Concurrent Algorithms and Data Structures

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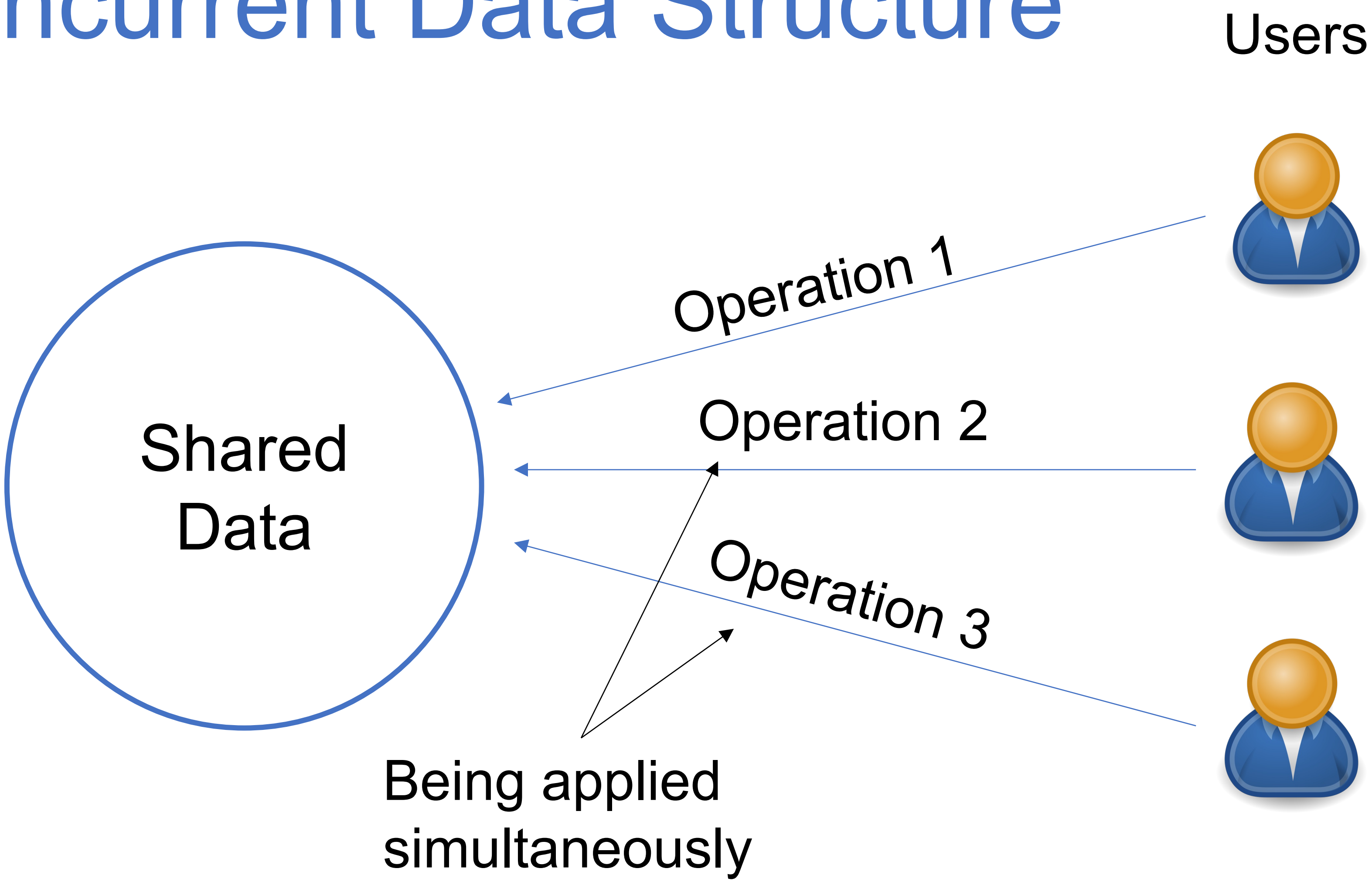
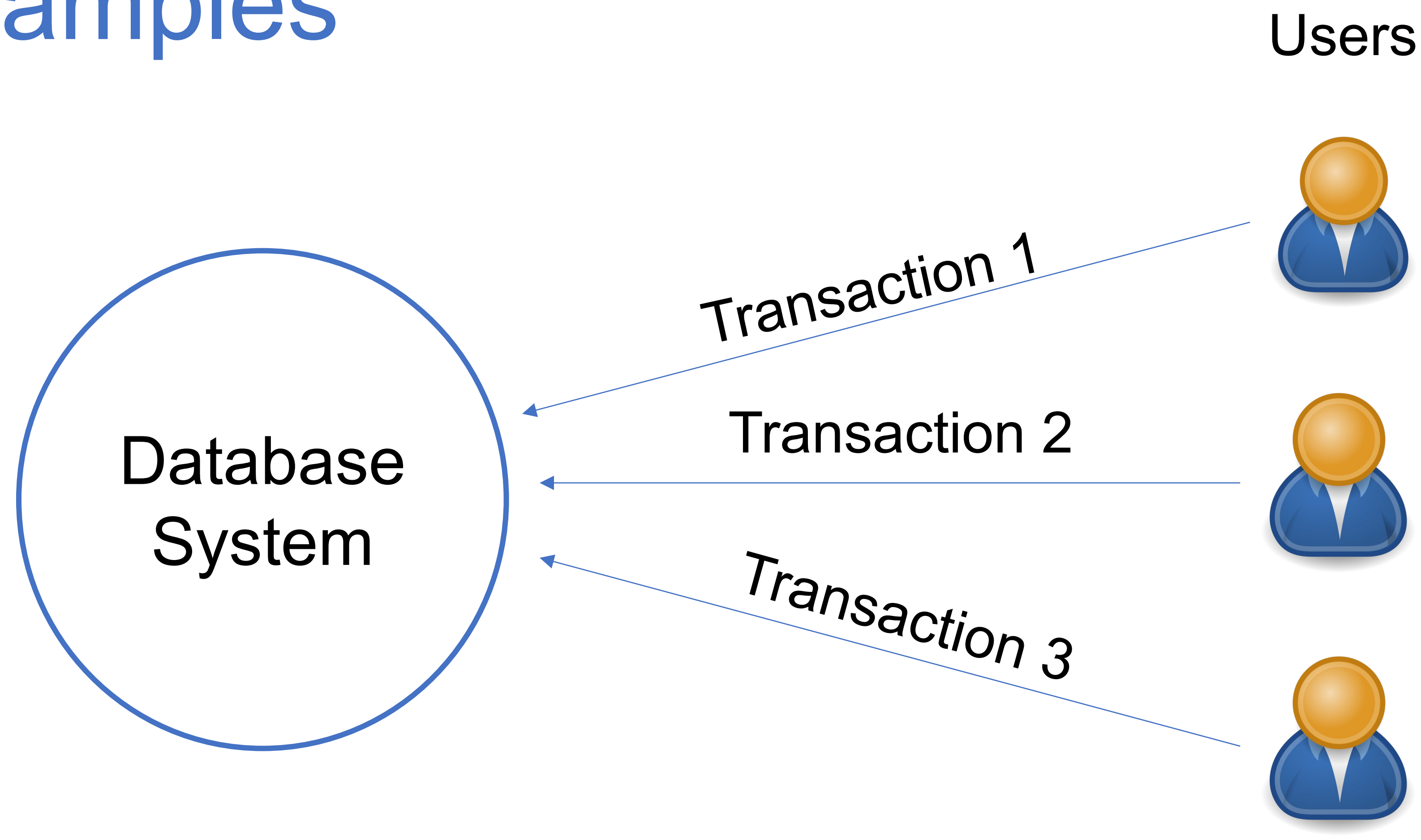
MIT 6.506 Algorithm Engineering

