Novel Dihedral-Based Control of Flapping-Wing Aircraft With Application to Perching

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Abstract—We describe the design of an aerial robot inspired by birds and the underlying theoretical developments leading to novel control and closed-loop guidance algorithms for a perching maneuver. A unique feature of this robot is that it uses wing articulation to control the flight path angle as well as the heading angle. It lacks a vertical tail for improved agility, which results in unstable lateral-directional dynamics. New closed-loop motion planning algorithms with guaranteed stability are obtained by rewriting the flight dynamic equations in the spatial domain rather than as functions of time, after which dynamic inversion is employed. It is shown that nonlinear dynamic inversion naturally leads to proportional-integral-derivative controllers, thereby providing an exact method for tuning the gains. The capabilities of the proposed bioinspired robot design and its novel closed-loop perching controller have been successfully demonstrated with perched landings on a human hand.

Index Terms—Nonlinear control systems, robot control, robot motion, unmanned aerial vehicles.

I. INTRODUCTION

The recent interest in bioinspired robotic aircraft has led to the development of several insect-size aircraft [1], [9]–[11], [35], as well as bird-size micro aerial vehicles (MAVs) [6], [14], [24]. These developments have been driven by the hypothesis that the maneuverability and robustness of bird and insect flight can be replicated in engineered flight by judiciously adapting their actuation and control principles. In this paper, we present a bird-scale aerial robot concept, which uses wing articulation for control of gliding flight, motivated by avian flight [22], [23], [25]. This concept is primarily meant to be used in flapping wing aircraft where the wings are inherently articulated. Moreover, the lift-to-drag (L/D) ratios of bird-sized aircraft are suitable for occasional gliding, which is indeed employed routinely by birds for soaring, descending, and landing. The use of wing articulation for control also eliminates the need for redundant traditional actuators.

A perched landing is arguably the most challenging among all maneuvers executed in gliding flight because of two reasons: 1) Its duration is shorter than or on the same order as the time constants of the fast modes of the aircraft dynamics; and 2) a high level of accuracy is required for a successful perched landing, particularly if only a small area is available for landing. The aerial robot concept proposed in this paper lacks a vertical tail for improved agility, similar to birds, which renders it dynamically unstable and exacerbates both challenges listed above. Consequently, we choose a perching maneuver to demonstrate the capabilities of our articulated-winged aircraft concept, novel guidance algorithms, and control design. The ability to perform perched landings on the human hand is one part of a broader range of capabilities required to operate around humans (see Fig. 1).
Fig. 2. Common notations for the body axes of the flying vehicle (see Table I). The up-and-down angle of the wing ($\delta_R, \delta_L$), called the dihedral angle, measures the elevation of the wing with respect to the $y_B$ axis.

In the interests of ensuring a coherent presentation, we now introduce the standard notation used on flight dynamics. Thereafter, we will review the literature on perching and highlight the contributions of our work.

### A. Nomenclature and Preliminaries

Aircraft motion is traditionally split into two “groups”: motion in its nominal plane of symmetry (the $x_Bz_B$ plane in Fig. 2), called the longitudinal motion, and motion outside the plane of symmetry, called the lateral-directional motion. Note that rotations about $x_B, y_B,$ and $z_B$ axes are called roll, pitch, and yaw, respectively. Fig. 2 shows some important flight dynamic parameters, which have been listed together with other longitudinal and lateral-directional variables in Table I.

### B. Motivation From Nature

Birds and some species of bats spend a considerable amount of their flight time in either low frequency flapping or gliding flight, particularly while soaring, descending, or executing a perched landing. Perching is routinely used by birds to land on objects such as tree branches, power wires, building ledges, etc. The design of a typical perching maneuver is inspired by that of birds. Fig. 3 shows some snapshots of an owl performing a perched landing, extracted from a reputed BBC documentary called “Life of Birds.” The time histories of the flight path angle $\gamma$ (the angle made by the velocity vector in Fig. 2 with the horizontal plane), the body axis pitch angle $\theta$ (the angle made by the $x_B$ axis in Fig. 2 and the horizontal plane), and the angle of attack $\alpha$, obtained using image processing tools in MATLAB, are shown in Fig. 3.

The perching maneuver in the snapshots consists of two phases: 1) a gliding phase to bring the owl to a suitable position with respect to the landing spot (Snapshots A and B); and 2) a rapid pitch up to a poststall angle of attack accompanied by an instantaneous climb and rapid deceleration (Snapshots C and D). Perching maneuvers described in the literature as well as in this paper follow this two-step profile. Although the perching maneuver essentially involves controlling longitudinal parameters ($V_\infty$ and $\gamma$), the control of lateral-directional dynamics cannot be ignored, particularly when they are rendered unstable by the lack of a vertical tail.

### C. Literature Review

Flapping-wing aircraft have been studied in the literature on flight mechanics as well as biology. The literature on the structure of bird wings and the use of even specific sets of feathers for particular maneuvers is quite extensive (see [3], [31], [32], and the references cited therein). It is also known that birds fold their wings during rapid rolls [3], and that the wing dihedral is used to control the lateral-directional stability and to control the motion in the longitudinal plane [28]–[30]. Although it has been argued that birds are stable despite the lack of a vertical tail [29], prior work by the authors [23] has shown otherwise.
In [2] and [15], a canted winglet concept whose actuation mechanism is similar to ours was demonstrated; however, its function was different in that it was used for roll control rather than yaw control.

A perching maneuver was studied analytically in [8], with the physically intuitive conclusion that a simple pitch up, with the elevator deflected upward to the maximum possible extent, is sufficient to achieve the rapid deceleration and flattening of the flight path required for perching. In other words, the profile shown in Fig. 3 was recovered analytically. It was demonstrated in [27] that perching aircraft do not lose their controllability even at the low flight speeds achieved during perching and are consequently able to reject gusts and disturbances. Improvements resulting from the use of variable wing twist and movable tail boom were studied in [33] and [34]. In our prior work [5], a linear quadratic Gaussian (LQG) controller was designed to execute a perching maneuver using a combination of wing twist and wing dihedral (see Fig. 2) as control inputs.

An optimized perching maneuver was demonstrated in [7] on a robotic glider. Interestingly, the optimized strategy yielded a maneuver profile similar to that seen in Fig. 3: a “nominal” glide followed by a pitch up with maximum upward elevator deflection. Recently, perching was demonstrated on a robotic aircraft which used spines attached to its legs to attach itself to a wall following a pitch up to a vertical attitude [12]. The region of attraction for ensuring a successful perching maneuver after incorporating the dynamics and constraints of the attachment mechanism was computed in [16].

