

Bayesian Nonparametric Modeling of Driver Behavior

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Abstract—Modern vehicles are equipped with increasingly complex sensors. These sensors generate large volumes of data that provide opportunities for modeling and analysis. Here, we are interested in exploiting this data to learn aspects of behaviors and the road network associated with individual drivers. Our dataset is collected on a standard vehicle used to commute to work and for personal trips. A Hidden Markov Model (HMM) trained on the GPS position and orientation data is utilized to compress the large amount of position information into a small amount of road segment states. Each state has a set of observations, i.e. car signals, associated with it that are quantized and modeled as draws from a Hierarchical Dirichlet Process (HDP). The inference for the topic distributions is carried out using an online variational inference algorithm. The topic distributions over joint quantized car signals characterize the driving situation in the respective road state. In a novel manner, we demonstrate how the sparsity of the personal road network of a driver in conjunction with a hierarchical topic model allows data driven predictions about destinations as well as likely road conditions.

I. INTRODUCTION

Vehicles are increasingly equipped with sensors and electronics to react dynamically to changing road conditions and to increase driver safety. As such, large volumes of driver-specific data related to driving conditions and driver behavior are generated. We are interested in analyzing this data to learn models of driving behavior. Such models could be used to anticipate dangerous situations, to improve the driving schedule of a person, and to tailor various aspects of the driving experience to the individual.

Here, we use data collected from one vehicle’s sensors over numerous trips to construct a Hierarchical Dirichlet Process (HDP) model of driving behavior and road conditions. HDPs are commonly used for topic modeling of text corpora [1], [2], [3] to uncover the set of topics that comprise each document in the corpus. In our case, the documents are road segments and the words are associated quantized sensor measurements. The topics in the HDP model are sensor distributions in the road segments; these distributions capture the driving conditions in each road segment as encountered by the driver as well as their driving behavior and common

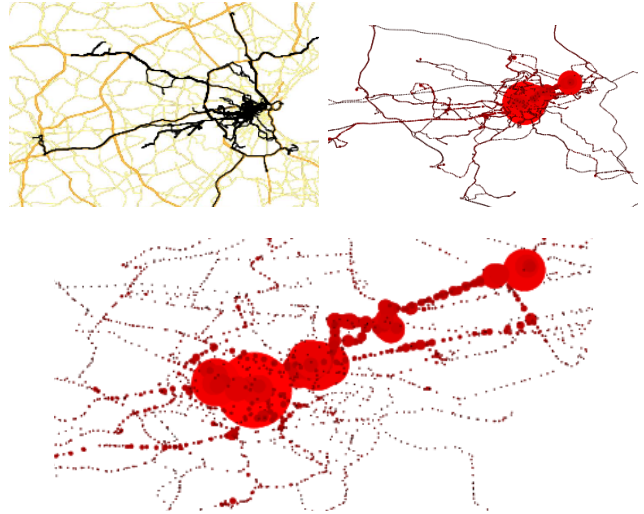


Fig. 1: Spatial distribution of the observations in the car dataset – zooming in from left to right. The leftmost plot shows the sparsity of the road states the driver is visiting compared to the full road network. The middle and right plot depict the distribution of the number of measurements taken in the respective road segment – the redder and larger the more. Clearly, this distribution is very imbalanced. A small number of states account for a majority of the measurements.

driving conditions. To our knowledge this is a new approach for modeling driving behavior. Unlike related work which is based on assumptions about the capabilities and behaviors of humans (i.e. see for an overview [4]), our model is purely data driven.

It is important to note that the hierarchy within the HDP model allows sharing of measurements across similar road segments. This is an appealing aspect of the model since it enables us to learn an expressive model for road segments which are visited rarely via similar road segments that are visited more often. In order to utilize an HDP model, we first organize the sensor data into “documents” (i.e., road segments and their associated quantized measurements). We consider the case in which a road map is not available, however, it is straightforward to incorporate such information. Additionally, typical drivers often traverse a small subset of the roads in the road network. We use a Hidden Markov Model (HMM) to learn the road segments. The HMM condenses position information from recorded trips into road segment states. The set of hidden states effectively corresponds to a sparse road network which consists only