Concurrent Data Structure

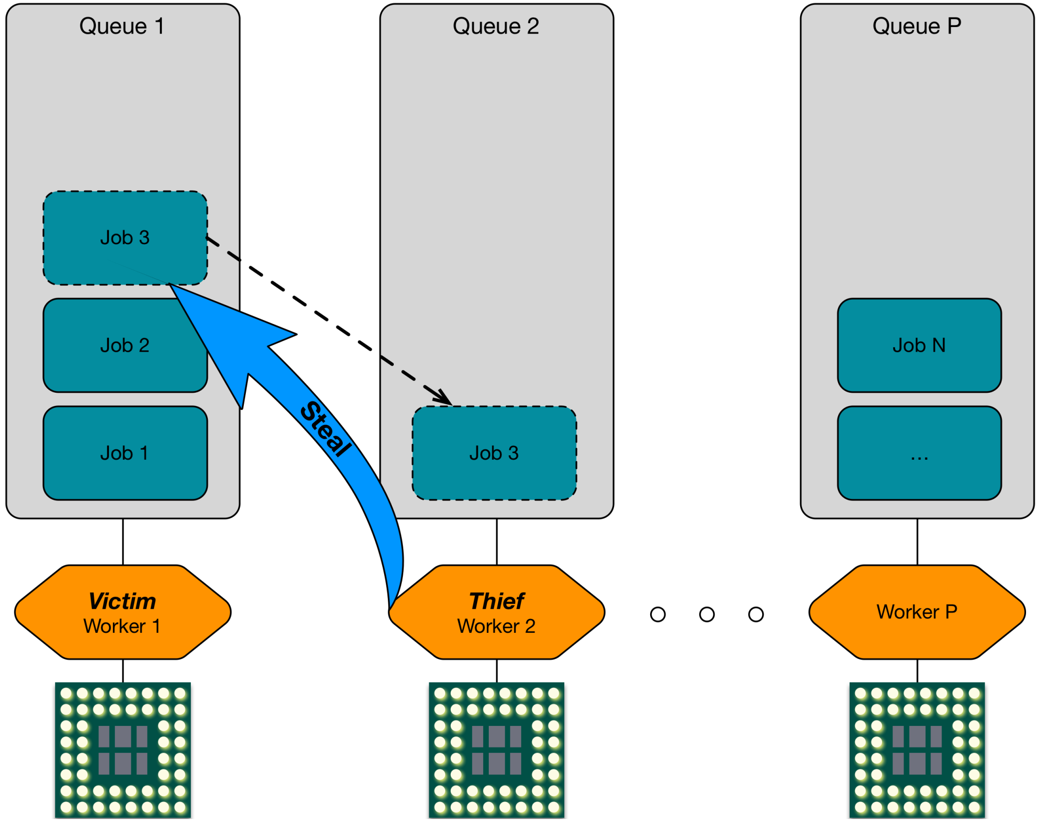


Examples



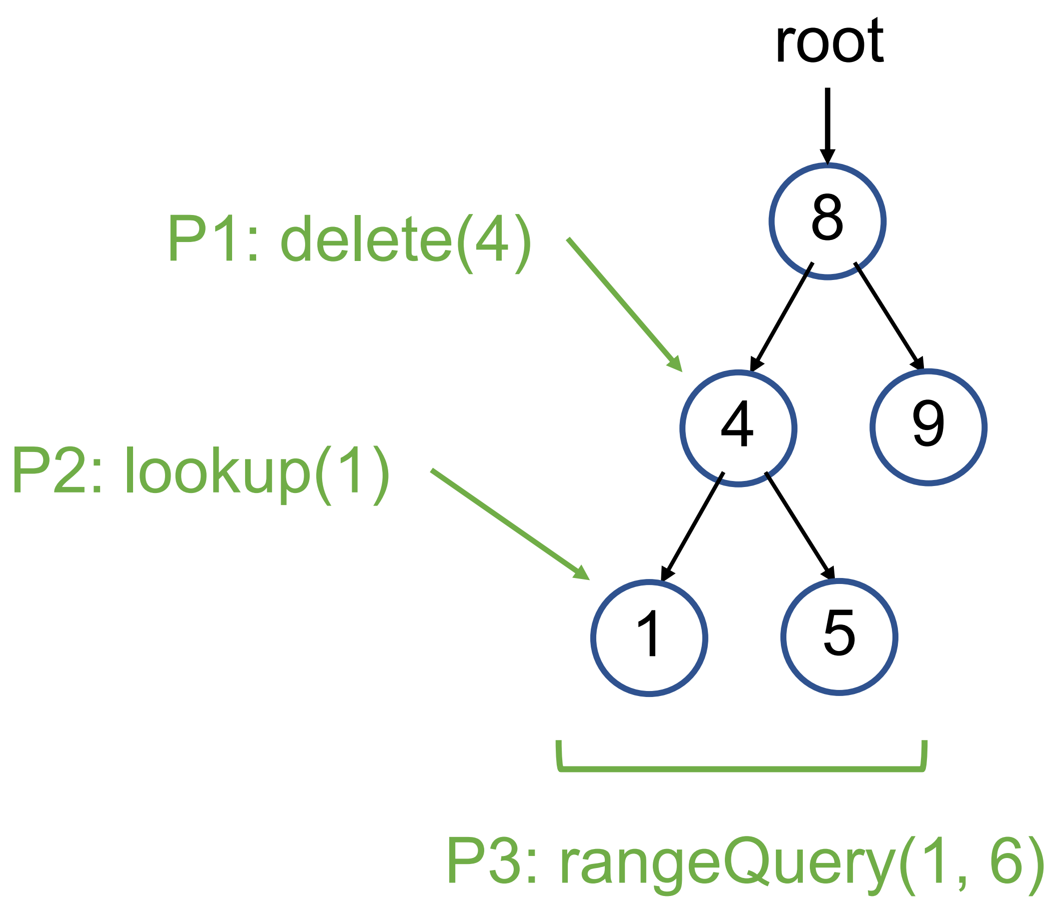
Other Applications: Operating Systems, Parallel Schedulers

Work Stealing – Concurrent Deque



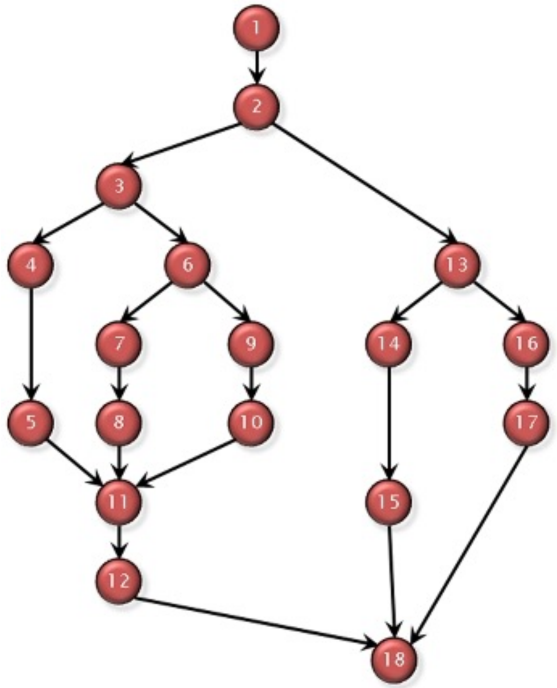
Source: https://actor-framework.readthedocs.io/en/0.17.5/Scheduler.html