Perching maneuvers considered in the literature [7], [8], [12], [27], [33], [34] were strictly longitudinal, and the aircraft considered therein were laterally and directionally stable. Consequently, they ignored the lateral-directional motion of the aircraft. While it is true that a perching maneuver fundamentally involves controlling longitudinal flight parameters, viz., the speed $V_{\infty}$ and the flight path angle $\gamma$, the success of the maneuver can be severely impeded by the lateral-directional motion, particularly when the perched landing has to be accomplished on a small surface such as an electric pole or a human palm (see Fig. 1). The need for controlling the lateral-directional motion becomes critical if the aircraft lacks a vertical tail, as birds do, and the lateral-directional dynamics are highly unstable with a time constant that matches the duration of a typical perching maneuver [13], [23]. In the absence of a vertical tail, the ability to change the wing dihedral angle is a promising capability, which can be used for both longitudinal and lateral-directional control [23].

Prior experiments by the authors [13], [26] demonstrated the feasibility of using the wing dihedral for longitudinal and lateral-directional flight control and covered the two key elements of perching: control of flight path ($\gamma$) with controlled lateral-directional control, and the pitch up. The theoretical foundations for the aircraft concept were laid in [22] and [23].

D. Objectives and Contributions

The primary objective of this paper is to demonstrate the practical viability of using wing dihedral (i.e., the “flapping” motion) for longitudinal as well as lateral-directional control during gliding maneuvers, particularly perching. The contributions of this paper are as follows.

1) We demonstrate lateral-directional control using asymmetric wing dihedral. The problem of nonuniformity in the sign of the yaw control effectiveness of antisymmetric wing dihedral was identified in [23], while Dorothy et al. [13] proposed a trailing edge flap-based approach to overcome it. This paper experimentally validates the proposition in [13], which is a unique application of both asymmetric wing dihedral and trailing edge flaps for flight control.

2) We design novel control and closed-loop guidance laws for perching. We present an equivalence relationship between dynamic inversion (DI)-based controllers for nonlinear systems, and the conventional proportional-integral-derivative (PID) controllers, along with an exact method to tune the gains. Novel closed-loop guidance algorithms are derived for the flight path angle $\gamma$ and heading $\chi$, whose dynamics are cast into a strict feedback form by rewriting them in the spatial domain rather than the time domain. The strict feedback form is amenable to applying DI and backstepping. The tracking performance and the stability of the closed loop are proven rigorously.

3) We show that it is sufficient to command the pitch up leading to a perched landing by using information only about the position of the aircraft, and independently of the flight speed and flight path angle at the time of commencing the pitch up. We demonstrate that after a pitch up is executed at a fixed altitude, the position of the landing point and the touch-down speed vary within a tolerable range for a broad range of speeds and flight path angles at the time of initiating the pitch up. Therefore, it suffices for the aforementioned guidance algorithm to achieve prescribed terminal coordinates at the end of the glide phase (i.e., at the time of commencing the pitch up) without needing to ensure any particular value of the speed and flight path angle at that instant.

This paper is organized as follows. The background material on flight dynamics with articulated wings [13], [23] is presented in Section II. The control law design is described in Section III. It is shown that nonlinear DI naturally leads to equivalent proportional-integral-derivative (PID) controllers, with exact gain tuning rules. Guidance and control laws for perching, successfully tested in the experiments, are described in Section IV. Experimental results are discussed in Section V.

II. FLIGHT MECHANICS OF WING ARTICULATION

In this section, we briefly review the theoretical underpinnings of the use of wing dihedral for longitudinal and lateral-directional control [23]. The equations of motion are presented next, with a brief review of the yaw dynamics and control. Finally, we present a novel scheme based on trailing edge flaps to overcome the controllability problems that arise from the use of wing dihedral for yaw control.
A. Using Wing Dihedral Angles (δL and δR) for Control

The equations of motion of articulated-wing aircraft are highly nonlinear and make the comparison highly case specific. The vertical tail plays a critical role in determining the yawing moment, which is more effective than the vertical tail and rudder at high angles of attack, particularly because the performance of the vertical tail degrades rapidly in the wake of the wing under those conditions. The longitudinal placement (i.e., the x-coordinate) of the CG plays a critical role in determining the yawing moment, which makes the comparison highly case specific. The vertical tail is strictly speaking redundant in a flapping-wing setting such as the one explored here. Flapping wings are themselves capable of ensuring roll and yaw control, which makes a vertical tail unnecessary.

B. Equations of Motion of Articulated-Wing Aircraft

The rigid body flight dynamics, together with the aerodynamics and kinematics of articulated wings, are highly nonlinear. The equations of motion, ignoring terms that arise from the angular velocity of the wing motion (due to flapping), have essentially the following structure [22], [23]:

\[
\begin{align*}
& m(\dot{\mathbf{u}}_B + S(\omega_B)\mathbf{u}_B + (S(\omega_B) + S^2(\omega_B))\mathbf{r}_{cg}) = \mathbf{F}_{\text{net}} \\
& \mathbf{J}\dot{\omega}_B + S(\omega_B)\mathbf{J}\omega_B + m(S(\mathbf{r}_{cg})\mathbf{u}_B \\
& \quad + S(\omega_B)S(\mathbf{r}_{cg})\mathbf{u}_B) = \mathbf{M}_{\text{net}}
\end{align*}
\]

where \( m \) denotes the mass of the aircraft, \( \mathbf{u}_B \) is the body velocity, \( \mathbf{r}_{cg} \) is the position of the center of gravity, \( \mathbf{F}_{\text{net}} \) and \( \mathbf{M}_{\text{net}} \) are the net forces and moments acting on the aircraft, \( \omega_B \) is the angular velocity of the aircraft, \( \mathbf{J} \) is the inertia tensor of the aircraft, and \( S(\mathbf{u}) \) denotes the lift, drag, and pitching moments due to lift, drag, and pitching moment coefficients, respectively.

The pressure distribution due to the flow around an airfoil produces forces and moments shown in Fig. 4. Lift is perpendicular to the local wind velocity and acts in the plane of the airfoil, while drag acts along the local wind velocity. The quarter-chord pitching moment \( M_{ac} \) is independent of the angle of attack. These quantities are typically written in terms of nondimensional coefficients \( C_L(\alpha) \), \( C_D(\alpha) \), and \( C_{m,ac} \) (called the coefficients of lift, drag, and quarter-chord pitching moment)

\[
\begin{align*}
L &= \frac{1}{2} \rho V^2_c c C_L(\alpha) \\
D &= \frac{1}{2} \rho V^2_c c C_D(\alpha) \\
M_{ac} &= \frac{1}{2} \rho V^2_c c^2 C_{m,ac}
\end{align*}
\]

where \( c \) denotes the chord length (see Fig. 4).

Fig. 5 illustrates the physics underlying the use of dihedral as a control input. The key point is that changing the wing dihedral reorients the lift vector with respect to the aircraft z-axis, thereby altering the net force acting in the body z-direction and generating a side force.

Increasing the wing dihedral reduces the net z force, which manifests in the form of a reduction in the net lift acting on the aircraft, accompanied by an incommensurately small reduction in the drag force. Thus, changing the wing dihedral angle alters the L/D ratio of the aircraft and offers the option of controlling the flight path angle and the aircraft speed independently of each other [23]. Fig. 6 shows the flight path angle as a function of the symmetric wing dihedral deflection. The points shown in the figure are equilibria computed at the same flight speed of 3 m/s.