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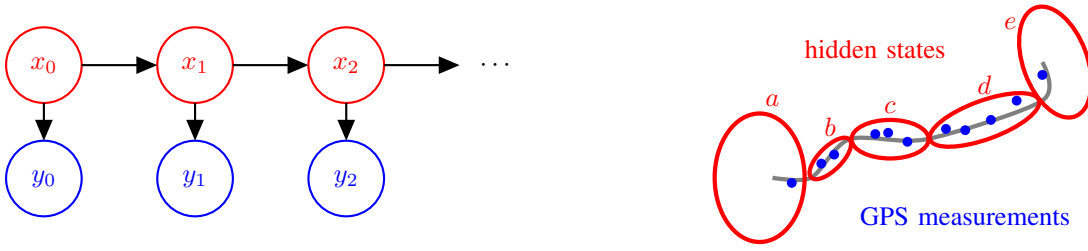


Fig. 2: (Left) A standard HMM where the hidden variables (in red) correspond to road segments and the observation variables (in blue) include position and heading measurements. (Right) A conceptual rendering of the HMM. The physical road is shown in gray while the HMM representation of the road states is shown in red. Position measurements are shown in blue.

of the roads which the driver has traversed. We then use the trained HMM to associate sensor measurements to road segments to produce "documents" for the HDP model.

In addition to organizing the data for the HDP model, the HMM also provides insight into driver behavior such as typical routes and probable destinations. Special hidden states are introduced in the HMM to represent starting locations (sources) and destinations. Consequently, identification of the most likely route between two states and finding the distribution over probable destinations become well-posed questions and allow us to make route and destination predictions.

The contributions of this paper are (1) to show how sparsity in the HMM transition matrix together with starting and absorption states lead to accurate long term predictions of driver routes and destinations and (2) the novel application of a HDP to model the joint distribution of quantized vehicle signals.

II. HIDDEN MARKOV MODEL

An HMM is used to model the trips that a driver takes through a road network. We explore two models for the HMM. In the first model, the hidden state corresponds to a road segment, a start location, or a destination. In this model, the future path is independent from the past path when conditioned on the current road segment. We expect this to be a poor model of driver behavior since this is likely an oversimplification; the past can provide considerable information about the future. For instance, drivers often do not return to a previously visited state within a trip (unless they are lost).

In our second model, we attempt to capture more of the trip history in the current state by augmenting the road states with the start location. Under this model, the road segment at the next time instance depends only on the current road segment and the start location. We will show that this model is more representative of driver behavior and provides accurate predictions of destinations and routes. We describe this second model below. The first model is a simplification of the described model.

A. Hidden States

Each hidden variable, x_t , in the HMM (see Fig. 3) takes on a value from the set of hidden states, \mathcal{X} . Source states, \mathcal{X}_S ,

destination states, \mathcal{X}_D , and road segment states augmented by the source state, $\mathcal{X}_R \times \mathcal{X}_S$, compose the set of hidden states: $\mathcal{X} = \mathcal{X}_S \cup \mathcal{X}_D \cup (\mathcal{X}_R \times \mathcal{X}_S)$. Destination states are absorbing states which are indicated by key-off events in the data. Similarly, source states are indicated by key-on events. The distribution over the initial state, $x_0 \in \mathcal{X}$, is parameterized as $p(x_0 = m) = \theta_m$. Conditioned on the current state, x_t , the distribution for the next state, x_{t+1} , is parameterized as

$$p(x_{t+1} = m | x_t = k) = \theta_{km}.$$

Since physically realizable transitions occur only between road segments in close proximity, we would expect most transition probabilities to be zero. We use a Dirichlet prior on the parameters with $\alpha < 1$ to favor a sparse transition matrix:

$$p(\theta_{k1}, \theta_{k2}, \dots, \theta_{k|\mathcal{X}|}) \propto \prod_{i=1}^{|\mathcal{X}|} \theta_{ki}^{\alpha-1}.$$

B. Observation Model

Each trip contains measurements of position, r_t and heading, h_t . When the vehicle has GPS, the recorded position is the GPS position; otherwise the reported position is obtained by dead reckoning, a process which estimates position by combining the previous position with aggregated incremental changes in a relative coordinate system. Positions which are inferred using dead reckoning are indicated by an inferred-position indicator, q_t ; for such measurements, we model a larger uncertainty associated with the measurement.