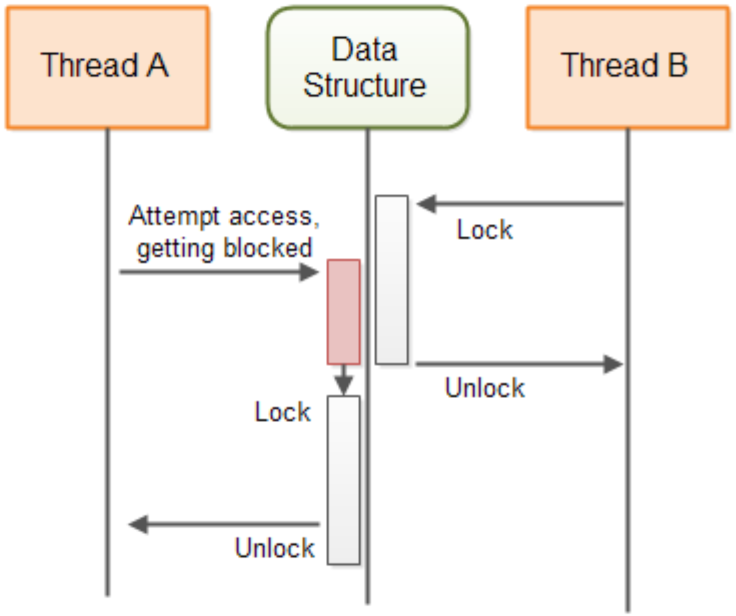
Example – Concurrent BST



Challenge:
- Ensuring data structure remains in a consistent state
- Return values are correct
- All processes make progress

Parallelism vs Concurrency





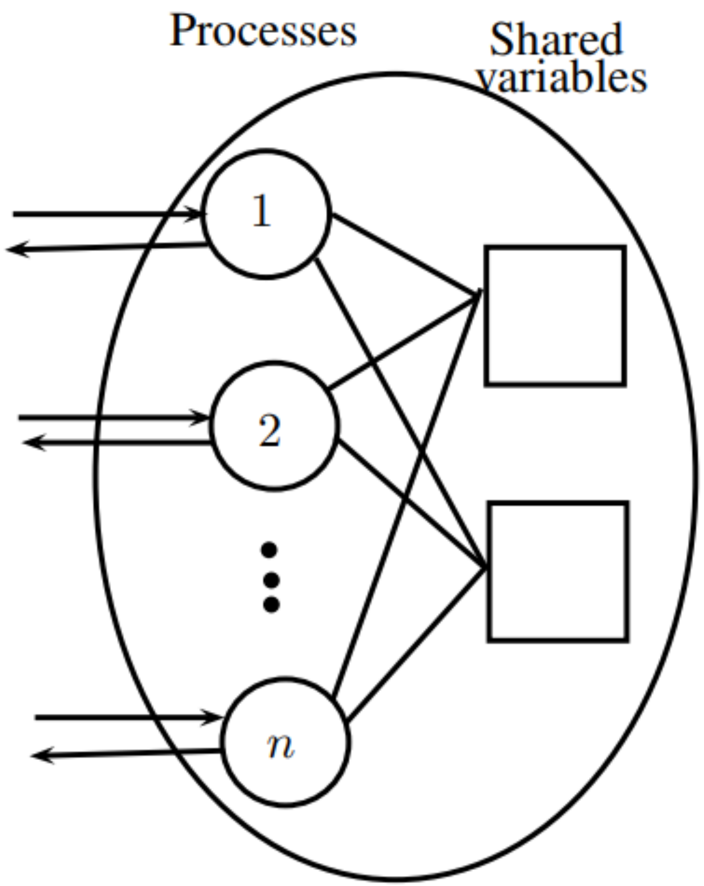
Source: https://www.cprogramming.com/parallelism.html