On the other hand, the side force can be used to provide the centripetal acceleration for turning, and as a source of yawing moment. In particular, if the CG is located behind the line of action of the side force, then a positive (rightward) side force produces a positive yawing moment and vice versa. It follows that a positive rolling moment (wherein the lift on the left wing is higher than the right wing) is accompanied by a positive yawing moment if the wings have a positive dihedral deflection. This effect mitigates a phenomenon known as adverse yaw, wherein a positive rolling moment is accompanied by a negative yawing moment. This is a common problem that is encountered by aircraft, which lacks a vertical tail and inhibits their lateral-directional performance if not addressed properly.

Conventional fixed-wing aircraft employ a vertical tail for lateral-directional control. It was argued in [23] that the dihedral is more effective than the vertical tail and rudder at high angles of attack, particularly because the performance of the vertical tail degrades rapidly in the wake of the wing under those conditions. The longitudinal placement (i.e., the x-coordinate) of the CG plays a critical role in determining the yawing moment, which makes the comparison highly case specific. The vertical tail is strictly speaking redundant in a flapping-wing setting such as the one explored here. Flapping wings are themselves capable of ensuring roll and yaw control, which makes a vertical tail unnecessary.
where \( m \) is the total mass of the aircraft, \( \mathbf{J} \) is the moment of inertia tensor for the aircraft, \( S(\cdot) \) denotes a vector product, and \( \mathbf{F}_{\text{net}} \) and \( \mathbf{M}_{\text{net}} \) represent the net external (aerodynamic + gravitational) force and moment on the aircraft, respectively. Furthermore, \( \mathbf{\omega}_B = [p, q, r]^T \) is the vector representation of the aircraft angular velocity of the aircraft, with components in the aircraft body axes. The net aerodynamic force depends on the wing orientation, as discussed in Section II-A. The position of the aircraft's center of gravity with respect to a zero dihedral configuration is denoted by \( \mathbf{r}_{cg} \) which is, in turn, approximated closely by

\[
\mathbf{r}_{cg} = \frac{m_w b}{4m} [0, (\cos \delta_R - \cos \delta_L), -(\sin \delta_L + \sin \delta_R)]^T
\]

where \( \delta_L \) and \( \delta_R \) are the dihedral angles of the left and right wings, \( m_w \) is the mass of each wing, and \( b \) is the total wing span (so that each wing has length \( b/2 \)).

The flight path angle \( \gamma \) is given by [23]

\[
\sin \gamma = \cos \alpha \cos \beta \sin \theta - \sin \beta \sin \phi \cos \theta - \sin \alpha \cos \beta \cos \phi \cos \theta
\]

while the wind axis heading angle \( \chi \) is calculated as follows:

\[
\sin \chi \cos \gamma = \cos \alpha \cos \beta \cos \psi \sin \psi + \sin \beta \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi
\]

\[
+ \sin \alpha \cos \beta (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi).
\]

The force and moment vectors \([\mathbf{F}_{\text{net}}, \mathbf{M}_{\text{net}}]\) in (2) depend strongly on the dihedral angles of the wings. In order to appreciate this point, we note, for example, that the yawing moment component of \( \mathbf{M}_{\text{net}} \) is given by [23]

\[
N = (Z_{w,L}(\alpha_L) x_a c + M_{ac,L}) \sin \delta_L - (Z_{w,R}(\alpha_R) x_a c + M_{ac,R}) \sin \delta_R
\]

where \( \alpha_L \) and \( \alpha_R \) are local angles of attack of the left and right wings (which vary as functions of the spanwise coordinate), \( x_a c \) (with \( x_a \) nondimensional) is the distance between the aerodynamic center of the wing and center of gravity, and \( Z_{w,L} \) and \( Z_{w,R} \) are the local \( z \)-forces on the two wings (see Fig. 5). The terms \( \alpha_L \) and \( \alpha_R \) themselves depend on \( \delta_L \) and \( \delta_R \), respectively [23]

\[
\alpha_R \approx \beta \sin \delta_R + \alpha \cos \delta_R + \frac{p y + r x_a c \sin \delta_R}{u}
\]

\[
+ \frac{r y}{u} \cos \delta_R + \frac{p r y^2 \cos \delta_R}{u^2}
\]

where the sideslip \( \beta \), roll rate \( p \), and yaw rate \( r \) were defined in Fig. 2. A similar expression can be written for \( \alpha_L \).

C. Use of Trailing Edge Flaps for Mitigating Control Effectiveness Problems

The yaw control effectiveness of the antisymmetric dihedral (\( \delta_{\text{asym}} := (\delta_L - \delta_R)/2 \), defined in Table I) depends not only on the angle of attack, but also on the angular rates. We define the yaw control effectiveness of antisymmetric dihedral as the partial derivative \( \frac{\partial N}{\partial \delta_{\text{asym}}^2} = N_{\delta_{\text{asym}}} \), with the yawing moment \( N \) given by (5) so that

\[
N_{\delta_{\text{asym}}} = (Z_{w:L}(\alpha_L) + Z_{w:R}(\alpha_R)) x_a c + M_{ac,L} + M_{ac,R}
\]

when \( \delta_L \) and \( \delta_R \) are nominally set to zero. Under conditions of symmetric flight, it follows that

\[
\text{sign}(N_{\delta_{\text{asym}}}) = \text{sign}(x_a C_L + C_{m,ac})
\]

where \( C_L \) and \( C_{m,ac} \) are the coefficients of lift and quarter chord pitching moment, introduced in (1). The term \( x_a \) denotes the nondimensional (with respect to chord length) distance between the center of gravity and the quarter-chord line. For positively cambered wings, \( C_{m,ac} < 0 \), and therefore, at small angles of attack, where \( C_L \) is small, the control effectiveness is negative. At higher angles of attack, the control effectiveness is positive. This disparity in the sign of the control effectiveness, or more precisely the negative control effectiveness, may be viewed as arising from a deficiency in the local angle of attack on the wing. Consequently, for an intermediate range of angles of attack, the sign is sensitive to the angular rates as well, as illustrated in Fig. 7 [see (6)]. The nonuniform sign of the control effectiveness can cause problems for yaw control, particularly when the angle of attack varies across the three regions in the course of a maneuver and can be mitigated by using trailing edge flaps.

Trailing edge flap deflection leads to a greater increase in \( C_L \) as compared with the reduction in \( C_{m,ac} \). Using thin airfoil theory [20], it can be shown that the change in \( C_L \) and \( C_{m,ac} \) due to a flap deflection \( \delta_f \) is given by

\[
\Delta C_L = (2(\pi - \theta_f) + 2 \sin \theta_f) \delta_f
\]

\[
\Delta C_{m,ac} = -\frac{\delta_f}{2} \sin \theta_f \cos(\theta_f - 1)
\]

where \( \theta_f \in [0, \pi] \) depends on the location of the flap (\( \mu_f \), nondimensionalized with respect to \( c \)) from the leading edge

\[
\cos \theta_f = 1 - 2\mu_f.
\]

The term \( \theta_f \) is defined purely for mathematical convenience in thin airfoil theory.