In addition to position and heading measurements, there is a key-on event at the start of each trip which indicates that hidden state must be from the set of source states. In the measurement model, we have a binary key-on indicator variable, k_t^{on} , which takes value 1 if a key-on event occurs at time t . Similarly, there is a key-off event and corresponding indicator variable, k_t^{off} , which indicates that the hidden state is from the set of destination states. This set of measurements comprise the observation $y_t = \{r_t, h_t, q_t, k_t^{off}, k_t^{on}\}$. Conditioned on the hidden state, the measurement model is as follows:

$$p(r_t, h_t, q_t, k_t^{on}, k_t^{off} | x_t) = p(r_t | q_t, x_t) p(h_t | x_t) p(q_t | x_t) \times p(k_t^{on} | x_t) p(k_t^{off} | x_t).$$

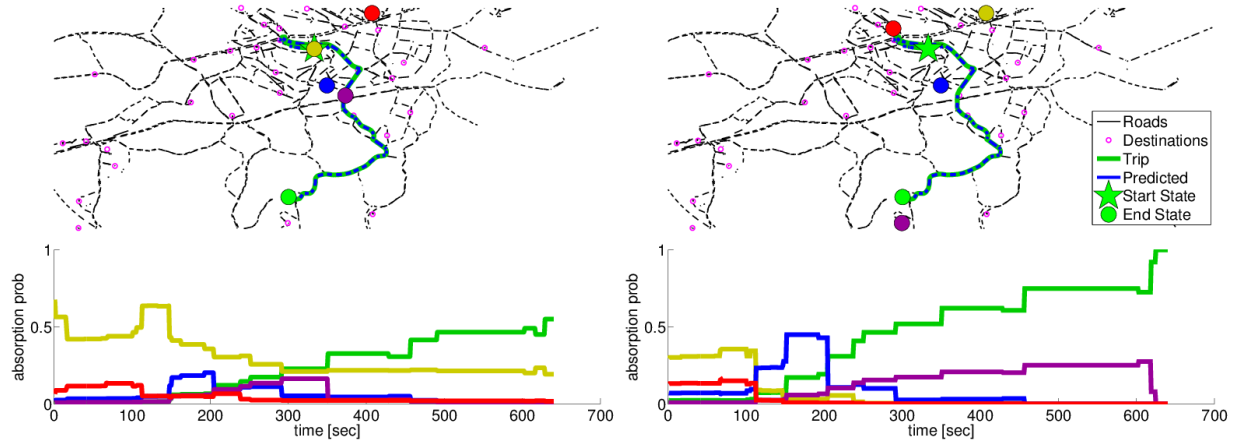


Fig. 3: Predicted routes and absorption probabilities corresponding to the five most likely destinations of the model without (left) and with (right) start locations. The most likely destinations differ between the two models; notably, in the left model, there is significant probability that the driver will return to his starting location (shown in yellow) at the start of the trip, while the starting location is not a likely destination in the model augmented with start location. Furthermore, the absorption probability of the true destination, shown in green, dominates over alternative possible destinations sooner in the bottom model.

Note that the conditional distribution for position depends on the value on the inferred-position indicator; a larger uncertainty is associated with the position when the position has been inferred. Position is Gaussian with state-dependent parameters:

$$p(r_t|q_t, x_t) = \begin{cases} \mathcal{N}(r_t; \mu_{r,x_t}, \Sigma_{r,x_t}) & q_t = 0 \\ \mathcal{N}(r_t; \mu_{r,x_t}, c \cdot \Sigma_{r,x_t}) & q_t = 1 \end{cases}$$

where $c > 1$ is a constant used to capture the increase in uncertainty of the inferred position.

Heading is also Gaussian with its own state-dependent parameters:

$$p(h_t|x_t) = \mathcal{N}(h_t; \mu_{h,x_t}, \Sigma_{h,x_t}).$$

The inferred position indicator has a Bernoulli distribution with parameter, p_{x_t} . The key-on and key-off measurements are indicators of source and destination states respectively: $k_t^{on} = \mathbb{1}(x_t \in \mathcal{X}_S)$, $k_t^{off} = \mathbb{1}(x_t \in \mathcal{X}_D)$ and have degenerate distributions.

We would expect measurements arising from the same physical location (road segment) to have parameters which do not depend on the source state. Therefore, the measurement parameters are independent of the source state when conditioned on the road segment state. That is, $\mu_{r,x_t} = \mu_{r,x_r}$ for $x_t \in \{x_r \times \mathcal{X}_S\}$, where $x_r \in \mathcal{X}_R$, and likewise for the other measurement parameters. When we estimate the measurement parameters for a road segment, this formulation allows us to aggregate observations from trips which start at different locations but share this physical road.

Similarly, there will be pairs of source and destination states will correspond to the same physical location. If a trip ends at a given location, the next trip will typically start from the same location. Since this pair of states share physical properties, these states will share measurement parameters.

