# Techniques for Inverted Index Compression 

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Problem and Background

## Inverted Index - Example

- Document 0 : here are some terms
- Document 1 : some more terms
- Document 2 : even more terms

Inverted Index:

- here : 0
- are : 0
- some : 0, 1
- terms : 0, 1, 2
- more : 1, 2
- even : 2


## Inverted Index

For each term $t$ in collection of documents, we want to keep track of the documents where the term appears in inverted lists

Wide range of applications, for example:

- large-scale search engines
- social networks
- data storage architectures
- database searching

Many indexed documents and heavy query loads - compression can help!

Inverted Index Compression

## Inverted Index Compression - Timeline

| 1949 | Shannon-Fano [32, 93] |
| :--- | :--- |
| 1952 | Huffman [43] |
| 1963 | Arithmetic [1] ${ }^{1}$ |
| 1966 | Golomb [40] |
| 1971 | Elias-Fano [30, 33]; Rice [87] |
| 1972 | Variable-Byte and Nibble <br> [101] |
| 1975 | Gamma and Delta [31] |
| 1978 | Exponential Golomb [99] |
| 1985 | Fibonacci-based [6, 37] |
| 1986 | Hierarchical bit-vectors [35] |
| 1988 | Based on Front Coding [16] |
| 1996 | Interpolative [65, 66] |
| 1998 | Frame-of-Reference (For) [39]; <br> modified Rice [2] |
| 2003 | SC-dense [11] |
| 2004 | Zeta [8, 9] |


| 2005 | Simple-9, Relative-10, and Carryover-12 [3]; <br> RBUC [60] |
| :--- | :--- |
| 2006 | PForDelta [114]; BASC [61] |
| 2008 | Simple-16 [112]; Tournament [100] |
| 2009 | ANS [27]; Varint-GB [23]; Opt-PFor [111] |
| 2010 | Simple8b [4]; VSE [96]; SIMD-Gamma [91] |
| 2011 | Varint-G8IU [97]; Parallel-PFor [5] |
| 2013 | DAC [12]; Quasi-Succinct [107] |
| 2014 | Partitioned Elias-Fano [73]; QMX [103]; <br> Roaring [15, 51, 53] |
| 2015 | BP32, SIMD-BP128, and SIMD-FastPFor [50]; <br>  <br> Masked-VByte [84] |
| 2017 | Clustered Elias-Fano [80] |
| 2018 | Stream-VByte [52]; ANS-based [63, 64]; <br>  <br> Opt-VByte [83]; SIMD-Delta [104]; <br> general-purpose compression libraries [77] |
| 2019 | DINT [79]; Slicing [78] |

## Goals

Survey of encoding algorithms useful for inverted index compression

- hierarchical division in three main classes

Characterize performance of inverted index through experimentation

- compression effectiveness
- query operation performance
- sequential decoding speed


## Inverted Index Compression - Organization

Techniques organized into three main classes:
Integer Codes: algorithms that compress a single integer
List Compressors: algorithms that compress lists of many integers together Index Compressors: algorithms that represent many lists together

Integer Codes

## Overview

Assign each integer x a uniquely decodable variable length code

- want to be able to decode without ambiguity from left to right Ideal codeword length for integer x is $\log 2(1 / P(x))$ bits
- we can derive optimal distribution for an encoding from this


## Prefix-Free Codes

## No codeword is a prefix of another codeword

Lexicographic ordering: codewords in same order as the integers, and this can help us speed up the encoding and decoding process

Table 2. Example Prefix-Free Code for the Integers 1..8, Along with Associated Codewords, Codeword Lengths, and Corresponding Left-Justified, 7-Bit Integers

| $(\mathrm{a})$ |  |  |  |  | (b) |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| $x$ | Codewords | Lengths | Values |  |  | Lengths |  |  | First | Values |
| 1 | 0 | 1 | 0 |  | 1 | 1 | 0 |  |  |  |
| 2 | 100 | 3 | 64 |  | 2 | 2 | 64 |  |  |  |
| 3 | 101 | 3 | 80 |  | 3 | 2 | 64 |  |  |  |
| 4 | 11000 | 5 | 96 |  | 4 | 4 | 96 |  |  |  |
| 5 | 11001 | 5 | 100 |  | 5 | 4 | 96 |  |  |  |
| 6 | 11010 | 5 | 104 |  | 6 | 8 | 112 |  |  |  |
| 7 | 11011 | 5 | 108 |  | 7 | 8 | 112 |  |  |  |
| 8 | 1110000 | 7 | 112 |  | - | 9 | 127 |  |  |  |
| - | - | - | 127 |  |  |  |  |  |  |  |

The codewords are left-justified to better highlight their lexicographic order. In (b), the compact version of the table in (a), used by the encoding/decoding procedures coded in Figure 1. The "values" and "first" columns are padded with a sentinel value (in gray) to let the search be well defined.