Note that the change in \( C_L \) in (8) is additive so that flap deflection effectively compensates for any deficiency in the local
angle of attack, including that which arises from unfavorable angular rates.

For the aircraft considered in this paper, shown in Fig. 12, \( \mu_f \approx 0.8 \) and \( \kappa_f = 0.25 \). Thus, \( \theta_f = 2.2143, \Delta C_L = 3.45\delta_f \), and \( \Delta C_{m,ac} = -0.145\delta_f \). Furthermore, from the data in [23], \( C_L(\delta_f = 0) = 0.28 + 2\alpha \) and \( C_{m,ac}(\delta_f = 0) = -0.1311 \).

It is of interest to find the flap deflection, as a function of \( \alpha \), which will guarantee a certain positive control effectiveness. For example, suppose that we need the effectiveness to be at least 0.025 (corresponding to an \( \alpha \) of 10° in Fig. 8). Then, substituting the expressions for \( \Delta C_L \) and \( \Delta C_{m,ac} \), it follows that

\[
\frac{C_L}{4} + C_{m,ac} + 0.72\delta_f = 0.025
\]

\[
0.07 + 0.5\alpha - 0.1311 + 0.72\delta_f = 0.025
\]

\[
\Rightarrow \delta_f = 0.12 - 0.69\alpha \text{[rad].}
\]

Thus, flap deflection of nearly 7° is required at \( \alpha = 0 \), and no flap deflection is required beyond \( \alpha = 10^\circ \).

The benefit of a uniformly positive control effectiveness, however, comes at a price. The aircraft is forced to fly in a high lift (it can be checked that \( C_L > 0.64 \), high drag configuration across the flight envelope, lowering the aircraft’s speed. However, the flight path angle can still be controlled effectively by changing the dihedral angle symmetrically.

### III. STABILITY THEOREMS FOR CONTROL LAW DESIGN

Consider the problem of controlling the yaw (\( \beta \) and \( r \)) dynamics. The yawing moment is given by (5) and (6). It is evident that the flight dynamics of aircraft with articulated wings are nonlinear and nonaffine in control. However, it is possible to use some knowledge about the flight dynamics to simplify control design. In particular, the pitch dynamics \( q, \alpha \) can be controlled entirely by the elevator, and almost always independently of the lateral-directional dynamics. This is true for most aircraft, except those that lack a horizontal tail. Moreover, the pitch dynamics of our aerial robot are stable, and the only source of instability is the lateral-directional \( (\beta, r) \) dynamics (due to the absence of a vertical tail). The roll dynamics are stable as well and much faster than the yaw and pitch dynamics. The rolling motion is not controlled directly in this paper.

For the purpose of controlling the yaw dynamics, (2) can be cast into the following control nonaffine form:

\[
\dot{\eta}(t) = f(t, \eta(t), \kappa(t), u(t))
\]

\[
\dot{\kappa}(t) = \zeta(t, \eta(t), \kappa(t), u(t), v(t))
\]

where \( \eta(t) = (\beta(t), r(t)) \in \mathbb{R}^2 \) represents the yaw dynamics, while \( u(t) \in \mathbb{R} \) is the yaw control input, viz., the asymmetric dihedral deflection \( (\delta_{asym}) \). The term \( v(t) \) represents other control inputs, namely the elevator deflection \( (\delta_e) \) and symmetric dihedral deflection \( (\delta_{sym}) \), which are used for longitudinal flight control. Finally, \( \kappa \in \mathbb{R}^6 = [V_x, \alpha, p, q, \theta, \phi] \) represents the rolling and pitching motion, as well as translation in the plane of symmetry (see Fig. 2). The flight dynamic modes corresponding to these six states are known to be stable [23]. One of the control objectives is to stabilize \( \eta = (\beta(t), r(t)) \) in (11), and ensure that it tracks a desired trajectory.

#### A. Dynamic Inversion and PI(D) Control

In this section, we show that a class of DI control laws can be simplified into traditional proportional-integral (PI) or PID controllers. Moreover, the process of simplification yields exact gain tuning laws, which allows the control gains to be linked explicitly to the desired convergence properties of the closed-loop system as well as the tracking error bound. Consider a general system described by (11), where \( \eta \) is no longer the yaw dynamics, but represents the state variables of interest for the purpose for control design. For now, we impose the additional condition \( \eta(t) \in D_\eta \subset \mathbb{R} \), where \( D_\eta \) is compact. We will consider the case \( \eta(t) \in \mathbb{R}^2 \) later in the section (see Theorem 2). Let \( e(t) = \eta(t) - \eta_\text{ref}(t) \) be the tracking error, where \( \eta_\text{ref}(t) \) denotes the reference signal. Then, the open-loop error dynamics are given by

\[
\dot{e}(t) = f(t, e(t) + \eta_\text{ref}(t), \kappa(t), u(t)) - \dot{\eta}_\text{ref}(t), e(0) = e_0
\]

\[
\dot{\kappa}(t) = \zeta(t, e(t) + \eta_\text{ref}(t), \kappa(t), u(t), v(t)), \kappa(0) = \kappa_0
\]

where \( f \) is assumed to be a continuously differentiable function of its arguments, and the unperturbed additional dynamics \( \dot{\kappa}(t) = \zeta(t, 0, \kappa(t), 0, v(t)) \) are assumed to be exponentially stabilized by the control input \( v(t) \) [see (11)]. We construct the DI controller

\[
e\dot{u}(t) = -\text{sign}\left( \frac{\partial f}{\partial \eta} \right) \tilde{f}(t, \eta, \kappa, u),
\]

with \( e > 0 \) sufficiently small, and

\[
\tilde{f}(t, \eta, \kappa, u) = f(t, e + \eta_\text{ref}, \kappa, u) - \dot{\eta}_\text{ref}(t) + a_m e(t)
\]

and \( a_m > 0 \) gives the desired rate of convergence of the closed-loop dynamics. We assume that the nonlinearity \( f(t, \eta(t), \kappa(t), u(t)) \) has an isolated root \( u_0(t, e + \eta_\text{ref}, \kappa) \) given by \( f_{\eta}(t, u_0(t, e + \eta_\text{ref}, \kappa)) = -a_m e(t) + \dot{\eta}_\text{ref}(t) \).

**Lemma 1** (see [17, Th. 1]): Given the system (12), the controller in (13) ensures that

1) the tracking error \( e(t) \sim \mathcal{O}(\epsilon) \); and
2) the control \( u(t) \) converges to the isolated root \( u_0 \) of (14); i.e., \( u(t) \) makes \( f(t, \eta(t), \kappa(t), u(t)) \to (-a_m \epsilon(t) + \hat{\eta}_d(t)) \).