C. EM Updates

Given the volume of data under consideration, we find that an EM formulation using explicit state assignments provides a tractable learning approach. This approach yields locally optimal values for the set of parameters $\psi = \{\theta_m, \theta_{km}, \mu_{r,m}, \Sigma_{r,m}, \mu_{h,m}, \Sigma_{h,m}, p_m\}$ for $m, k \in \mathcal{X}$ using measurements from N trips. The EM updates consist of iteratively finding the most likely assignment for the hidden states given previous parameter estimates, then using these assignments to improve the parameter estimates. The reader is referred to [5] for an introduction to the EM algorithm.

To initialize the parameters, we run DP means [6] to cluster the measurements based on position and heading; a state is created from each cluster. DP means allows us to initialize the model without pre-specifying the number of states. Measurements assigned to the cluster (state) are used to calculate initial values for measurement model parameters. The transition matrix is initialized as a full matrix with higher probability for states which are closer together. The distribution for the first state is initialized as a uniform distribution.

D. Predicting Routes and Destinations

Using the HMM model, we can predict a driver *route* from state a to state b by identifying the sequence of states $\{x_1^* = a, x_2^*, \dots, x_N^* = b\}$ with the highest likelihood:

$$\prod_{i=1}^{N-1} p(x_{i+1}^*|x_i^*) = \max_n \max_{\{x_1, \dots, x_n\} \in \mathcal{X}} \prod_{i=1}^{n-1} p(x_{i+1}|x_i)$$

It is well known, [7], that this can be formulated as a shortest path problem by defining a graph on the hidden states with edge weights $w_{ij} = -\log p(x_j|x_i)$.

Additionally, from any road segment state, i , we can find the probability of reaching any destination state, j ;

this is known as the absorption probability. The absorption probability, a_{ij} , is the probability of reaching absorbing state, j , if the chain starts from state, i , and can be found by solving the following set of equations:

$$\begin{aligned} a_{jj} &= 1 \quad \forall j \in \mathcal{X}_D \\ a_{ji} &= 0 \quad \forall j \in \mathcal{X}_D \quad \forall i \neq j \\ a_{ij} &= \theta_{ij} + \sum_{k \in \mathcal{X} \setminus j} \theta_{ik} a_{kj} \quad \forall j \in \mathcal{X}_D, \forall i \in \mathcal{X}_R \end{aligned}$$

This gives us a probability distribution over destinations when we start from a given road state.

E. Bayesian Nonparametric Topic Modeling of Car Signals

Thus far, we have formulated an HMM model for driving behavior which has predictive aspects. The model is also used to organize the data into "documents" so that we can perform HDP topic modeling on the dataset. In this section we discuss how we combine a standard HDP model with the use of an HMM to discover documents. We also relate our HDP model for car signals to the classical HDP topic model.

To bridge the gap between the classical HDP topic modeling of text corpora and the modeling of car signals, such as velocity, acceleration and rotational speed, note the following correspondences:

$$\begin{aligned} \text{word} &\leftrightarrow \text{car signals at one instance in time} \\ \text{document} &\leftrightarrow \text{road segment} \\ \text{corpus} &\leftrightarrow \text{map} \end{aligned}$$

Each learned road segment from the HMM is used as a document in the HDP model. To obtain a set of sensor measurements associated with a road segment, or a set of words from the document, we perform ML assignment of road states for the trips and assign the corresponding sensor measurements to those road states.

Since the car signals are continuous quantities, we quantize them and use a discrete base measure equivalently to the classical text corpus topic model. This means, a word is described by a multidimensional vector, which amounts to modeling the joint distribution over all signals.

In the next section we briefly summarize the HDP model and describe how we use a standard variational approximation to perform inference with discretized car measurements. For a more detailed presentation the reader is referred to the original papers [1] and [3].

F. HDP model

The HDP model describes a set of D documents, which contain N_d words $w_{d,n}$ each. In this context, the documents correspond to trips and the words in a document correspond to quantized sensor measurements from the trip.

The distribution of these words $w_{d,n}$ is modeled as a mixture of so called topic distributions. These topics are Categorical distributions parameterized by β_k with a Dirichlet distribution prior with parameter ν .

We utilize a construction of the HDP which applies Sethuraman's stick breaking construction of a DP [8] twice.

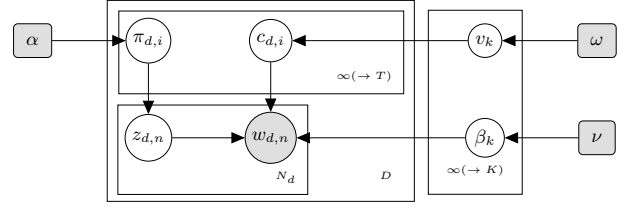


Fig. 4: The HDP model utilizing a two level stick breaking construction. The variational approximation truncates the stick breaking at the corpus level to K and on the document level to T .

