High Resolution Celestial Imaging: An Algorithm and Dataset for VLBI Image Reconstruction

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Recently, there has been an international effort to create an Event Horizon Telescope (EHT) capable of imaging a black hole's event horizon for the first time. Imaging distant celestial sources, such as the event horizon of a black hole, with high resolving power (i.e. small angular resolution) requires telescopes with prohibitively large diameters due to the inverse relationship between angular resolution and telescope diameter.Although a single telescope is unrealizable, by simultaneously collecting data from an array of telescopes located around the Earth, it is possible to emulate samples from a single telescope with a diameter equal to the maximum distance between telescopes in the array. Using multiple telescopes in this manner is referred to as very long baseline interferometry (VLBI).

The challenges in reconstructing images from high resolution VLBI data are immense. The data is extremely sparse and noisy, thus requiring statistical image models such as those designed in the computer vision community. Current methods used for VLBI image reconstruction either make use of simple priors unsuitable for a wide range of possible source emissions, or require significant parameter tuning that may be prohibitively difficult for a new user to master. We present a novel, robust Bayesian approach for VLBI image reconstruction that requires minimal parameter tuning. The success of our method is demonstrated on realistic synthetic experiments as well as publicly available real data. We also provide a dataset and website to allow for controlled comparisons across algorithms. This dataset will foster development of new, robust methods by making VLBI easily approachable to computer vision researchers.

A Brief Introduction to VLBI The *Van Cittert-Zernike Theorem* states that, for ideal sensors, the time-averaged correlation of the measured signals from two telescopes, i and j, for a single wavelength, λ, can be approximated as:

$$\Gamma_{i,j}(u,v) \approx \int_\ell \int_m e^{-i2\pi(u\ell+vm)} I_\lambda(\ell,m)\, d\ell dm \tag{1}$$

where $I_\lambda(\ell,m)$ is the emission of wavelength λ traveling from the direction $\hat{s} = (\ell, m, \sqrt{1-\ell^2-m^2})$. The dimensionless coordinates (u,v) (measured in wavelength) are the projected baseline, B, orthogonal to the line of sight. Notice that Eq. 1 is just the Fourier transform of the source emission image, $I_\lambda(\ell,m)$. Thus, $\Gamma_{i,j}(u,v)$ provides a single complex Fourier component of I_λ at position (u,v) on the 2D spatial frequency plane. *We refer to these measurements, $\Gamma_{i,j}$, as visibilities.* As the Earth rotates, the direction that the telescopes point towards the source ($\hat{s}$) changes. Assuming a static source, this results in measuring different visibilities along elliptical paths in the (u,v) frequency plane.

Inhomogeneities in the atmosphere cause the light to travel at different velocities towards each telescope. These delays have a significant effect on the phase of measurements, and renders the phase unusable for image reconstructions at wavelengths less than 3 mm.Although absolute phase measurements cannot be used, by multiplying the visibilities from three different telescopes, we obtain an expression (refered to as the *bispectrum*) that is invariant to the atmosphere.

$$\Gamma^{\text{meas}}_{i,j}\Gamma^{\text{meas}}_{j,k}\Gamma^{\text{meas}}_{k,i} = e^{i(\phi_i-\phi_j)}\Gamma^{\text{ideal}}_{i,j} e^{i(\phi_j-\phi_k)}\Gamma^{\text{ideal}}_{j,k} e^{i(\phi_k-\phi_i)}\Gamma^{\text{ideal}}_{k,i} = \Gamma^{\text{ideal}}_{i,j}\Gamma^{\text{ideal}}_{j,k}\Gamma^{\text{ideal}}_{k,i} \tag{2}$$

Method The image that we wish to recover, $I_\lambda(\ell,m)$, is defined over the continuous space of angular coordinates l and m. Many algorithms assume a discretized image of point sources during reconstruction and subsequently blur the final image to account for the expected instrumental resolution.Instead, we parameterize a *continuous* image using a discrete number of terms, $\hat{I}_\lambda(\mathbf{X})$, in a way that reduces the continous integration necessary to compute a visiblity to a linear matrix operation similar to a DTFT. This parameterization allows us to easily evaluate the likelihood of measuring a set of visibilities from an estimate of the continuous image during optimization.