Source: https://jenkov.com/tutorials/java-concurrency/non-blocking-algorithms.html

Outline

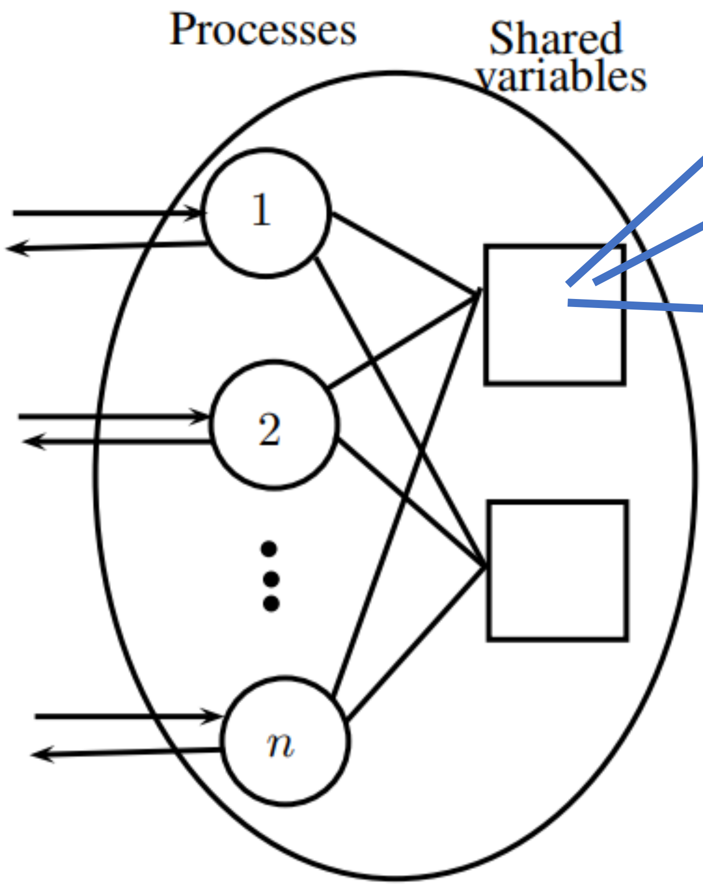
- Asynchronous Shared Memory Model
- Correctness Conditions – Linearizability, Serializability
- Lock-based techniques
  - Hand-over-hand locking
  - Lock-free searches
  - Optimistic locking
- Lock-free techniques
  - Helping
  - Harris linked list

Asynchronous Shared Memory



- Processes **communicate** through **shared variables**
- **Adversarial scheduler** interleaves steps by the processes
- Processes can be **arbitrarily slow** or **crash** (never scheduled again)

Shared Variables



Read-Write Variable: **Read**(X), **Write**(X, value)

Lock: Lock(L), Unlock(L)

Compare-and-Swap (CAS) Variable:
Read(X), **CAS**(X, oldValue, newValue)

```
if Read(X) == oldValue
    Write(X, newValue)
    return true
else return false
```

Example: Concurrent Counter

Increment():

```
y = read(C)    //  C is a shared variable, initially 0
write(C, y+1)
```

Thread 1

```
For i = 1 to 10:
    Increment()
```

Thread 2

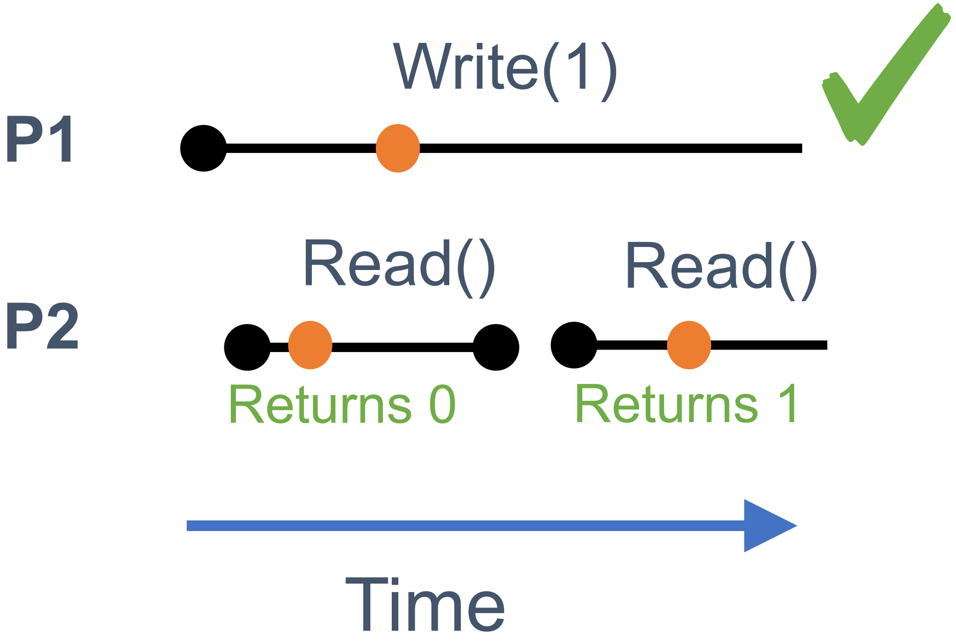
```
For i = 1 to 10:
    Increment()
```