## Prefix-Free Codes

1 Encode $(x)$ :
2 determine $\ell$ such that first $[\ell] \leq x<\operatorname{first}[\ell+1]$
3 offset $=x-\operatorname{first}[\ell]$
4 jump $=1 \ll(M-\ell)$
$5 \quad$ Write $(($ values $[\ell]+$ offset $\times j u m p) \gg(M-\ell), \ell)$
1 Decode() :
2 determine $\ell$ such that values $[\ell] \leq$ buffer $<$ values $[\ell+1]$
3 offset $=($ buffer - values $[\ell]) \gg(M-\ell)$
$4 \quad$ buffer $=(($ buffer $\ll \ell) \&$ MASK $)+$ Take $(\ell) \quad \triangleright$ MASK is the constant $2^{M}-1$.
5
return first $[\ell]+$ offset
5 L return first $[\ell]+$ offset

Fig. 1. Encoding and decoding procedures using two parallel arrays first and values of $M+1$ values each.

## Unary Coding

Table 3. Integers $1 . .8$ as Represented with Several Codes

## $x-1$ ones followed by a single zero

length is $x$
optimal when

$$
P(x)=2^{\wedge}-x
$$

| $x$ | $\mathrm{U}(x)$ | $\mathrm{B}(x)$ | $\gamma(x)$ | $\delta(x)$ | $\mathrm{G}_{2}(x)$ | $\mathrm{ExpG}_{2}(x)$ | $\mathrm{Z}_{2}(x)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0. | 0. | 0.0 | 0.00 | 0.0 |
| 2 | 10 | 1 | 10.0 | 100.0 | 0.1 | 0.01 | 0.10 |
| 3 | 110 | 10 | 10.1 | 100.1 | 10.0 | 0.10 | 0.11 |
| 4 | 1110 | 11 | 110.00 | 101.00 | 10.1 | 0.11 | 10.000 |
| 5 | 11110 | 100 | 110.01 | 101.01 | 110.0 | 10.000 | 10.001 |
| 6 | 111110 | 101 | 110.10 | 101.10 | 110.1 | 10.001 | 10.010 |
| 7 | 1111110 | 110 | 110.11 | 101.11 | 1110.0 | 10.010 | 10.011 |
| 8 | 11111110 | 111 | 1110.000 | 11000.000 | 1110.1 | 10.011 | 10.1000 |

The "." symbol highlights the distinction between different parts of the codes and has a purely illustrative purpose: it is not included in the final coded representation.

## Binary Coding

Table 3. Integers $1 . .8$ as Represented with Several Codes

## representation of $x-1$ in binary <br> length is k, where the integers are bounded by 2^k optimal when

$$
P(x)=2^{\wedge}-k
$$

| $x$ | $\mathrm{U}(x)$ | $\mathrm{B}(x)$ | $\gamma(x)$ | $\delta(x)$ | $\mathrm{G}_{2}(x)$ | $\operatorname{ExpG}_{2}(x)$ | $\mathrm{Z}_{2}(x)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0. | 0. | 0.0 | 0.00 | 0.0 |
| 2 | 10 | 1 | 10.0 | 100.0 | 0.1 | 0.01 | 0.10 |
| 3 | 110 | 10 | 10.1 | 100.1 | 10.0 | 0.10 | 0.11 |
| 4 | 1110 | 11 | 110.00 | 101.00 | 10.1 | 0.11 | 10.000 |
| 5 | 11110 | 100 | 110.01 | 101.01 | 110.0 | 10.000 | 10.001 |
| 6 | 111110 | 101 | 110.10 | 101.10 | 110.1 | 10.001 | 10.010 |
| 7 | 1111110 | 110 | 110.11 | 101.11 | 1110.0 | 10.010 | 10.011 |
| 8 | 11111110 | 111 | 1110.000 | 11000.000 | 1110.1 | 10.011 | 10.1000 |

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## Gamma Coding

Table 3. Integers $1 . .8$ as Represented with Several Codes

## U(|bin(x)|) followed by last |bin(x)-1| bits from bin( $x$ )

length is $2|\mathrm{bin}(x)|-1$ optimal when
$P(x)=1 / 2 k^{\wedge} 2$

| $x$ | $\mathrm{U}(x)$ | $\mathrm{B}(x)$ | $\gamma(x)$ | $\delta(x)$ | $\mathrm{G}_{2}(x)$ | $\mathrm{ExpG}_{2}(x)$ | $\mathrm{Z}_{2}(x)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0. | 0. | 0.0 | 0.00 | 0.0 |
| 2 | 10 | 1 | 10.0 | 100.0 | 0.1 | 0.01 | 0.10 |
| 3 | 110 | 10 | 10.1 | 100.1 | 10.0 | 0.10 | 0.11 |
| 4 | 1110 | 11 | 110.00 | 101.00 | 10.1 | 0.11 | 10.000 |
| 5 | 11110 | 100 | 110.01 | 101.01 | 110.0 | 10.000 | 10.001 |
| 6 | 111110 | 101 | 110.10 | 101.10 | 110.1 | 10.001 | 10.010 |
| 7 | 1111110 | 110 | 110.11 | 101.11 | 1110.0 | 10.010 | 10.011 |
| 8 | 11111110 | 111 | 1110.000 | 11000.000 | 1110.1 | 10.011 | 10.1000 |

