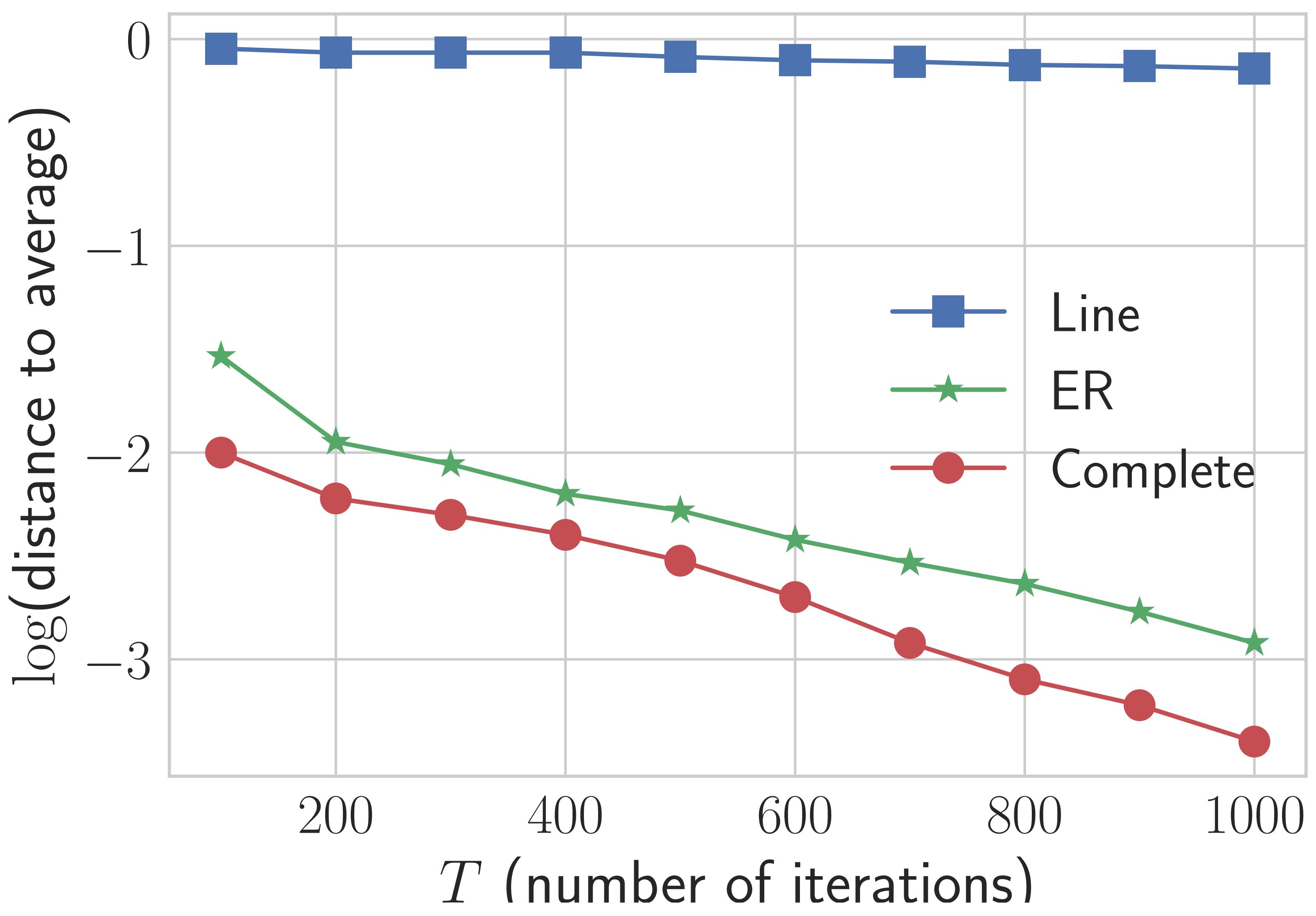