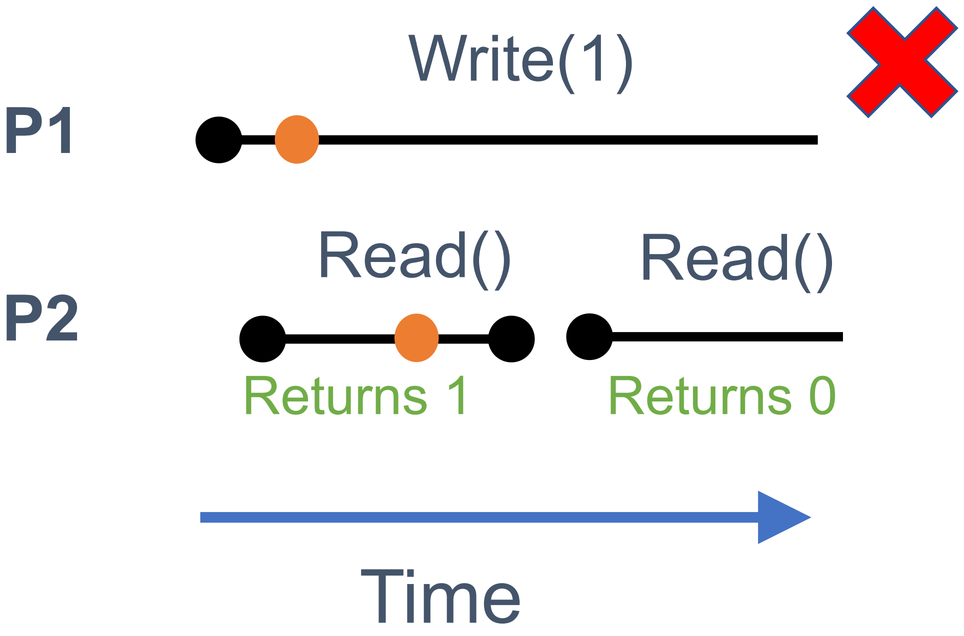
When both threads complete, what are the possible values of C?

Correctness: Linearizability

A concurrent data structure is **linearizable** if we can assign linearization points to each operation such that:

1. The linearization point of each operation lies between the invocation and response of that operation
2. The operations appear to be applied sequentially, ordered by their linearization points