The proof of this theorem is based on Tikhonov’s theorem ([18, Th. 11.1]) and may be found in [17]. We now state the main result relevant to our control design.

**Theorem 1:** The control law in (13) is equivalent to a PI controller with proportional \((k_p)\) and integral \((k_I)\) gains tuned to satisfy \( k_p = 1/\epsilon \) and \( k_I = a_m/\epsilon \), where \( a_m \) is the desired time constant for the closed-loop dynamics.

**Proof:** Since \( \dot{\eta} = f(t, \eta, \kappa, u) \), and \( \dot{e} = \hat{\eta} - \dot{\eta}_d \), we write the controller as

\[
e e(t) = -\text{sign}(\partial f/\partial u) f(t, \eta, \kappa, u) \]

\[
= -\text{sign}(\partial f/\partial u) (\dot{e}(t) + a_m e(t)) .
\]

Integrating both sides yields a PI controller of the form

\[
u(t) = u(0) - \text{sign}(\partial f/\partial u) \frac{1}{\epsilon} (e(t) - e(0) + a_m \int_0^t e(t) dt) .
\]

We choose \( u(0) = -\text{sign}(\partial f/\partial u) e(0)/\epsilon \). If \( k_p \) and \( k_I \) denote the proportional and integral gains of the PI controller, then they should be chosen to satisfy

\[
k_I = a_m/\epsilon, \text{ and } k_p = 1/\epsilon
\]

so that

\[
u(t) = -\text{sign}(\partial f/\partial u) \left(k_p e(t) + k_I \int_0^t e(t) dt\right).
\]

This completes the proof of Theorem 1. 

Consider a second-order system, \( \dot{\eta}(t) = f(t, \eta, \kappa, u) \), and suppose that the control objective is to design \( u(t) \) so that \( \eta(t) \) tracks a smooth reference signal \( \eta_d(t) \). We can write it in the form

\[
\dot{\eta}_1(t) = \eta_2(t)
\]

\[
\dot{\eta}_2(t) = f_2(t, \eta_1(t), \eta_2(t), \kappa(t)) + g_2(t) u(t)
\]

with \( \eta = \eta_1 \) as the output. The equation for \( \eta_2 \) is affine in \( u(t) \). The existence of \( f_2(\cdot) \) and \( g_2(\cdot) \) for a continuously differentiable \( f(\cdot) \) was shown in [4]. Note that \( \text{sign}(g_2) = \text{sign}(\partial f/\partial u) \).

Define the desired value of \( \eta_2(t) \) as

\[
\eta_{2,d}(t) = -a(\eta_1(t) - \eta_d(t)) + \dot{\eta}_d(t)
\]

where \( \dot{\eta}_d(t) \) is the reference trajectory for \( \eta(t) = \eta_1(t) \), and \( a > 0 \). Define the error state for \( \eta_{2,d}(t) \) as \( e_2(t) = \eta_2(t) - \eta_{2,d}(t) \), whose dynamics are given by

\[
\dot{e}_2(t) = f_2(t, \eta_1(t), \eta_2(t), \kappa(t)) + g_2(t) u(t) - \dot{\eta}_{2,d}(t).
\]

From Theorem 1, the controller

\[
u(t) = -\text{sign}(g_2) \left(k_p e_2(t) + k_I \int_0^t e_2(t) dt\right)
\]

ensures that \( \eta_2 \) tracks \( \eta_{2,d} \), where the gains \( k_p \) and \( k_I \) are chosen as per the guidelines of Theorem 1. It remains to simplify the controller to the PID form. Note that \( e_2(t) = \eta_2(t) + ae_1(t) - \dot{\eta}_d(t) = \dot{e}_1(t) + ae_1(t) \), where \( e_1(t) = \eta_1(t) - \eta_d(t) \). Substituting into (21), we get

\[
u(t) = -\text{sign}(g_2) \left(k_p \dot{e}_1(t) + (ak_p + k_I)e_1(t) + ak_I \int_0^t e_1(t) dt\right)
\]

which is a PID controller.

**Theorem 2:** The second-order system (18) can be stabilized using the PID controller (22), and moreover, it can be ensured that the tracking error between \( \eta(t) \) and the reference signal \( \eta_d(t) \) is bounded.

**Proof:** Theorem 1 guarantees that the control law (21) ensures that the tracking error \( e_2(t) \) of the (20) is bounded. Thus, \( \|\eta_2 - \eta_{2,d}\| < C(\epsilon) \) for some \( C > 0 \). Consider now the first equation \( \dot{\eta}(t) = \eta_1(t) = \eta_2(t) \). Since \( \eta_2(t) = e_2(t) + \eta_{2,d}(t) \), we can write

\[
\dot{\eta}(t) = \eta_1(t) = \eta_2(t) = -ae_1(t) + e_2(t).
\]

Since the unperturbed \( e_1 \) dynamics (obtained by setting \( \epsilon = 0 \)) are globally exponentially stable, it follows from the Comparison Lemma (18, Lemma 9.1) that \( e_1(t) \) is bounded, and \( e_1 \sim C(\epsilon) \). This completes the proof.

**Remark 1:** The following observations summarize the results in this section.

1) PI and PID controllers can be employed for nonlinear systems of the form (11) provided the additional \( \kappa \) dynamics are stable.
2) The DI procedure yields a systematic gain tuning procedure [see (17) and (22)].
3) Tighter bounds on the tracking error are obtained by increasing the gains (i.e., by reducing \( \epsilon \)), but the upper limit on the gains is set by considerations of robustness, particularly the time delay margin and noise attenuation. Note that modeling uncertainties are implicitly accommodated by the present approach. Formally, the small gain theorem can be used to derive the bounds on the control gains, together with the addition of a low-pass filter, as described in [21].

**IV. GUIDANCE AND CONTROL LAWS FOR PERCHING**

The objectives of controlled perching are as follows.

1) Design a control law for the symmetric dihedral deflection \((\delta_{\text{sym}} = (\delta_L + \delta_R)/2)\), which ensures that the projection of the flight path on the \(xz\) plane, given by \( z(x) \), tracks the desired profile \( z_d(x) \), which is a straight line connecting the initial point to the desired final point \((x_f, z_f)\) (see Fig. 10).

2) Design a control law for the antisymmetric dihedral deflection \((\delta_{\text{asym}} = (\delta_L - \delta_R)/2)\), which ensures that \( y(x) \to 0 \) as \( x \to x_f \), the desired final point. The stabilization of the yaw \( (\gamma) \) dynamics is a part of this objective.