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## Delta Coding

## $\gamma(|\operatorname{bin}(x)|)$ followed by last |bin(x)-1| bits from $\operatorname{bin}(x)$ <br> length is $\mid \gamma$ (|bin(x)|)|+|bin(x)|-1 <br> optimal when $P(x)=$ 1/(2x $\left(\log 2(x)^{\wedge} 2\right)$ <br> k-gamma, SIMD delta codes

Table 3. Integers $1 . .8$ as Represented with Several Codes

| $x$ | $\mathrm{U}(x)$ | $\mathrm{B}(x)$ | $\gamma(x)$ | $\delta(x)$ | $\mathrm{G}_{2}(x)$ | $\mathrm{ExpG}_{2}(x)$ | $\mathrm{Z}_{2}(x)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0. | 0. | 0.0 | 0.00 | 0.0 |
| 2 | 10 | 1 | 10.0 | 100.0 | 0.1 | 0.01 | 0.10 |
| 3 | 110 | 10 | 10.1 | 100.1 | 10.0 | 0.10 | 0.11 |
| 4 | 1110 | 11 | 110.00 | 101.00 | 10.1 | 0.11 | 10.000 |
| 5 | 11110 | 100 | 110.01 | 101.01 | 110.0 | 10.000 | 10.001 |
| 6 | 111110 | 101 | 110.10 | 101.10 | 110.1 | 10.001 | 10.010 |
| 7 | 1111110 | 110 | 110.11 | 101.11 | 1110.0 | 10.010 | 10.011 |
| 8 | 11111110 | 111 | 1110.000 | 11000.000 | 1110.1 | 10.011 | 10.1000 |

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## Golomb

unary encoding of quotient then minimal binary of remainder optimal when
$P(x)=p(1-p)^{\wedge}(x-1)$
gaps between integers drawn at random follows geometric distribution

Table 3. Integers 1.8 as Represented with Several Codes

| $x$ | $\mathrm{U}(x)$ | $\mathrm{B}(x)$ | $\gamma(x)$ | $\delta(x)$ | $\mathrm{G}_{2}(x)$ | $\mathrm{ExpG}_{2}(x)$ | $\mathrm{Z}_{2}(x)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0. | 0. | 0.0 | 0.00 | 0.0 |
| 2 | 10 | 1 | 10.0 | 100.0 | 0.1 | 0.01 | 0.10 |
| 3 | 110 | 10 | 10.1 | 100.1 | 10.0 | 0.10 | 0.11 |
| 4 | 1110 | 11 | 110.00 | 101.00 | 10.1 | 0.11 | 10.000 |
| 5 | 11110 | 100 | 110.01 | 101.01 | 110.0 | 10.000 | 10.001 |
| 6 | 111110 | 101 | 110.10 | 101.10 | 110.1 | 10.001 | 10.010 |
| 7 | 1111110 | 110 | 110.11 | 101.11 | 110.0 | 10.010 | 10.011 |
| 8 | 11111110 | 111 | 1110.000 | 11000.000 | 1110.1 | 10.011 | 10.1000 |

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## Rice

Table 3. Integers $1 . .8$ as Represented with Several Codes


| $x$ | $\mathrm{U}(x)$ | $\mathrm{B}(x)$ | $\gamma(x)$ | $\delta(x)$ | $\mathrm{G}_{2}(x)$ | $\mathrm{ExpG}_{2}(x)$ | $\mathrm{Z}_{2}(x)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0. | 0. | 0.0 | 0.00 | 0.0 |
| 2 | 10 | 1 | 10.0 | 100.0 | 0.1 | 0.01 | 0.10 |
| 3 | 110 | 10 | 10.1 | 100.1 | 10.0 | 0.10 | 0.11 |
| 4 | 1110 | 11 | 110.00 | 101.00 | 10.1 | 0.11 | 10.000 |
| 5 | 11110 | 100 | 110.01 | 101.01 | 110.0 | 10.000 | 10.001 |
| 6 | 111110 | 101 | 110.10 | 101.10 | 110.1 | 10.001 | 10.010 |
| 7 | 1111110 | 110 | 110.11 | 101.11 | 1110.0 | 10.010 | 10.011 |
| 8 | 11111110 | 111 | 1110.000 | 11000.000 | 1110.1 | 10.011 | 10.1000 |

The "." symbol highlights the distinction between different parts of the codes and has a purely illustrative purpose: it is not included in the final coded representation.