Unlike the original construction by Teh et. al [1], this allows the derivation of stochastic online variational inference as proposed by Wang et. al. [2] (see also [3] for a more detailed derivation).

- On the corpus level we have an infinite number of topics β_k with proportions v_k :

$$\begin{aligned} \beta_k &\sim H(\nu) \quad \text{for } k \in \{1, \dots, \infty\} \\ v_k &\sim \text{Stick}(\omega) \quad \text{for } k \in \{1, \dots, \infty\} \end{aligned}$$

- For each document in the corpus we draw an infinite number of indicators c_d pointing to the corpus level topics as well as mixing proportions π_d

$$\begin{aligned} c_{d,i} &\sim \text{Cat}(v) \quad \text{for } i \in \{1, \dots, \infty\} \\ \pi_{d,i} &\sim \text{Stick}(\alpha) \quad \text{for } i \in \{1, \dots, \infty\} \end{aligned}$$

- Finally, for each word we draw a pointer to the document level topics. The word $w_{d,n}$ in document d is then drawn from the topic selected by the two level indicator $c_{d,z_{d,n}}$.

$$\begin{aligned} z_{d,n} &\sim \text{Cat}(\pi_d) \quad \text{for } n \in \{1, \dots, N_d\} \\ w_{d,n} &\sim \beta_{c_{d,z_{d,n}}} \quad \text{for } n \in \{1, \dots, N_d\} \end{aligned}$$

The stick-breaking distribution $\text{Stick}(\alpha)$ is defined as:

$$\begin{aligned} \pi'_k &\sim \text{Beta}(1, \alpha) \\ \pi_k &= \pi'_k \prod_{j=1}^{k-1} (1 - \pi'_j) \end{aligned}$$

G. Stochastic Variational Inference for the HDP

Because of the large quantity of data, we resort to the variational approximation of the HDP described in [2] and [3] for manageable inference. Utilizing the mean-field approximation we obtain the following variation approximation to the true joint distribution $p(\beta, \mathbf{v}, \pi, \mathbf{c}, \mathbf{z})$:

$$q(\beta, \mathbf{v}, \pi, \mathbf{c}, \mathbf{z} | \lambda, \mathbf{a}, \zeta, \gamma, \phi) = \left(\prod_{k=1}^K q(\beta_k | \lambda_k) q(v_k | a_k) \right) \left(\prod_{d=1}^D \prod_{i=1}^T q(c_{d,i} | \zeta_{d,i}) q(\pi_{d,i} | \gamma_{d,i}) \prod_{n=1}^{N_d} q(z_{d,n} | \phi_{d,n}) \right)$$

In the following we will refer to all variables as $\mathcal{Z} = \{\beta, \mathbf{v}, \pi, \mathbf{c}, \mathbf{z}\}$ and to their parameters in the variational approximation as $\Lambda = \{\lambda, \mathbf{a}, \zeta, \gamma, \phi\}$. Besides the assumption that all variables involved are independent, this approximation also truncates the stick-breaks to K on the corpus level and T on the document level. In practice this is no problem,