The angle of attack is controlled by the elevator \((\delta_e)\). The trailing edge flaps \((\theta_f)\) in Section II-C are deflected to a constant
angle of 10°. The controller block diagram is shown in Fig. 9. A novel feature of the guidance algorithms presented in this section is that they are derived in the spatial domain, i.e., as functions of a spatial variable instead of time. Rewriting in the spatial domain recasts the dynamics into a strict feedback form, thereby permitting the use of DI presented in Section III. The guidance problems have been illustrated schematically in Fig. 10.

A. Angle of Attack Control

With a large horizontal tail, and the CG located approximately c/3 behind the wing aerodynamic center, our aircraft is sufficiently stable in pitch. Given the excellent open-loop stability characteristics, the angle of attack is not controlled by feedback laws in the experiments that are presented in this paper. Rather, on the basis of open-loop glide tests, the elevator deflection is set as a function of the commanded angle of attack

$$\delta_e = \frac{5}{3} (15 - \alpha_e)$$

where $\delta_e$ and $\alpha_e$ have been specified in degrees.

B. Control of Flight Path ($\gamma$)

The motion in the $xz$ plane (see Fig. 10) can be isolated from (2) to obtain

$$\dot{x} = V_\infty \cos \gamma, \quad \dot{z} = V_\infty \sin \gamma$$

$$\gamma = \frac{\rho V_\infty SC_L (\alpha \cos \delta_{sym}) \cos \delta_{sym} - \frac{g}{V_\infty^2} \cos \gamma}{2m}$$

where $m$ is the aircraft mass, $S$ is the wing area, $g$ is the gravitational constant, and $\delta_{sym}$ is the symmetric dihedral deflection ($\delta_{sym} = (\delta_L + \delta_R)/2$). In deriving the above equation, we have neglected the lateral-directional dynamics, and specifically assumed that $\cos \chi \approx 1$. The coefficient of lift $C_L$ depends on the local angle of attack on the wing, given by $\alpha \cos \delta_{sym}$, as in (6), and the net lift is further scaled by $\cos \delta_{sym}$. Since

$$\frac{d\gamma}{dx} = \frac{\dot{\gamma}}{x} = \frac{\dot{\gamma}}{V_\infty \cos \gamma}$$

we obtain

$$\frac{dz}{dx} = \tan(\gamma(x))$$

$$\frac{d\gamma}{dx} = \frac{\rho SC_L (\alpha \cos \delta_{sym}) \cos \delta_{sym} (x) - \frac{g}{V_\infty^2}}{2m \cos \gamma}$$

where $\delta_{sym}$ is the control input. The system in (26) can be recast in the form (18) except that derivatives and functions are defined with respect to $x$, not $t$. Hence, by replacing $t$ and $dt$ in (22) with $x$ and $dx$, the following controller is designed (see Theorem 2):

$$\delta_{sym}(x) = -\left( k_p \frac{de_z}{dx} + (ak_p + k_I)e_z (x) \right)$$

$$+ ak_I \int_0^x e_z (x)dx$$

where $e_z (x) = z(x) - z_d(x)$, and $a$ is the desired rate of convergence of $z(x)$ to the desired trajectory $z_d(x)$. The above controller ensures that $z(x)$ tracks the commanded trajectory $z_d(x)$ in Fig. 10. The stability of the controller is guaranteed by Theorem 2.

C. Outer Loop Heading Control

The motion in the $xy$ plane in Fig. 10 is given by

$$\dot{x} = V_\infty \cos \gamma \cos \chi, \quad \dot{y} = V_\infty \cos \gamma \sin \chi.$$  

The inner loop yaw controller in Fig. 9, described in Section IV-D, ensures that $r$ converges rapidly to $r_c$ so that

$$r_c \approx r = \dot{\chi} \cos \theta \cos \phi.$$  

This is a consequence of the time scale separation between the fast yaw ($r$) dynamics and the slower heading ($y, \chi$) dynamics. A similar approach for controlling turning flight of MAVs, where the turn rate is mapped to the corresponding yaw rate for the purpose of feedback and control, was used in [19]. Thus, we get

$$\frac{dy}{dx} = \tan \chi(x)$$

$$\frac{d\chi}{dx} = \frac{\dot{\chi}(t)}{V_\infty \cos \gamma \cos \chi} = \frac{r_c(x)}{V_\infty \cos \gamma \cos \chi \cos \theta \cos \phi}$$

where $\phi$ is the bank angle of fuselage. Note that $\cos \gamma$, $\cos \chi$, $\cos \theta$, and $\cos \phi$ are all positive since they generally lie in $[-\pi, \pi]$, which implies that the control coefficient
\[ V_\infty \cos \gamma \cos \theta \cos \phi \] is uniformly positive. The control problem is very similar to that encountered for the flight path in the \(xz\) plane, and we derive a controller similar to (27)

\[
r_z(x) = - \left( k_p \tan(\chi(x)) + (ak_p + k_I)g(x) + ak_I \int_0^x g(x)dx \right). \tag{31}
\]

The stability of this controller is also guaranteed by Theorem 2. Next, we describe the design of the inner yaw control loop in Fig. 9.

Remark 2: The choice of \(y = 0\) as the desired path can be replaced by any suitable path \(y_d(x)\), such as a straight line connecting the initial point and the desired final point, as for the flight path angle guidance law.

D. Inner Loop Yaw Control

The objective of the inner yaw control loop in Fig. 9 is to command the antisymmetric wing dihedral \(\delta_{asym}\) so that the yaw rate \(r\) tracks the yaw rate \(r_z\) commanded by the outer loop (see (29)).

In the simplest form, the yaw dynamics are given by the following set of equations:

\[
\dot{r} = \frac{I_x - I_y}{I_z} \rho q + N(V_\infty, \alpha, \beta, p, r, \delta_{asym})
\]

\[
\dot{\beta} = p \sin \alpha - r \cos \alpha + Y(V_\infty, \alpha, \beta, p, r, \delta_{asym}) \tag{32}
\]

where \(I_x, I_y,\) and \(I_z\) are the principal moments of inertia, while \(Y()\) and \(N()\) denote the side force and yawing moment, respectively. The other symbols have been defined in Table I. We can now differentiate the yaw rate \(r\) dynamics to get

\[
\ddot{r} = R(\alpha, p, q, r, \dot{r}, \delta_{asym}) \tag{33}
\]

for some function \(R\), and the dynamics of \(V, \alpha, q, p\) are input-to-state stable [18] with respect to the yaw dynamics. The yaw dynamics (33) are of the form (12), and they are nonaffine in the control input (unlike the guidance dynamics). Therefore, a PID controller can be designed for the yaw dynamics, with stability guaranteed by Theorem 2.

However, in experiments described in Section V, yaw control is achieved using a PI (instead of a PID) controller given by

\[
\delta_{asym} = k_p(r_z(t) - r(t)) + k_I \int_0^t (r_z(t) - r(t))dt. \tag{34}
\]

The proportional and integral gains were set to \(k_p = k_I = 2\) during the experiments. The use of a PI was helpful especially because it helped do away with the need to differentiate noisy yaw rate signals. This control design problem also illustrates the usefulness of the DI-based control scheme presented in this paper, in that a convenient and easy-to-implement PI(D) controller can be designed for a highly nonlinear system such as (32) without sacrificing theoretical stability guarantees.