## Exponential Golomb

Unary encoding of bucket identifier followed by binary encoding of bucket offset

$$
B=\left[0,2^{k}, \sum_{i=0}^{1} 2^{k+i}, \sum_{i=0}^{2} 2^{k+i}, \sum_{i=0}^{3} 2^{k+i}, \ldots\right]
$$

## Zeta

Exponential Golomb with these buckets:

$$
\left[0,2^{k}-1,2^{2 k}-1,2^{3 k}-1, \ldots\right]
$$

optimal for power law distribution with small exponent, i.e.
$P(x)=1 /\left(\zeta(a) x^{\wedge} a\right)$ where $\zeta$ is Riemann zeta function

## Fibonacci

every positive integer has unique representation as sum of some non-adjacent Fibonacci numbers

1 for if i-th Fibonacci number used in sum, 0 otherwise, final control 1 bit
optimal when $\mathrm{P}(\mathrm{x})$ is approximately $1 /\left(2 x^{\wedge} 1.44\right)$
can generate lexicographic codewords from the lengths

Table 4. Integers $1 . .8$ as Represented with Fibonacci-Based Codes

| (a) "Original" Codewords |  |  |  |  |  |  | (b) Lexicographic Codewords |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | $\mathrm{F}(x)$ |  |  |  |  |  | $x$ | $\mathrm{F}(x)$ |  |  |  |  |  |
| 1 | 1 | 1 |  |  |  |  | 1 | 0 | 0 |  |  |  |  |
| 2 | 0 | 1 | 1 |  |  |  | 2 | 0 | 1 | 0 |  |  |  |
| 3 | 0 | 0 | 1 | 1 |  |  | 3 | 0 | 1 | 1 | 0 |  |  |
| 4 | 1 | 0 | 1 | 1 |  |  | 4 | 0 | 1 | 1 | 1 |  |  |
| 5 | 0 | 0 | 0 | 1 | 1 |  | 5 | 1 | 0 | 0 | 0 | 0 |  |
| 6 | 1 | 0 | 0 | 1 | 1 |  | 6 | 1 | 0 | 0 | 0 | 1 |  |
| 7 | 0 | 1 | 0 | 1 | 1 |  | 7 | 1 | 0 | 0 | 1 | 0 |  |
| 8 | 0 | 0 | 0 | 0 | 1 | 1 | 8 | 1 | 0 | 0 | 1 | 1 | 0 |
| $F_{i}$ | 1 | 2 | 3 | 5 | 8 | 13 |  |  |  |  |  |  |  |

In (a), the final control bit is highlighted in bold font and the relevant Fibonacci numbers $F_{i}$ involved in the representation are also shown at the bottom of the table. In (b), the "canonical" lexicographic codewords are presented.

## Variable-Byte

Codes previously described are bit-aligned, but byte or word-aligned codes can have better decoding speed

MSB signals continuation of byte sequence, rest of the bits used for data
Optimal when $\mathrm{P}(\mathrm{x})$ approximately $\mathrm{x}^{\wedge}(-8 / 7)$ for byte-aligned

## Example: 00000100.10000001. 11111110

## Branch Prediction

Varint-GB groups control bits together to reduce the probability of a branch misprediction for higher throughput

Assume largest represented integer fits in 4 bytes, then we have only four different byte-lengths, which only requires two bits

So groups of four such integers requires only one control byte

## SIMD Parallelism

Varint-G8IU: one control byte for variable number of integers in 8-byte segment, for groups of between two and eight compressed integers

Masked-VByte: decoding by first gathering MSBs of consecutive bytes with SIMD instruction, and permuting data bytes accordingly

Stream-VByte: separate encoding of control data bits into separate streams, allows decoding multiple control bits separately and reduces data dependencies

## SC-Dense

Variable-Byte has $2^{\wedge} 7$ as separator between stoppers and continuers, we can generalize this to adapt to distribution in question.

Table 5. Integers $1 . .20$ as Represented by SC(4,4)- and SC (5, 3)-Dense Codes, Respectively

| $x$ | SC ( $4,4, x$ ) | $\mathrm{SC}(5,3, x)$ | $x$ | SC( $4,4, x$ ) | SC ( $5,3, x$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 000 | 000 | 11 | 101.010 | 110.000 |
| 2 | 001 | 001 | 12 | 101.011 | 110.001 |
| 3 | 010 | 010 | 13 | 110.000 | 110.010 |
| 4 | 011 | 011 | 14 | 110.001 | 110.011 |
| 5 | 100.000 | 100 | 15 | 110.010 | 110.100 |
| 6 | 100.001 | 101.000 | 16 | 110.011 | 111.000 |
| 7 | 100.010 | 101.001 | 17 | 111.000 | 111.001 |
| 8 | 100.011 | 101.010 | 18 | 111.001 | 111.010 |
| 9 | 101.000 | 101.011 | 19 | 111.010 | 111.011 |
| 10 | 101.001 | 101.100 | 20 | 111.011 | 111.100 |



## Summary

For our inverted indexes with sorted inverted lists, compression on gaps of the sequence works well

Tuned parametric codes can be good, but tuning not always possible

Some different relative strengths at smaller and larger values


## List Compressors

## Overview

Information-theoretic lower bound gives minimum bits to represent list of n strictly increasing integers drawn at random from a universe

In practice, compressors often take advantage of inverted lists having clusters of close integers, which are more compressible, to use less than the bound

These can arise because of indexed documents often being clustered by sharing the same set of terms