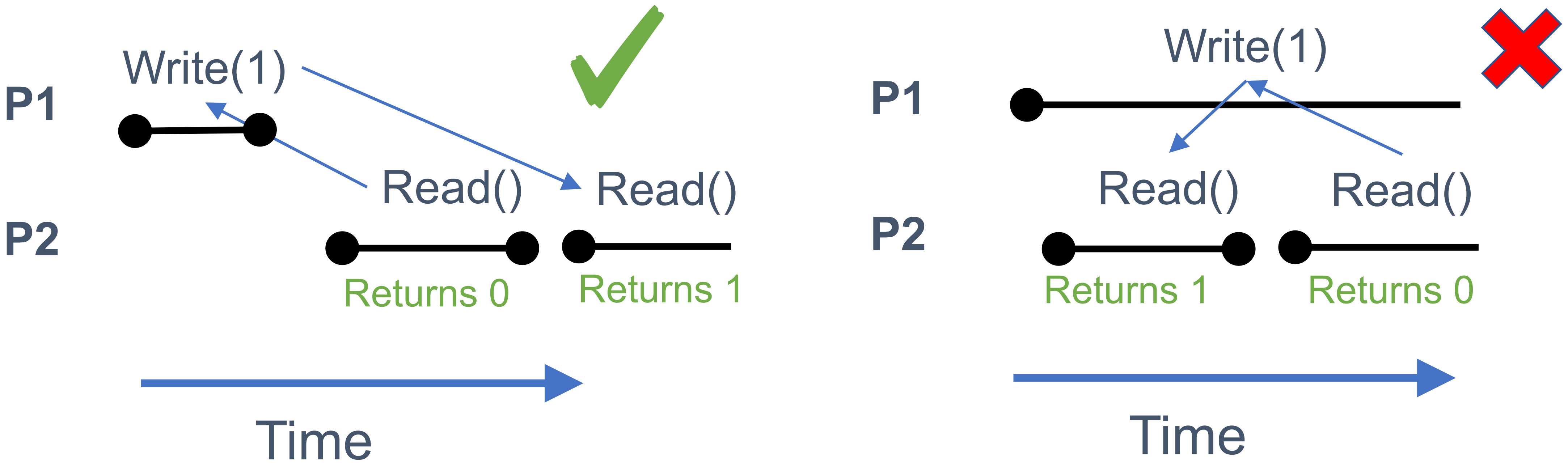




Correctness: Sequential Consistency

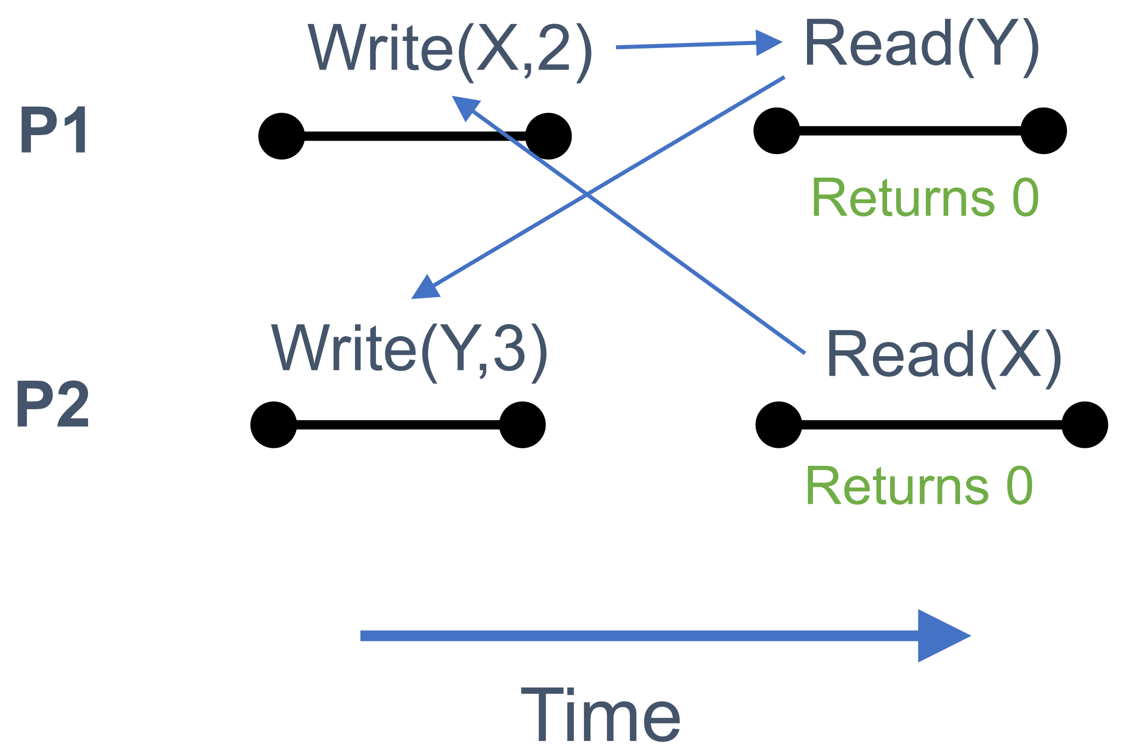
A concurrent data structure is **sequentially consistent** if we can order the operations such that:

1. The operations appear to be applied sequentially, according to this order
2. This order is consistent with the program order of each process



Linearizability vs Sequential Consistency

- Aka "strict serializability" vs "serializability" in database community



Sequential Consistency is not composable but Linearizability is!

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- Lock-based Linked Lists
  - Hand-over-hand locking
  - Lock-free searches
  - Optimistic locking
- Lock-free Linked Lists
  - Helping
  - Harris linked list

Set Abstract Data Type

```
public interface Set<T> {
  boolean add(T x);
  boolean remove(T x);
  boolean contains(T x);
}
```

- We will cover several ways of implementing this using a concurrent linked list
- These techniques generalize to other pointer-based data structures like binary trees (balanced and unbalanced), b-trees, radix trees, etc

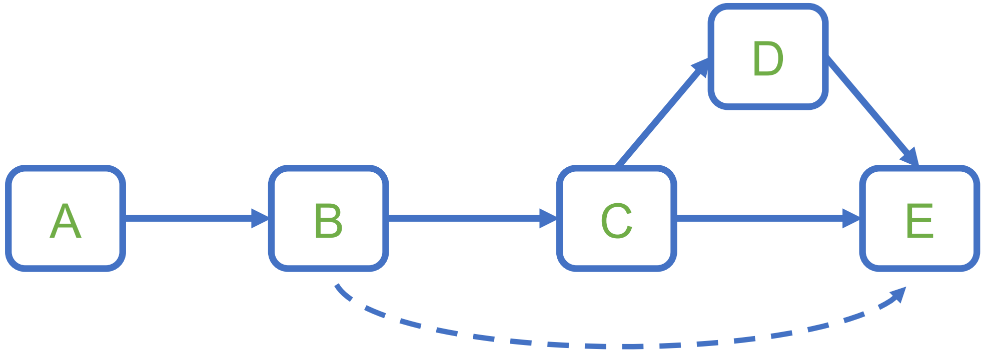
Concurrent Linked List: Challenges

Consider a Linked List where:

- Process 1 wants to remove C
- Process 2 wants to add D

Asynchrony: no assumption about relative speed of processes

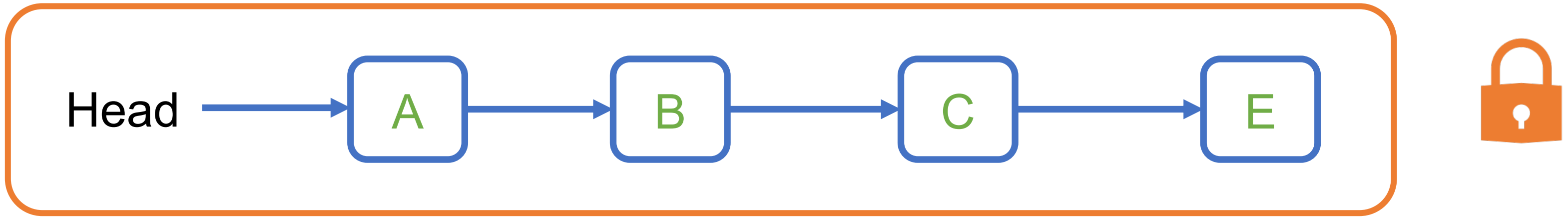
2. **Process 2** adds node D



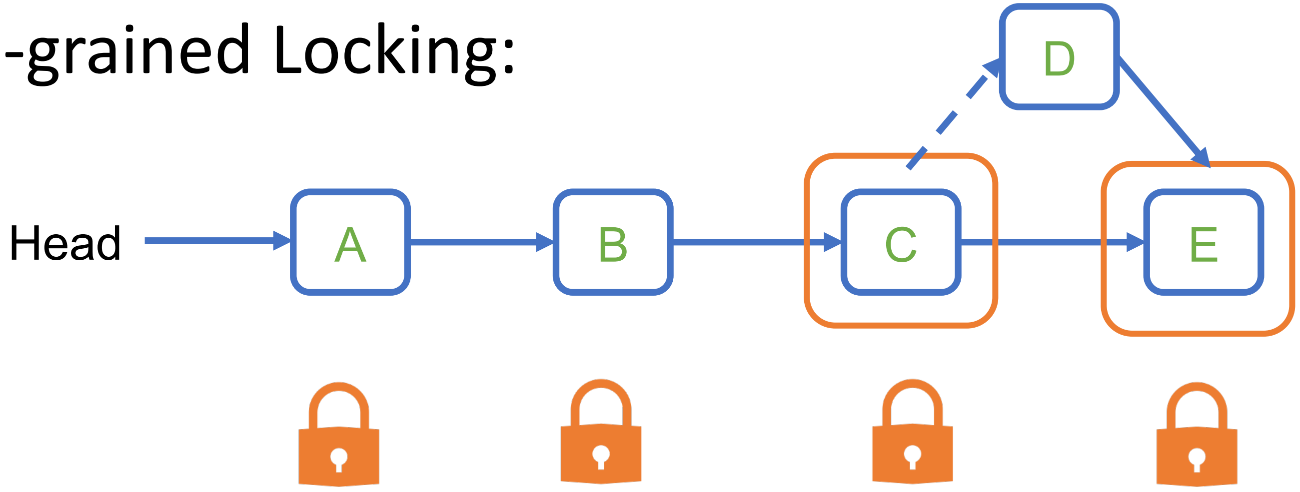
1. **Process 1** pauses right before writing the pointer to E
3. **Process 1** unpauses and accidently removes D as well as C. Incorrect!