E. Timing of the Pitch Up for Perching

The guidance laws derived above guide the aircraft to a suitable point at which it pitches up in order to slow down for perched landing. It is difficult, in general, to obtain an analytical expression for the pitch-up point given the coordinates of the landing point because the fourth-order longitudinal dynamics have to be considered in their entirety with no scope for simplification, such as those that can be made using time scale arguments. This is largely due to the fact that the pitch-up maneuver typically lasts less than a second; this also precludes the use of any traditional tracking or stabilization metrics to guide and assess control design.

Because the flight path flattens considerably toward the end of the perching maneuver [see Fig. 15(c)], albeit instantaneously, and is accompanied by rapid deceleration to low speeds, it is reasonable to expect that the coordinates of the landing point would depend primarily on the position of the aircraft at the time of the pitch up. In other words, we claim that given the location of the target, the location of the point for commencing the pitch up can be chosen solely based on its distance from the target independently of the flight speed and flight path angle at the time of the pitch up.

In order to test this hypothesis and measure the errors that arise from ignoring the initial (at the time of commencing the pitch up) flight speed \(V_{in}\) and flight path angle \(\gamma_{in}\), we performed a series of numerical simulations on a longitudinal model of the MAV, obtained from (2). Simulation results can also help determine the feasibility of perching, i.e., given constraints on \(V_f\) (which may arise from the choice of the landing mechanism), we can determine admissible values of the flight speed and the flight path angle at the time of the pitch up.

In this section, we use the subscript “in” to denote the value of the associated variables at the start of the pitch up. Without loss of generality, we set the initial \(x\) and \(z\) coordinates to \(x_{in} = 0\) and \(z_{in} = 0.8\) m. The elevator deflection is set to the maximum upward value, while the wing dihedral angles were set to zero during the pitch up. The objective of the simulations is to determine the final landing speed \(V_f\) and \(x\)-coordinate \(x_f\) as functions of \(V_{in}\) and \(\gamma_{in}\), with \(z_f = 0\). Contour plots of \(x_f\) and \(V_f\) are given in Fig. 11(a) and (b), respectively.

From Fig. 11(a), we deduce that the aircraft lands at \(z_f = 0\) within a 30 cm error radius about \(x_f = 2\) m when the initial flight speed \(V_{in} > 4.0\) m/s and \(\gamma_{in} < -0.3\) rad, which is the typical speed and flight path angle range of our aircraft. The value of \(x_f\), or alternatively, the distance from the landing point at which the pitch up should be initiated, is seen to be sensitive to drag (which controls deceleration) and the moment of inertia of the aircraft (which controls the time constant of the pitch dynamics). These values, therefore, need to be calculated accurately, either computationally or experimentally, before being employed for the design of guidance and control laws.

From Fig. 11(b), it is evident that the terminal flight speed \(V_f\) reduces with increasing magnitude of \(\gamma_{in}\) for all values of \(V_{in}\). The admissible values of the landing speed \((V_f)\) would depend on the grasping mechanisms employed for landing. Fig. 11(b) can be used to identify the safe regions of the \((V_{in}, \gamma_{in})\) envelope,
and they can be fed back into the guidance laws used for the glide phase leading to the pitch up.

In summary, the pitch up can be commanded based only on the distance of the aerial robot from the landing point; for a wide range of flight speeds and flight path angles at the time of the pitch up, the aircraft lands within a tolerable radius of the desired landing point. For the particular aerial robot considered here, the guidance algorithm in Sections IV-B and IV-C should bring the aircraft to a point 2 m away and 0.8 m above the desired landing point. Thereafter, a pitch up, with the elevator deflected to the maximum upward position and wings brought to level, would bring the aircraft to within 30 cm of the desired landing point.

V. RESULTS OF FLIGHT TESTS

A. Experimental Setup

The experiments described in this paper were performed on the aerial robot shown in Fig. 12. Note that the aerial robot lacks a vertical tail. The original wing was cut to facilitate hinging of the outboard 60% of the wing. It has five control surfaces.

1) An elevator, which is a movable flap attached to the horizontal tail, and whose deflection is denoted by $\delta_e$.
2) The dihedral angles ($\delta_R$ and $\delta_L$) of the outboard segments of the right and left wing can be changed independently of each other. The actuators for changing the wing dihedral angle were attached on the lower surface of the center (nonrotating) wing section.
3) The outboard segments are equipped with flaps which are capable of being actuated independently. In our experiments, both flaps were deflected by the same amount $\delta_f$ for ensuring uniform yaw control effectiveness, as explained in Section II-C. Note that these flaps, when deflected in an antisymmetric manner, can also act as the traditional ailerons.

The geometric properties of the MAV are listed in Table II.

Both wings can rotate from a maximum 45° dihedral to minimum $-15^\circ$ for a total arc range of 60°. Digital actuators with a torque rating of 0.29 kg cm are used to maneuver the wings. The time required for the wings to rotate from the minimum $-15^\circ$ to maximum 45° is about 0.05 s, and the actuators have a time delay of 0.2 s. Actuator saturation is addressed by limiting the maximum signal commanded by the controller.

---

### Table II

<table>
<thead>
<tr>
<th>Property</th>
<th>Metric Measurement</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>Mass (m)</td>
<td>44.0</td>
<td>g</td>
</tr>
<tr>
<td>Wing span (S)</td>
<td>41.8</td>
<td>cm</td>
</tr>
<tr>
<td>Wing chord (c)</td>
<td>9.5</td>
<td>cm</td>
</tr>
<tr>
<td>Wing incidence angle</td>
<td>6.0</td>
<td>deg</td>
</tr>
<tr>
<td>Wing dihedral (left and right)</td>
<td>controlled-variable</td>
<td></td>
</tr>
<tr>
<td>MAV length</td>
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<td>cm</td>
</tr>
<tr>
<td>Elevator area</td>
<td>39.12</td>
<td>cm²</td>
</tr>
</tbody>
</table>

---

Fig. 11. Plots showing the landing position $x_f$ and the landing speed $V_f$ as functions of, and in the space of, the speed and flight path angle ($V_{in}$ and $\gamma_{in}$) at the moment of commencing the pitch up. The color bar (shown in the plots) gives the values of $x_f$ and $V_f$ in the respective plots. (a) $x_f(V_{in}, \gamma_{in})$. (b) $V_f(V_{in}, \gamma_{in})$.
Flight data were measured using the Vicon motion-capture system consisting of 16 infrared cameras that track reflective markers attached to the various articulated parts of the robot with an accuracy of 1 mm. A recording rate of 100 Hz is used to capture the position and orientation data. In practice, the operating frequency is 60 Hz after allowing for offline computation, and control signals are transmitted at 20 Hz.