## Binary Packing

Partition sequence into blocks of fixed or variable length and encode them separately

- if sequence has clusters of close integers, values are likely to be of similar magnitude

Compute bit width of max element in block and represent integers in block with that number of bits, gaps between integers can be computed to lower width

- variable sized blocks usually preferable

Many variants of this approach, including word-aligned version

## Simple

split sequence into fixed-memory units, and pack as many integers as we can fit into a unit
selector code gives information on how elements are packed in segment
typically provides good decoding speed and good compression

Table 6. Nine Different Ways of Packing Integers in a 28 -Bit Segment as Used by Simple9

| 4-Bit Selector | Integers | Bits per Integer | Wasted Bits |
| :---: | :---: | :---: | :---: |
| 0000 | 28 | 1 | 0 |
| 0001 | 14 | 2 | 0 |
| 0010 | 9 | 3 | 1 |
| 0011 | 7 | 4 | 0 |
| 0100 | 5 | 5 | 3 |
| 0101 | 4 | 7 | 0 |
| 0110 | 3 | 9 | 1 |
| 0111 | 2 | 14 | 0 |
| 1000 | 1 | 28 | 0 |

## PForDelta

Space inefficiency of block-based strategies like simple when there is just one large value in the block
"patched" frame of reference chooses a range that fits most of the integers, and encodes exceptions separately with different algorithm

$$
[3,4,7,21,9,12,5,16,6,2,34]
$$

$$
[1,2,5, *, 7,10,3, *, 4,0, *]-[21,16,34]
$$

## Elias-Fano

For n sorted integers in range $[1, \mathrm{U}]$
Split them into I=floor(log2(U/n)) low bits and ceil(log2(U))-I high bits
Low bits are encoded separately with nl size bitvector directly
High bits are encoded separately with $2 n$ bits: for high bit h_i, we set the bit in position h_i+i, so unary encoding of how many integers have h_i equal to value

## Elias-Fano - Example

Table 7. Example of Elias-Fano Encoding Applied to the Sequence

$$
\mathcal{S}=[3,4,7,13,14,15,21,25,36,38,54,62]
$$

| $\mathcal{S}$ | 3 | 4 | 7 | 13 | 14 | 15 | 21 | 25 | 36 | 38 |  | 54 | 62 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| high | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
|  | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| low | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |  | 1 | 1 |
|  | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |  | 1 | 1 |
|  | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |  | 0 | 0 |
| $H$ | 1110 |  |  |  |  |  |  |  |  |  |  |  | 1110 |
| L | 011.100 .111 | 101.110 .111 | 101 | 001 | 100.110 |  | 110 | 110 |  |  |  |  |  |

## Elias-Fano - Random Accesses

How to decode an individual integer $S[i]$ in $O(1)$ time:

- need data structure to get i-th bit set to b in H (high bits bitvector) in $\mathrm{O}(1)$, it turns out this only requires $o(n)$ bits, small compared to encoding size
- call the query above Select_b(i), then high bits is Select_1(i)-i, since we unary encoded how many integers share same high part, so there is 1 for each integer and 0 for each distinct high part
- read low bits directly from L (low bits bitvector)
- concatenate high and low bits


## Elias-Fano - Successor Queries

How to find smallest integer at least x , for some integer x :

- let h_x be high bits of $x$
- $i=$ Select_0(h_x)-h_x+1 (for $h \_x>0$, use $i=0$ otherwise) says that there are $i$ integers in $S$ with high bits less than h_x
- $j=$ Select_ $0\left(h \_x+1\right)-h \_x$ is start position for elements with larger high bits
- then only the range from i to $j$ needs to be searched
- runs in $O(1+\log (U / n))$ time
- this successor query is also called NextGEQ


## Elias Fano - Partitioning

High and low bit split can be chosen arbitrarily, for a non-parametric split
Roaring partitions $U$ <= $2^{\wedge} 32$ into chunks of $2^{\wedge} 16$ values each, and encodes chunks depending on if they are sparse, dense, or very dense

- sorted array for sparse, bitmap for dense, runs for very dense

Slicing also continues encoding recursively for the sparse chunks
Main idea in these is to find dense regions for bitmap encodings but treat the sparse regions differently

## Interpolative

Binary Interpolative Code (BIC)
Recursively split list in half and encode middle element with minimal bits
Fully make use of runs of consecutive integers

## Interpolative - Example



Fig. 4. The recursive calls performed by the Binary Interpolative Coding algorithm when applied to the sequence $[3,4,7,13,14,15,21,25,36,38,54]$ with initial knowledge of lower and upper bound values $l=0$ and $h=62$. In bold font, we highlight the middle element being encoded.