Lock-based Solutions

- Corse-grained Locking:



- Fine-grained Locking:



Hand-over-hand Locking

```
public boolean add(T item) {
  int key = item.hashCode();
  head.lock();
  Node pred = head;
  try {
    Node curr = pred.next;
    curr.lock();
    try {
      while (curr.key < key) {
        pred.unlock();
        pred = curr;
        curr = curr.next;
        curr.lock();
      }
      if (curr.key == key) {
        return false;
      }
      Node newNode = new Node(item);
      newNode.next = curr;
      pred.next = newNode;
      return true;
    } finally {
      curr.unlock();
    }
  } finally {
    pred.unlock();
  }
}
```

Hand-over-hand Locking

- **Invariants:** our two locked nodes are always adjacent and guaranteed to be reachable from the root

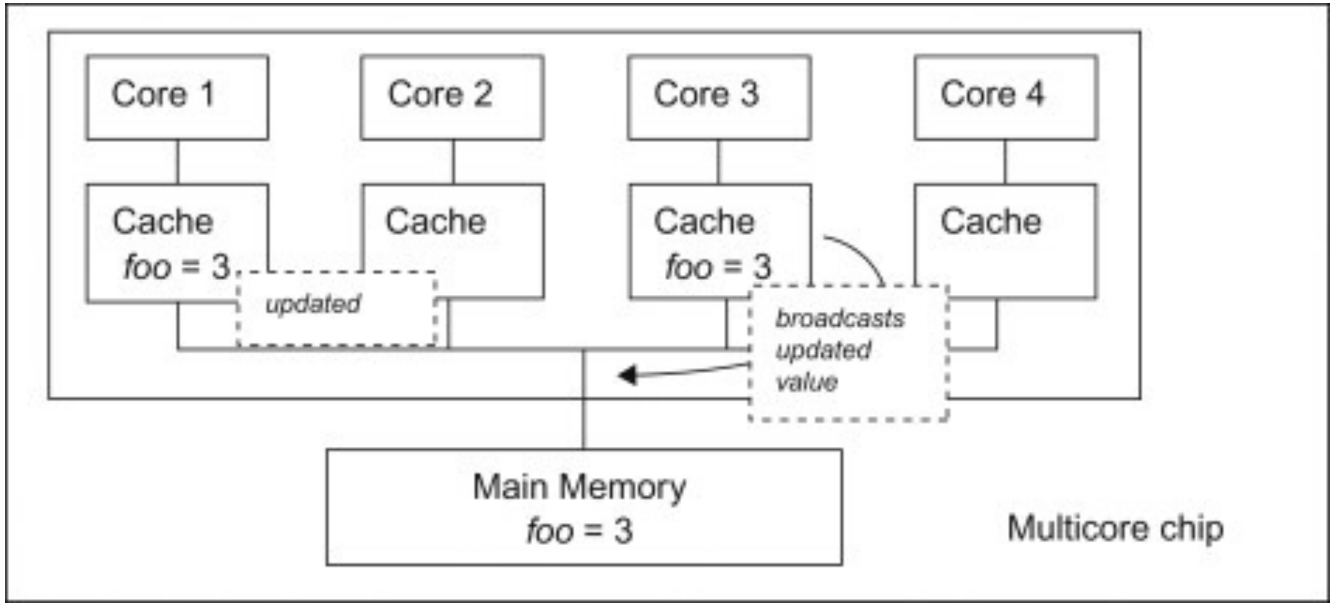
- At first glance, this looks fairly scalable, many operations can proceed in parallel
- What’s the problem?

Principles for Efficient Locking

1. Don’t hold too many locks
2. Don’t hold a lock for too long
3. Only lock the locations you plan to write to

Deadlock prevention: acquire locks in a consistent order

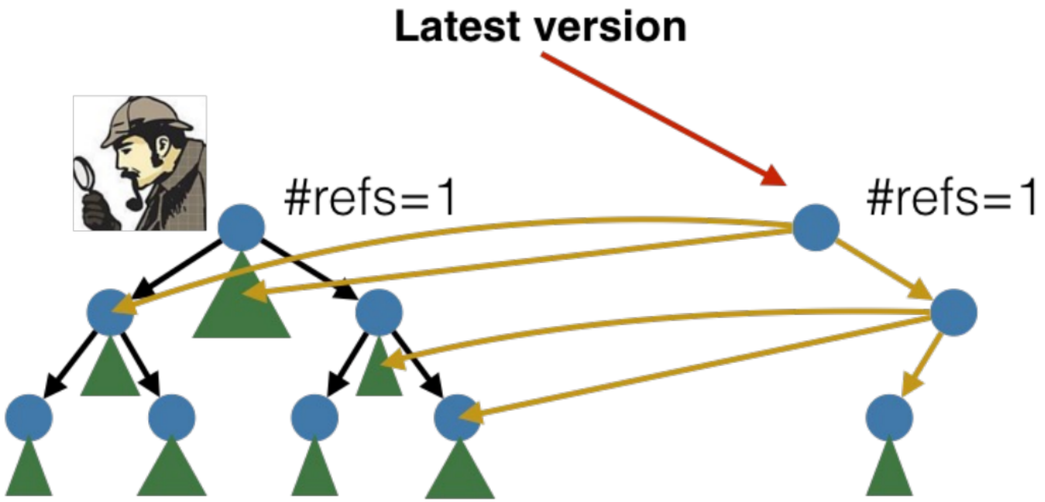
Cache Coherency



Source: https://www.sciencedirect.com/topics/engineering/cache-coherence

Path-copy runs into a similar issue

Immutability Enables Concurrency



Lock-free contains()