Bispectrum measurements are invariant to atmospheric inhomogeneity. Therefore, we choose to express an image's "likelihood" in terms of the bispectrum, rather than visibility, measurements. Let Y_k be a noisy, complex bispectrum measurement corresponding to visibilities $k_{1,2}$, $k_{2,3}$, and $k_{1,3}$ for telescopes $1, 2$ and 3. Ideal bispectrum values, ξ, can be extracted from $\hat{I}_\lambda(\mathbf{X})$ using the following polynomial equation:

$$\xi_k(\mathbf{X}) = A_{k_{1,2}}\mathbf{X}A_{k_{2,3}}\mathbf{X}A_{k_{1,3}}\mathbf{X} = \xi_k^{\Re}(\mathbf{X}) + i\xi_k^{\Im}(\mathbf{X}) \tag{3}$$

where complex, row vector $A_{k_{m,n}}$ extracts the 2D frequency, (u,v), corresponding to the baseline between telescopes m and n from $\hat{I}_\lambda(\mathbf{X})$. By assuming Gaussian noise (Σ_k) on the complex bispectrum measurements, we evaluate the measurement "likelihood" as

$$\sum_{i=k}^{M} \left[\frac{\alpha_k}{2} \begin{pmatrix} \xi_k^{\Re}(X) - Y_k^{\Re} \\ \xi_k^{\Im}(X) - Y_k^{\Im} \end{pmatrix}^T \Sigma_k^{-1} \begin{pmatrix} \xi_k^{\Re}(X) - Y_k^{\Re} \\ \xi_k^{\Im}(X) - Y_k^{\Im} \end{pmatrix} \right] \tag{4}$$

To account for the independence of each constraint, we set $\alpha_k = \frac{3\gamma}{T_k}$ for T_k telescopes observing at the time corresponding to the k-th bispectrum value.

We impose a Gaussian mixture model (GMM) patch prior to regularize our solution of $\hat{I}_\lambda(\mathbf{X})$ [4]. We maximize the EPLL by maximizing the probability of each of the N overlapping pulse patches in $\hat{I}_\lambda(\mathbf{X})$ using "Half Quadratic Splitting" in a mutliscale framework. To reduce computation time, we perform a 2nd order Taylor expansion around our current solution to obtain an approximation of the local minimum in closed-form.

Dataset and Results We introduce a novel dataset and website (vlbiimaging.csail.mit.edu) for evaluating the performance of VLBI image reconstruction algorithms. In Figure 1 we compare reconstruction results from our algorithm with the three state-of-the-art algorithms using synthetic data corresponding to parameters of the EHT telescope array when pointed towards the black hole in M87 [1, 2, 3].

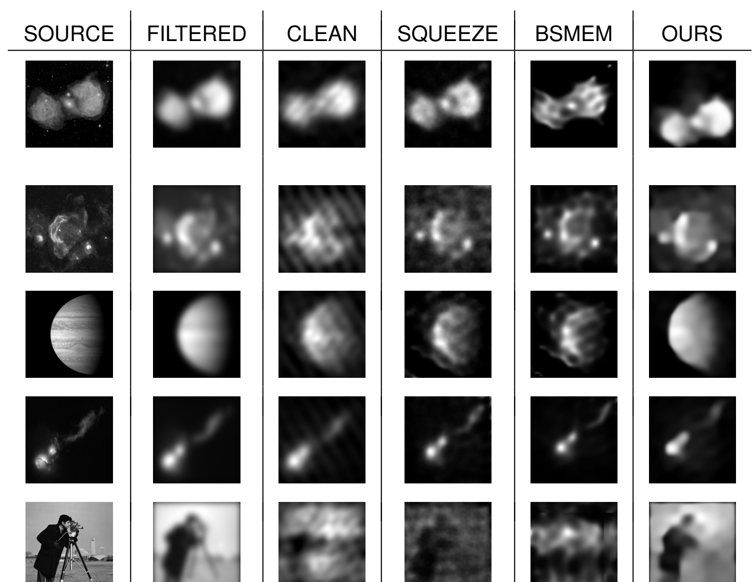

Figure 1: **Method Comparison:** Normalized reconstruction of a variety of celestial and natural source images with a total flux density (sum of pixel intensities) of 2 Janskys and a 183.82 μ-arcsecond FOV. Since absolute position is lost when using the bispectrum, shifts in the reconstructed source location are expected. The original source images ('SOURCE') are used to synthetically generate realistic VLBI measurements. These measurements impose an intrinsic maximum resolution, illustrated by the 'FILTERED' images. We compare our algorithm, 'OURS' to three state-of-the-art methods: 'CLEAN', 'SQUEEZE', and 'BSMEM'.

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