## Directly-Addressable Codes

Reduce problem of random access to ranking over a bitmap

## Hybrid Approaches

Hybrid approaches can use different compressors for blocks of a list
Example: collect access statistics for blocks

- represent rarely accessed blocks with a more space-efficient compressor
- represent frequently accessed blocks with a more time-efficient compressor

Some algorithms for optimal partitioning into blocks for these hybrid approaches

## Entropy Coding

Usually not as competitive for efficiency and simplicity of implementation Huffman, Arithmetic, and Asymmetric Numeral Systems (ANS)

Index Compressors

## Clustered

Inverted lists grouped into clusters of lists sharing as many integers as possible For each cluster, we have a reference list, where for integers in reference list and list in cluster, they can be represented as position occupied in reference list

Any compressor can be used for the intersections between the reference list and cluster lists but PEF (partitioned Elias-Fano) was used by authors

Time/space tradeoffs from varying size of the reference lists

## ANS Based

Alphabet size of ANS method may be too large for representing our integers even if we only work with gaps

VByte+ANS: this can be adapted by preprocessing with Variable-Byte to reduce input list, and then applying ANS on the sequence of bytes

## Dictionary Based

## Store most frequent $2^{\wedge} \mathrm{b}$ patterns in dictionary, and use it to encode subsequences of gaps, as there are often very repetitive patterns across the whole inverted index



Fig. 6. A dictionary-based encoded stream example, where dictionary entries corresponding to $\{1,2,4,8,16\}$ long integer patterns, runs, and exceptions are labeled with different shades. Once provision has been made for such a dictionary structure, a sequence of gaps can be modeled as a sequence of codewords $\left\{c_{k}\right\}$, each being a reference to a dictionary entry, as represented with the encoded stream in the picture. Note that, for example, codeword $c_{9}$ signals an exception, and therefore the next symbol $e$ is decoded using an escape mechanism.

## Experiments

## Experimental Setup

Table 9. Different Tested Index Representations

## Mainly focused on comparing some selected representations over being completely exhaustive, some further comparisons in their benchmark repository

| Method | Partitioned by | SIMD | Alignment | Description |
| :--- | :---: | :---: | :---: | :--- |
| VByte | Cardinality | Yes | Byte | Fixed-size partitions of 128 |
| Opt-VByte | Cardinality | Yes | Bit | Variable-size partitions |
| BIC | Cardinality | No | Bit | Fixed-size partitions of 128 |
| $\delta$ | Cardinality | No | Bit | Fixed-size partitions of 128 |
| Rice | Cardinality | No | Bit | Fixed-size partitions of 128 |
| PEF | Cardinality | No | Bit | Variable-size partitions |
| DINT | Cardinality | No | 16-bit word | Fixed-size partitions of 128 |
| Opt-PFor | Cardinality | No | 32-bit word | Fixed-size partitions of 128 |
| Simple16 | Cardinality | No | 64-bit word | Ffixed-size partitions of 128 |
| QMX | Cardinality | Yes | 128-bit word | Fixed-size partitions of 128 |
| Roaring | Universe | Yes | byte | Single span |
| Slicing | Universe | Yes | byte | Multi-span |

Table 10. Datasets Used in the Experiments
(a) Basic Statistics

|  | Gov2 | ClueWeb09 | CCNews |  | Gov2 | ClueWeb09 | CCNews |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lists | 39,177 | 96,722 | 76,474 | Queries | 34,327 | 42,613 | 22,769 |
| Universe | 24,622,347 | 50,131,015 | 43,530,315 | 2 terms | 32.2\% | 33.6\% | 37.5\% |
| Integers | 5,322,883,266 | 14,858,833,259 | 19,691,599,096 | 3 terms | 26.8\% | 26.5\% | 27.3\% |
| Entropy of the gaps | 3.02 | 4.46 | 5.44 | 4 terms | 18.2\% | 17.7\% | 16.8\% |
| $\left\lceil\log _{2}\right\rceil$ of the gaps | 1.35 | 2.28 | 2.99 | 5+ terms | 22.8\% | 22.2\% | 18.4\% |

## Compression Effectiveness

Table 11. Space Effectiveness in Total GiB and Bits per Integer, and Nanoseconds per Decoded Integer

| Method | Gov2 |  |  | ClueWeb09 |  |  | CCNews |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\overline{\mathrm{GiB}}$ | Bits/int | ns/int | GiB | Bits/int | ns/int | GiB | bits/int | ns/int |
| VByte | 5.46 | 8.81 | 0.96 | 15.92 | 9.20 | 1.09 | 21.29 | 9.29 | 1.03 |
| Opt-VByte | 2.41 | 3.89 | 0.73 | 9.89 | 5.72 | 0.92 | 14.73 | 6.42 | 0.72 |
| BIC | 1.82 | 2.94 | 5.06 | 7.66 | 4.43 | 6.31 | 12.02 | 5.24 | 6.97 |
| $\delta$ | 2.32 | 3.74 | 3.56 | 8.95 | 5.17 | 3.72 | 14.58 | 6.36 | 3.85 |
| Rice | 2.53 | 4.08 | 2.92 | 9.18 | 5.31 | 3.25 | 13.34 | 5.82 | 3.32 |
| PEF | 1.93 | 3.12 | 0.76 | 8.63 | 4.99 | 1.10 | 12.50 | 5.45 | 1.31 |
| DINT | 2.19 | 3.53 | 1.13 | 9.26 | 5.35 | 1.56 | 14.76 | 6.44 | 1.65 |
| Opt-PFor | 2.25 | 3.63 | 1.38 | 9.45 | 5.46 | 1.79 | 13.92 | 6.07 | 1.53 |
| Simple16 | 2.59 | 4.19 | 1.53 | 10.13 | 5.85 | 1.87 | 14.68 | 6.41 | 1.89 |
| QMX | 3.17 | 5.12 | 0.80 | 12.60 | 7.29 | 0.87 | 16.96 | 7.40 | 0.84 |
| Roaring | 4.11 | 6.63 | 0.50 | 16.92 | 9.78 | 0.71 | 21.75 | 9.49 | 0.61 |
| Slicing | 2.67 | 4.31 | 0.53 | 12.21 | 7.06 | 0.68 | 17.83 | 7.78 | 0.69 |