The real-time data stream provided by the Vicon motion-capture system includes the global reference position \((x, y, z)\) and the orientation (Euler angles) of each object (the fuselage and the two wings). The flight path angle \(\gamma\), heading angle \(\chi\), and the yaw rate \(r\) required by the controller (see Fig. 9) are computed using a finite-difference scheme. In particular, \(\gamma\) and \(\chi\) are determined from the position coordinates, while Euler angles are used to compute the yaw rate \(r\).

The availability of tracking data is contingent upon the visibility of the objects. For time-steps where no data were available due to some part of the robot being outside the field of view of the required number of cameras, a linear fit was used to estimate the missing data. Such out-of-frame events were rarely (once in several flights) seen to comprise of consecutive frames. Out-of-frame events typically occurred less than once per flight. Experiments were performed within the effective volume of capture of \(6 \times 4 \times 2.5\) m. Since Vicon provides only position and attitude information, a second-order Lagrangian polynomial was used to compute velocities and angular rates, which were then filtered to eliminate noise.

### B. Experimental Results

The experiments consisted of a series of flight tests of the aerial robot, each of which started with a hand launch of the robot from a height of approximately 2.5 m. The flaps were deflected to 10° in order to ensure yaw controllability, as explained in Section II-C. Control signals were computed offline and transmitted to the robot only after it entered the field of view of the cameras. For experiments involving perched landing on the hand, the guidance laws were provided the \((x, y, z)\) coordinates of the point at which the pitch up was to be commenced, as explained in Section IV-E. The pitch-up command consisted of deflecting the elevator to the maximum upward position while simultaneously setting the wing dihedral to zero.

A montage of snapshots taken from the video recordings of two successful perched landings on a human hand are shown in Fig. 13. Flight parameters recorded during nine representative tests are shown in Figs. 14 and 15. For each flight, we plot the trajectory in the 3-D space, and the time histories of the flight speed and the angle of attack. The following observations can be made, which are common to both sets.

1) The entire maneuver lasts just over 1.5 s and only the fast dynamics settle entirely within this range. The success of the guidance loop, on the other hand, can be severely compromised if the initial heading offset from the desired path to the target is more than approximately 30°, because the dynamics of the translational and the directional motion have a time constant on the same order as the duration of the maneuver.

2) The angle of attack settles down to a nominal value of approximately 10° within 0.5 s during the gliding phase, and increases to a peak value around 50° during the pitch up which terminates in a perched landing.

3) During the pitch up and perching phase, the flight speed drops significantly from an average peak value of 4.7 to under 2.5 m/s at the time of landing, a reduction to nearly 50% of the original speed, as predicted in Section IV-E.

4) The aircraft is directionally unstable. Moreover, the aircraft does not enter the control volume of the cameras right away, and it starts occasionally with a mildly asymmetric wing configuration. These factors cause a divergence of nearly 1 m during the first few moments of flight, but the guidance algorithms rapidly correct the course and bring the aircraft to an appropriate point for executing the pitch up.

The polar plot in Fig. 16 shows the spread of the landing points from 29 flight tests in the \(xy\) plane. The radius denotes the distance from the target, while the angle denotes the bearing of the landing point with respect to the target. The mean radial distance of the landing point from the target is 22 cm. The spread...
of the radial distances about the mean is important. The mean radial offshoot of the landing point for the 20 most accurate landings is 14 cm. Note that the landing happened inside a 40 cm disc in nearly 70% of the cases. Errors in the landing position arose primarily due to the duration of the maneuver and the actuator time delay. The average duration of the maneuver is 1.6 s, while the translational dynamics of the aircraft, which directly impact the accuracy of the landing, have a time period of approximately 2–3 s. The aircraft has between a 0.5 and 0.75 modal cycle to regulate the error in the terminal position. A pin-point landing is, therefore, very difficult to achieve, and the difficulty is further exacerbated by actuator time delays. The control law becomes active only when the aircraft enters the field of view of the cameras. Due to the instability in the lateral-directional dynamics, the control law typically gets approximately 0.5 s to correct the flight path before the aircraft either flies outside the field of view of the cameras, or diverges into an unrecoverable spin. It is possible to obtain a higher degree of precision by flapping the wings at the time of landing to make last-moment corrections, or by using grasping mechanisms such as those in [7] and [16].

The timing of the pitch-up command leading to the perched landing is critical, and some tuning is required to accommodate the actuator time delays. In our case, the command was sent when the altitude was nearly 1.5 m to account for the 0.2 s actuator time delay, whereas the altitude calculated using the approach in Section IV-E was 0.8 m.

In summary, we have demonstrated all three elements of perching stated earlier: 1) closed-loop flight control; 2) control of the lateral-directional dynamics; and 3) significant speed reduction following a rapid pitch up leading up to the landing.

C. Experimental Observations and Design Pointers

Due to the slow time scale of the translational and directional dynamics, it is important to restrict the initial sideward deviation and heading offset. In our case, an initial sideward offset of 1 m (which amounts to 20% of the total length of the flight path) was compensated for by the controller, although it occasionally required aggressive maneuvering. This limitation is purely physical and arises due to the absence of thrust. It is not related to the design of the controllers.

Although dihedral-based yaw control is capable of sustaining turn rates (\(\dot{\chi}\)) as high as 100°/s [23], it is advisable to restrict the maximum commanded turn rate if ailerons are absent. Without
ailerons, roll rate is produced purely in reaction to the yaw rate and sideslip. Consequently, a large transient sideslip is produced in the process of achieving a large turn rate. Since tailless aircraft lack directional stability [23], it takes significant control effort and time to stabilize the yaw dynamics about the commanded flight states. Moreover, the absence of ailerons impedes recovery from a turn and turn reversal. Therefore, we restricted the maximum commanded turn rate to 30°/s by restricting the maximum commanded yaw rate. It will be noted that this is not a drawback of dihedral-based yaw control, but a limitation arising from the absence of ailerons, which can be easily added to an aerial robot. As argued in [23], use of wing dihedral results in a greater agility than the use of a vertical tail when flying at low flight speeds and high angles of attack.

VI. CONCLUSION

This paper reported the first perching demonstration on a laterally unstable aerial robot. The lateral-directional motion of the aircraft was controlled actively using asymmetric wing dihedral and without a vertical tail. This study advances the state-of-the-art experiments reported in the literature, which were concerned almost exclusively with the longitudinal motion of stable aircraft. We used variable, asymmetric wing dihedral, effectively to control the flight path as well as the heading of the aerial robot. Trailing edge flaps were used to ensure that the wing dihedral provided uniform yaw control effectiveness across the flight envelope. Novel closed-loop flight guidance laws were designed for perching by rewriting the equations of motion in the spatial domain and applying DI-motivated PID control. The ability to perch on the human hand, such as the successful demonstration in this paper, would enhance the ability of aerial robots to operate around humans. Future work should focus on installing a take-off and go-around capability, using wing flapping, to accommodate failures during perching attempts.

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