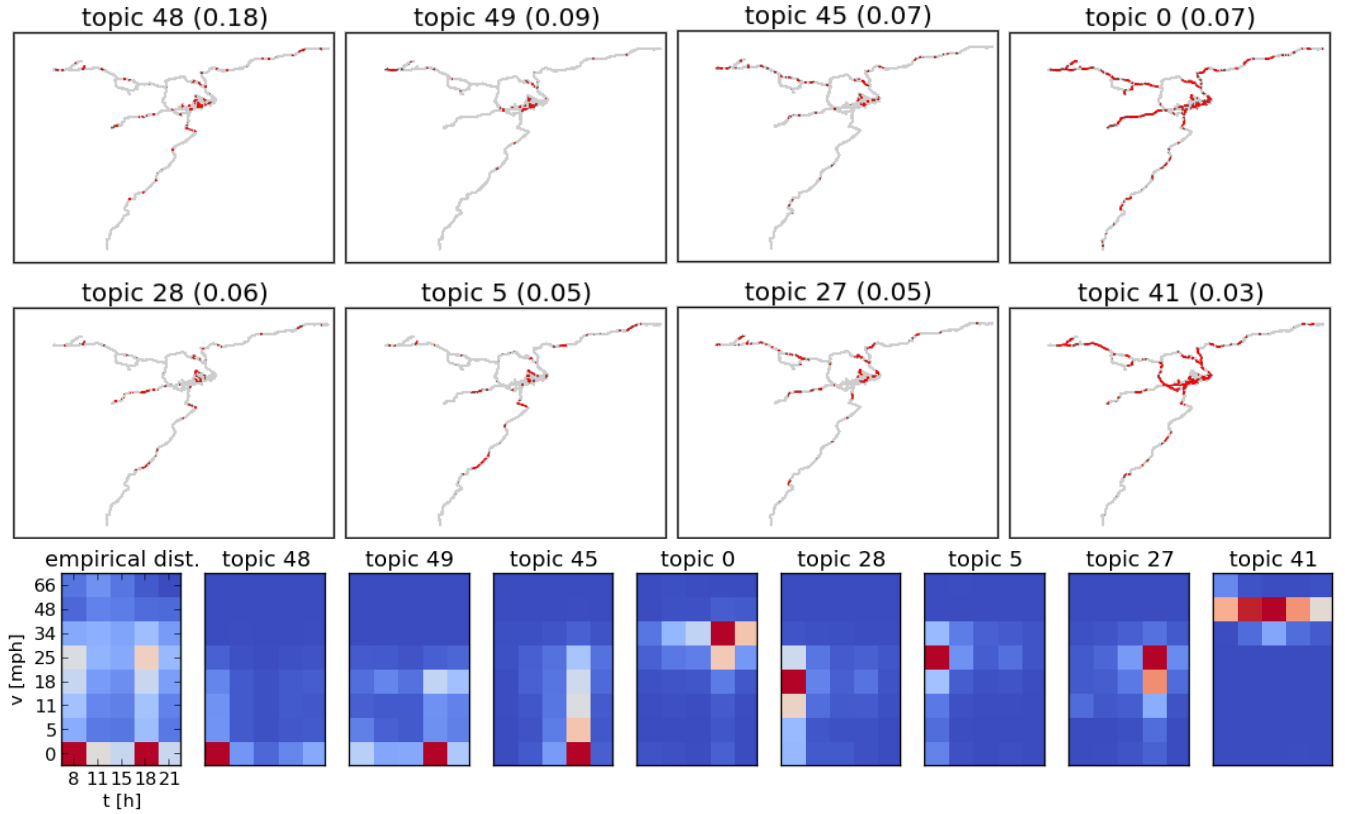


Fig. 5: Plots in the first two rows depict the road states in red, in which the respective topic has the highest likelihood. In the third row, the respective topics are shown. These are categorical distributions over the joint quantized speed and time-of-day measurements. The probabilities are color-coded from blue (low) to red (high).

since the truncations can be set high enough to allow the HDP to adapt to the complexity of the data freely.

The objective of mean-field variational inference is to find the distribution q which is closest to the true posterior distribution in the KL-Divergence sense:

$$\begin{aligned}
 q(\mathcal{Z}|\Lambda^*) &= \arg \min_{\Lambda} D_{\text{KL}}(q(\mathcal{Z}|\Lambda) || p(\mathcal{Z}|\mathbf{D})) \\
 &= \arg \min_{\Lambda} \mathbb{E}[q(\mathcal{Z}|\Lambda)] - \mathbb{E}[p(\mathcal{Z}, \mathbf{D})] + \log p(\mathbf{D}) \\
 &= \arg \min_{\Lambda} -\mathcal{L}(q) + \log p(\mathbf{D}),
 \end{aligned}$$

where $\log p(\mathbf{D})$ is constant with respect to the parameters. Therefore minimizing the KL-divergence amounts to maximizing the so called Evidence Lower Bound (ELBO) $\mathcal{L}(q)$.

The full derivation of stochastic variational inference and the algorithm can be found in [3]. For the discretized car signals, we use the online stochastic variational algorithm presented in [3] with a Dirichlet base measure to perform inference. We use the learning-rate $\rho_t = (1 + t)^{-\kappa}$; $\kappa \in (0.5, 1]$, where t is the iteration number. As suggested in the original paper, we also implemented mini-batch updates. This means, the corpus level parameters are updated with the average updates from the set of documents in a mini-batch.