## Sequential Decoding

Table 11. Space Effectiveness in Total GiB and Bits per Integer, and Nanoseconds per Decoded Integer

| Method | Gov2 |  |  | ClueWeb09 |  |  | CCNews |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\overline{\mathrm{GiB}}$ | Bits/int | ns/int | GiB | Bits/int | ns/int | GiB | bits/int | ns/int |
| VByte | 5.46 | 8.81 | 0.96 | 15.92 | 9.20 | 1.09 | 21.29 | 9.29 | 1.03 |
| Opt-VByte | 2.41 | 3.89 | 0.73 | 9.89 | 5.72 | 0.92 | 14.73 | 6.42 | 0.72 |
| BIC | 1.82 | 2.94 | 5.06 | 7.66 | 4.43 | 6.31 | 12.02 | 5.24 | 6.97 |
| $\delta$ | 2.32 | 3.74 | 3.56 | 8.95 | 5.17 | 3.72 | 14.58 | 6.36 | 3.85 |
| Rice | 2.53 | 4.08 | 2.92 | 9.18 | 5.31 | 3.25 | 13.34 | 5.82 | 3.32 |
| PEF | 1.93 | 3.12 | 0.76 | 8.63 | 4.99 | 1.10 | 12.50 | 5.45 | 1.31 |
| DINT | 2.19 | 3.53 | 1.13 | 9.26 | 5.35 | 1.56 | 14.76 | 6.44 | 1.65 |
| Opt-PFor | 2.25 | 3.63 | 1.38 | 9.45 | 5.46 | 1.79 | 13.92 | 6.07 | 1.53 |
| Simple16 | 2.59 | 4.19 | 1.53 | 10.13 | 5.85 | 1.87 | 14.68 | 6.41 | 1.89 |
| QMX | 3.17 | 5.12 | 0.80 | 12.60 | 7.29 | 0.87 | 16.96 | 7.40 | 0.84 |
| Roaring | 4.11 | 6.63 | 0.50 | 16.92 | 9.78 | 0.71 | 21.75 | 9.49 | 0.61 |
| Slicing | 2.67 | 4.31 | 0.53 | 12.21 | 7.06 | 0.68 | 17.83 | 7.78 | 0.69 |

## Boolean AND Queries

Table 12. Milliseconds Spent per AND Query by Varying the Number of Query Terms

| Method | Gov2 |  |  |  |  | ClueWeb09 |  |  |  |  | CCNews |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5+ | avg. | 2 | 3 | 4 | 5+ | avg. | 2 | 3 | 4 | 5+ | avg. |
| VByte | 2.2 | 2.8 | 2.7 | 3.3 | 2.8 | 10.2 | 12.1 | 13.7 | 13.9 | 12.5 | 14.0 | 22.4 | 19.7 | 21.9 | 19.5 |
| Opt-VByte | 2.8 | 3.1 | 2.8 | 3.2 | 3.0 | 12.2 | 13.3 | 14.0 | 13.6 | 13.3 | 16.0 | 23.2 | 19.6 | 20.3 | 19.8 |
| BIC | 6.8 | 9.7 | 10.4 | 13.2 | 10.0 | 31.7 | 44.2 | 51.5 | 53.8 | 45.3 | 45.6 | 79.7 | 76.9 | 88.8 | 72.8 |
| $\delta$ | 4.6 | 6.3 | 6.5 | 8.2 | 6.4 | 20.9 | 28.3 | 33.5 | 34.5 | 29.3 | 28.6 | 50.9 | 48.0 | 55.6 | 45.8 |
| Rice | 4.1 | 5.6 | 5.8 | 7.3 | 5.7 | 19.2 | 25.7 | 30.2 | 31.1 | 26.6 | 26.5 | 46.5 | 43.5 | 50.1 | 41.6 |
| PEF | 2.5 | 3.1 | 2.8 | 3.2 | 2.9 | 12.3 | 13.5 | 14.4 | 13.8 | 13.5 | 17.2 | 24.6 | 21.0 | 21.9 | 21.2 |
| DINT | 2.5 | 3.3 | 3.3 | 4.1 | 3.3 | 11.9 | 14.6 | 16.5 | 17.1 | 15.0 | 16.9 | 27.3 | 24.6 | 28.1 | 24.2 |
| Opt-PFor | 2.6 | 3.5 | 3.5 | 4.3 | 3.5 | 12.8 | 15.9 | 18.0 | 18.3 | 16.3 | 16.6 | 27.2 | 24.3 | 27.1 | 23.8 |
| Simple 16 | 2.8 | 3.7 | 3.7 | 4.6 | 3.7 | 12.8 | 16.3 | 18.4 | 18.9 | 16.6 | 17.6 | 28.8 | 26.3 | 29.5 | 25.5 |
| QMX | 2.0 | 2.6 | 2.5 | 3.0 | 2.5 | 9.6 | 11.5 | 13.0 | 13.1 | 11.8 | 13.3 | 21.5 | 18.8 | 20.8 | 18.6 |
| Roaring | 0.3 | 0.5 | 0.7 | 0.8 | 0.6 | 1.5 | 2.5 | 3.1 | 4.3 | 2.9 | 1.1 | 2.0 | 2.6 | 4.1 | 2.5 |
| Slicing | 0.3 | 1.0 | 1.2 | 1.6 | 1.0 | 1.5 | 4.5 | 5.4 | 6.7 | 4.5 | 1.8 | 4.3 | 5.1 | 6.0 | 4.3 |