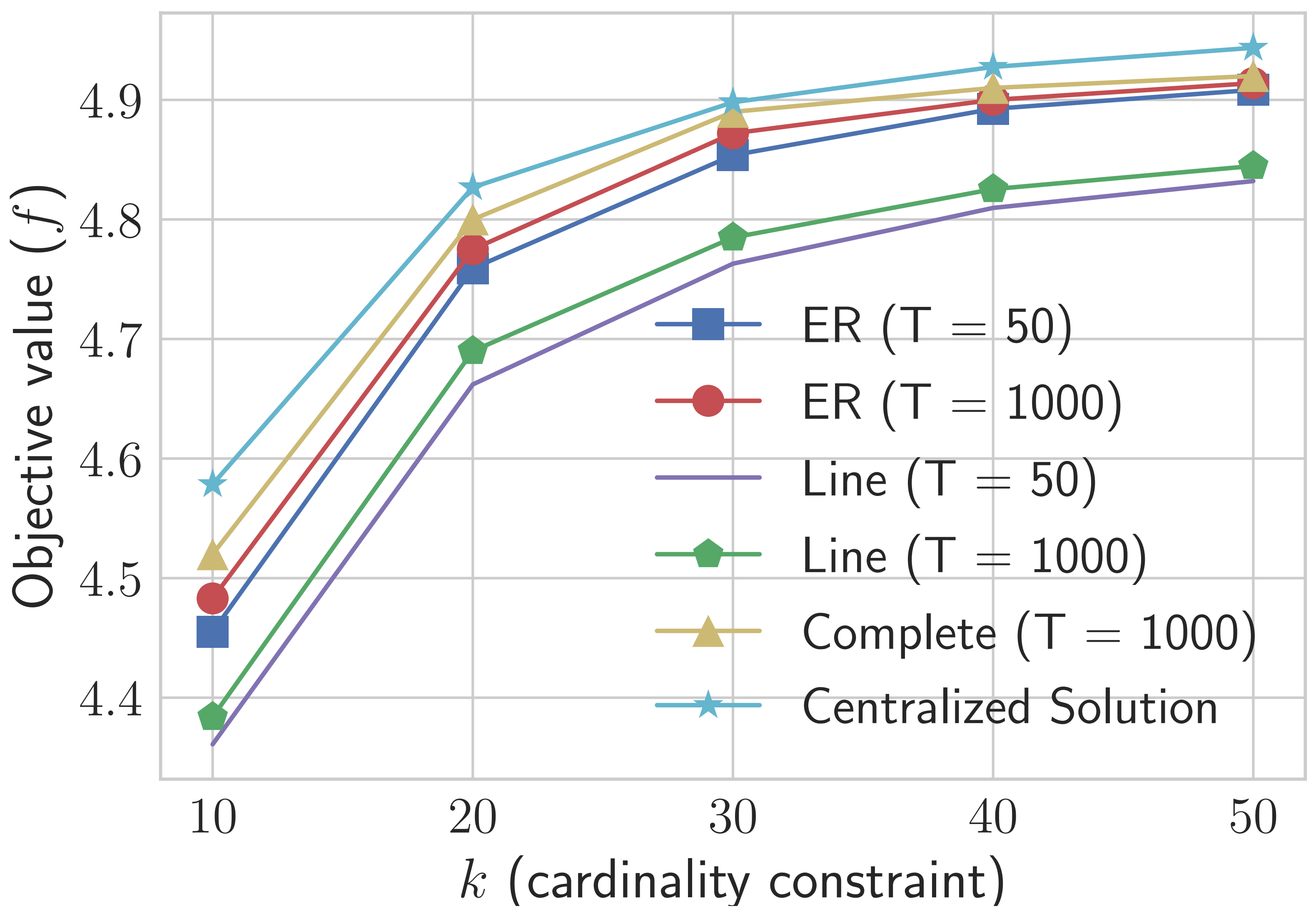

Thank you!