H. Performance Evaluation

To evaluate the performance of the HDP model, we are computing the average log predictive probability of held-out

words. To compute this probability, we split a test document into two sets: held out words w^{ho} and observed words w^{obs} . Then we update the model using the observed words. This gives us the posterior parameters $\{\zeta^{\text{obs}}, \lambda^{\text{obs}}\}$ for the test document which we in turn use to find ϕ^{ho} for the held-out words. Now we can compute the probability of a held-out words as given all training data \mathbf{D} as well as the observed words w^{obs} in this document:

$$p(w_n^{\text{ho}}|\mathbf{D}, w^{\text{obs}}) = \sum_{i=1}^T q(z_n = i|\phi^{\text{ho}}) \sum_{k=1}^K q(c_i = k|\zeta^{\text{obs}}) p(w_n^{\text{ho}}|z_n = i, c_i = k, \lambda^{\text{obs}}) \quad (1)$$

where $p(w_n^{\text{ho}}|z_n^i, c_i^k, \lambda^{\text{obs}}) = \frac{\lambda_k^{\text{obs}}(w_n^{\text{ho}})}{\sum_w \lambda_k^{\text{obs}}(w)}$ is the conditional distribution of a held-out word under the posterior distribution of words in this document.

As a model to compare the HDP to, we utilize a non-hierarchical model that assumes a Categorical distribution with a Dirichlet prior for the words in each road-state. These distributions are modeled completely independent – not connected via a hierarchy like in the HDP model. This allows us to compute posterior Categorical distributions given the observed words in each road-state.

III. RESULTS

In the following we will first give results for the predictive power of the HMM model before we describe a topic

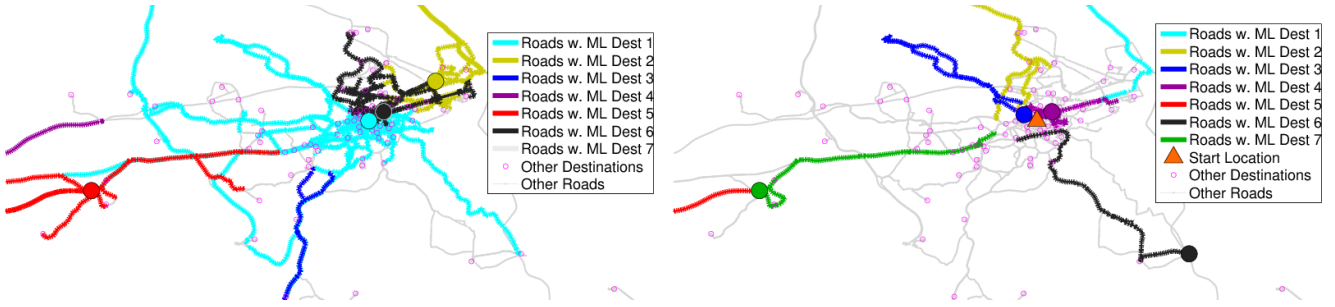


Fig. 6: Most likely destination for each road segment without (left) and with (right) start location. The seven most popular destination locations are indicated by large colored circles. In the model augmented with start location, the start location for this plot is shown in red. Only roads whose most likely destination belongs to the set of seven most popular destinations in each model are plotted in the color corresponding to the destination. In the right plot, we see that the model is able to capture the phenomenon that the most likely destination is not a destination which the driver has already passed, or equivalently, the destination will not be on any typical path between the start location and the road segment

model for the joint distribution of speed and time-of-day measurements.

A. Dataset Description

Our dataset comprises 1K trips recorded from a standard car used by a single driver. The routes are mostly commuting to work but also some longer range trips outside the city.

The GPS position and heading measurements of the car are used to train the HMM model. From various other signals of the car we selected quantized car velocity and time of day for the HDP topic model. These were selected, since they contain interesting information both about the driving behavior as well as the driving situation in a road state.

B. Predicting Routes and Destinations

To evaluate the quality of the learned HMM, we examine the ability of the HMM to predict the destination for 20 held-out trips under the two different models. Additionally, we compare the path of the held-out trips against the most likely route obtained from the transition matrix for the HMM.

Fig. 3 shows the performance of the two models on a held-out trip. The plots under the maps in the figure show the absorption probabilities for the probable destinations as a function of time. The maps above show the trip, the most likely route between the source and destination state, and the locations of the probable destinations. While the most likely path between source and destination from both models agrees with the observed trip trajectory, we observe that the augmented model is able to identify the correct destination sooner than the first model. In fact, the first model is able to correctly predict the destination after 10% of the trip for only 3 of the held-out trips while the augmented model is able to do so for 11 of the trips.

In Fig. 6, we show the most likely destination for each road segment. For the augmented model, since each state associated with a road segment also has a start location, we’ve chosen a particular start location to illustrate the differences between these two models. In particular, when starting from the specified start location, we see that trips which traverse beyond destination 7 in Fig. 6 (bottom) are

more likely to terminate at a destination which is further from the starting location. The unaugmented model is unable to make this distinction, so trips which traverse road segments near destination 5 in Fig. 6 on the top (which corresponds to destination 7 in Fig. 6 on the bottom) are likely to terminate at that destination.