- Even lock-based data structures at least want their read-only operations to be lock-free
- A sequential contains() algorithm basically works in the concurrent setting

```
public boolean contains(T item) {
  int key = item.hashCode();
  Node curr = head;
  while (curr.key < key)
    curr = curr.next;
  return curr.key == key
}
```

Lock-free contains(): Linearizability

- There are times during the contains() where curr is not reachable from the root
- Therefore, we cannot linearize the contains() when it returns
- What’s the correct linearization point?

```java
public boolean contains(T item) {
  int key = item.hashCode();
  Node curr = head;
  while (curr.key < key)
    curr = curr.next;
  return curr.key == key
}
```

Speeding-up Updates: Optimistic Locking

1. Traverse optimistically without locks until you reach a node you wish to update
2. Lock neighborhood of node
3. Validate neighborhood
4. Perform updates
5. Release Locks

Validation is necessary to make sure the nodes you locked are still reachable.
A “removed” bit is added to check reachability.

Appeared as early as the 1980s, re-invented many times since then.
Go-to technique for almost all pointer-based data structures.

Speeding-up Updates: Optimistic Locking

```java
public boolean add(T item) {
  int key = item.hashCode();
  while (true) {
    Node pred = head;
    Node curr = pred.next;
    while (curr.key <= key) {
      pred = curr; curr = curr.next;
    }
    pred.lock(); curr.lock();
    try {
      if (validate(pred, curr)) {
        if (curr.key == key) {
          return false;
        } else {
          Node node = new Node(item);
          node.next = curr;
          pred.next = node;
          return true;
        }
      }
    } finally {
      pred.unlock(); curr.unlock();
    }
  }
}
```

```java
private boolean validate(Node pred, Node curr) {
  return !pred.marked && !curr.marked && pred.next == curr;
}
```

Updated Lock-free contains()

```
public boolean contains(T item) {
  int key = item.hashCode();
  Node curr = head;
  while (curr.key < key)
    curr = curr.next;
  return curr.key == key && !curr.marked;
}
```

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Progress Guarantee: Lock-freedom

Definition: some operation eventually completes regardless of how processes are scheduled

- Must hold for an adversarial scheduler
- Disallows a process from waiting for another process to take a step

History of Lock-freedom

Lock-free programming is hard!

1965 • Dijkstra introduces mutual exclusion

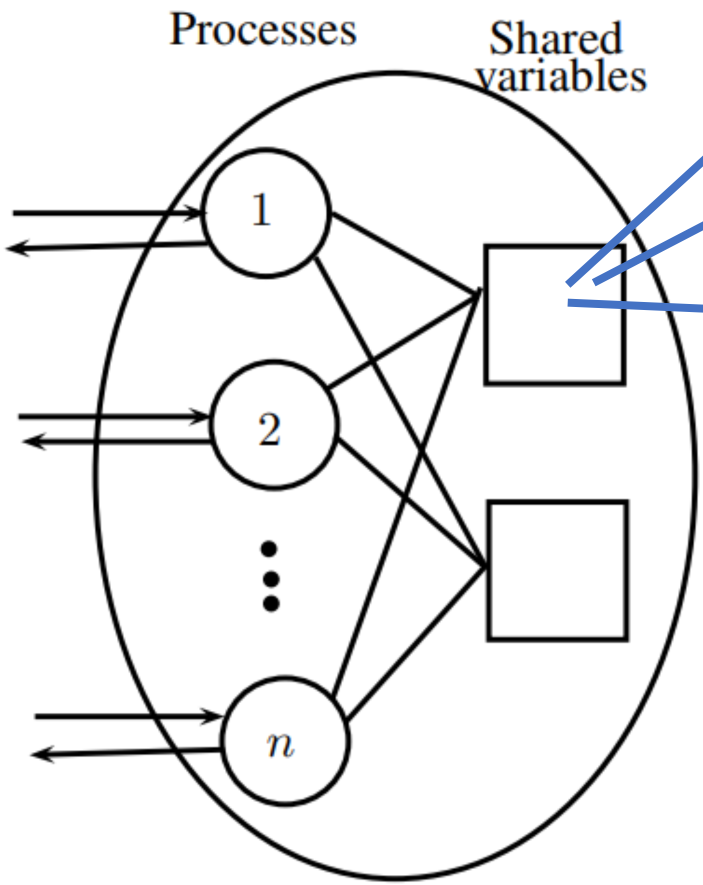
1980 • Scalable lock-based binary search trees developed

…. Lots of work on lock-freedom

….

2010 • First scalable lock-free binary search tree

Shared Variables



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if Read(X) == oldValue
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```

Lock-freedom: Key Ideas

- Most shared writes should be done with CAS
- Update operations should become visible with a single CAS
  - E.g. instead of updating the fields of a node one by one, create a new copy of the node and install it atomically
- Helping: updates operations might temporarily leave data structure in an inconsistent state, if you see this, help complete their operation.

Lock-free Linked List

- Lock-free contains() the same as before
- If we only need to support lock-free add(), then a sequential implementation with some writes replaced with CAS would be sufficient
- The tricky part is supporting delete()s

Source: https://concurrencyfreaks.blogspot.com/2014/03/harriss-linked-list.html

Lock-free Linked List

- Key idea: deletes "freeze" the node being deleted before physically removing it

- This "freeze" prevents any other process from making modifications to it

- If other processes come across a frozen node, they have to help remove it to prevent it from blocking their progress

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These techniques can be applied to a wide range of data structures

Topic I didn’t get to cover

- Concurrent Memory Management
- Weak Memory Models
- Proving Correctness
- Consensus Hierarchy
- More complex concurrent data structures

- Lots of open problems in this area