## Boolean OR Queries

Table 13. Milliseconds Spent per OR Query by Varying the Number of Query Terms

| Method | Gov2 |  |  |  |  | ClueWeb09 |  |  |  |  | CCNews |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5+ | avg. | 2 | 3 | 4 | 5+ | avg. | 2 | 3 | 4 | 5+ | avg. |
| VByte | 6.8 | 24.4 | 54.7 | 131.7 | 54.4 | 20.1 | 71.3 | 156.0 | 379.5 | 156.7 | 24.4 | 94.5 | 178.8 | 391.4 | 172.3 |
| Opt-VByte | 11.0 | 35.7 | 77.4 | 176.0 | 75.0 | 31.3 | 101.4 | 213.4 | 500.1 | 211.6 | 36.4 | 128.0 | 232.0 | 510.4 | 226.7 |
| BIC | 16.7 | 50.3 | 105.0 | 238.8 | 102.7 | 49.9 | 145.3 | 290.4 | 668.2 | 288.4 | 64.4 | 193.8 | 332.6 | 692.5 | 320.8 |
| $\delta$ | 12.6 | 40.8 | 87.9 | 202.5 | 85.9 | 34.9 | 112.9 | 236.7 | 557.7 | 235.6 | 42.2 | 144.9 | 263.8 | 571.3 | 255.5 |
| Rice | 13.4 | 43.1 | 93.3 | 211.3 | 90.3 | 36.8 | 118.2 | 248.5 | 576.6 | 245.0 | 43.6 | 149.3 | 270.5 | 585.6 | 262.2 |
| PEF | 10.2 | 33.0 | 71.7 | 164.2 | 69.8 | 31.1 | 99.7 | 208.5 | 492.3 | 207.9 | 37.6 | 127.5 | 232.6 | 507.1 | 226.2 |
| DINT | 8.5 | 28.5 | 63.7 | 147.6 | 62.1 | 24.9 | 84.1 | 178.8 | 424.3 | 178.0 | 30.6 | 109.2 | 200.4 | 432.7 | 193.2 |
| Opt-PFor | 8.9 | 31.1 | 69.4 | 161.4 | 67.7 | 27.0 | 90.8 | 194.0 | 453.5 | 191.3 | 31.3 | 113.2 | 209.0 | 447.2 | 200.2 |
| Simple16 | 7.8 | 26.2 | 58.3 | 138.2 | 57.6 | 23.7 | 78.0 | 165.5 | 394.7 | 165.5 | 28.7 | 101.5 | 185.3 | 397.8 | 178.4 |
| QMX | 6.6 | 23.8 | 53.4 | 128.1 | 53.0 | 19.7 | 70.0 | 153.2 | 377.9 | 155.2 | 24.0 | 92.6 | 175.2 | 382.4 | 168.6 |
| Roaring | 1.2 | 2.8 | 4.3 | 6.4 | 3.7 | 4.7 | 9.0 | 12.0 | 15.7 | 10.3 | 3.8 | 7.6 | 10.5 | 15.1 | 9.2 |
| Slicing | 1.3 | 4.0 | 6.3 | 9.2 | 5.2 | 5.0 | 12.8 | 18.1 | 25.3 | 15.3 | 5.8 | 12.9 | 17.3 | 23.0 | 14.8 |

## Space/Time Tradeoffs



Fig. 7. Space/time trade-off curves for the ClueWeb09 dataset.

## Future Work

Simpler compression formats that can be decoded faster using low-latency instructions and minimal branches

Making full use of superscalar execution and SIMD instructions
Dynamic compressed representations for integer sequences that can support additions and deletions, a specific case of more general dictionary problem Implementations with good practical performance

# Techniques for Inverted Index Compression 

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