The results show that the most likely route obtained by the models frequently align exactly with the path of the held-out trips. This can be explained through the sparsity of the transition matrices; since each state can only transition to few states, and very often just one state, long term predictions in this model are quite accurate.

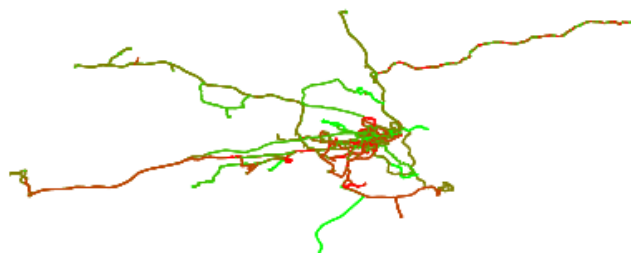
C. HDP Model

We are quantizing velocity and time-of-day measurements to words that can be fed into the HDP inference algorithm. The speed in a range from 0 to 81 mph is quantized into eight bins whereas the time-of-day is quantized to five discrete values. Quantization is performed by standard k-means clustering over the individual signals. The number of quantizations was selected such that the underlying distribution is well captured.

The speed measurements arrive at a rate of 1 Hz from the GPS sensor. There are 696k joint observations – velocity/time-of-day pairs – across all 12k road states. As can be seen in Fig. 1, these observations are distributed non-uniformly – we get a lot of measurements on the daily commute route and few on highways leading outside the city. This means that the road-state corpus has very imbalanced document sizes when compared to text corpus modeling. However, our results demonstrate, that this presents no issue to the inference algorithm.

We empirically found the following set of parameters: a mini-batch size $S = 50$, $\kappa = 0.7$, $\omega = 10.0$ and $\alpha = 0.1$. We set the truncations of the HDP to $K = 50$ corpus-level topics and $T = 15$ document-level topics to allow the HDP to adapt to the complexity of the car dataset.

Fig. 5 demonstrates that the hierarchy in the HDP is able to pool measurements from different road-states to obtain a descriptive topic for these. For each road state we obtain



(a) ML estimate of time-of-day per road state computed from empirical distribution. Color-coded from green (early in the day) to red (late at night).



(b) ML estimate of speed per road state computed from empirical distribution. Color-coded from green (slow speeds) to red (fast driving).



(c) ML estimate of time-of-day per road state computed via inferred HDP model.



(d) ML estimate of speed per road state computed via inferred HDP model.

Fig. 7: Plots of the Maximum Likelihood (ML) estimates of time-of-day and speed computed from the inferred HDP model (bottom row) and the empirical distribution (top row).

the maximum likelihood (ML) topic assignment and plot the respective road states in red. This pooling of observations can for example be observed for topics 0 and 41, which consist of almost all highway road states as can be seen in the ML topic assignment plots (compare the red road segments to the highways depicted in map in Fig. 1).

Using the inferred mixture of topics for each state, we can now compute the ML estimate of the marginals for the individual sensor signals and plot them color-coded for each road state. Fig. 7 shows this for the marginal over speed and time-of-day. Comparing the spatial distribution of the ML speed estimates computed from the inferred HDP model in Fig. 7d with ML estimates obtained from the empirical distribution depicted in Fig. 7b, we can see that the HDP model is able to capture the distribution of the input data.

Additionally, It is clear from the spatial distribution of the ML estimates of driving speeds, that the HDP model captures for example the fact, that inner city driving is slower than highway driving. The ML estimates of time-of-day (Fig. 7c and 7a) show that the trips outside the city were not undertaken in the morning or evening.

IV. CONCLUSION

We have shown that the inherent sparsity of the learned personal road network allows accurate long term predictions of driver routes. Additionally, augmenting the model with start location yields a more representative model which provides better destination predictions. Exploiting the hierarchy of the HDP topic model, we are able to learn expressive topic distributions despite the fact that the number of car signal measurements differs widely between different road states. The combination of both types of models allows us to model the driving behavior of an individual driver.

This type of model can for example assist in optimizing the daily commute route or help predict traffic jams. As a next step it would be interesting to explore adding several more car signals to allow additional predictions beyond expected speed distribution at a certain time of day. Additionally, we would like to compare the driver models for different drivers to allow driver classification based on their driving behavior.

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