#### Brain MRI Segmentation Using an Expectation-Maximization Algorithm

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### **MRI of the brain**

- Magnetic Resonance (MR) imaging of the brain:
  - Three-dimensional (3-D)
  - High soft tissue contrast
  - High spatial resolution
  - Possibly multi-spectral











# Segmentation of brain MRI

- Radiotherapy planning
- Surgical planning
- Image-guided interventions
- Visualizations
- Studying brain diseases
- Clinical drug trials



### How to segment brain MRI?

#### Manual delineation by a human expert

- difficult to accurately delineate complex 3-D structures
- extremely time-consuming
- considerable inter- and intra-rater variability
- multi-spectral input is hard to interpret
  - Routine analysis is impractical



Need for automated procedures

#### **Overview**

- The mixture model and the EM algorithm
- A probabilistic brain atlas
- Modeling MR bias fields
- Multiple Sclerosis lesion segmentation
- Partial volume segmentation
- Discussion and future directions

#### **Overview**

#### The mixture model and the EM algorithm

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#### The Gaussian mixture model





- Intensity distributions of white matter, gray matter and CSF are modeled as Gaussian distributions
  - mean = average intensity
  - variance = variation around the average intensity

#### The Gaussian mixture model





Once the mean and variance of each tissue type is known, voxels can be classified based on their intensity

#### How to obtain model parameters?

- Interactively select representative voxels for each tissue type
  - Train model once and apply it to hundreds of scans
  - Needs to be re-done for every new pulse-sequence
  - Not fully reproducible
  - In clinical trials: inter-scan variations in the intensity distributions of the tissue types
    - Hardware fluctuations of MR scanners over time
    - Multi-center trials may involve different scanners

Can we estimate the model parameters automatically from each individual scan?

## More formal image model



- Fotal model parameters:  $\Phi = \{\Phi_Y, \Phi_L\}$
- ▷ Overall model:  $f(\mathbf{Y} \mid \Phi) = \sum_{\mathbf{x}} f(\mathbf{Y} \mid \mathbf{L}, \Phi_{\mathbf{Y}}) f(\mathbf{L} \mid \Phi_{\mathbf{L}})$

#### **Spatial model**



- Assume that the label of each voxel is drawn independently from the labels of other voxels, with probability \u03c0k k for tissue type k
- > The spatial model parameters are then:  $\Phi_L = \{\pi_k\}$

## **Intensity model**

spatial model

 $f(L \mid \Phi_L)$ 







 $\boldsymbol{Y} = \{\boldsymbol{y}_{i}\}$ 

- Assume that the intensity of a voxel is conditionally independent from the intensity of other voxels, given its tissue label: f(Y | L, Φ<sub>Y</sub>) = Πf(y<sub>j</sub> | l<sub>j</sub>, Φ<sub>Y</sub>)
- > Assume that the intensity distribution of tissue type k is normally distributed with mean  $\mu_k$  and covariance  $\Sigma_k$

The intensity model parameters are then:  $\Phi_Y = \{\mu_k, \Sigma_k\}$ Unifying Statistical Classification and Geometrical Models – MICCAI 2003

#### **Parameter estimation**

- Given an image, estimate the so-called Maximum-Likelihood parameters
  - = parameters that maximize  $\log f(\mathbf{Y} \mid \Phi)$
  - = parameters that best explain the data
- Cannot be solved with closed-form expressions
- Expectation-Maximization (EM) algorithm [Dempster et al., 1977] provides a very intuitive iterative parameter estimation scheme

## **Expectation-Maximization algorithm**



spatial model

"Missing data"

Observed data

- If the tissue labels ("missing data") were known, parameter estimation would be straightforward
- EM algorithm iteratively fills in the missing data and updates the parameters accordingly

### **Expectation-Maximization algorithm**

#### Iterative optimization algorithm

Expectation step: find the function

Likelihood with the missing tissue labels filled in

$$Q(\Phi \mid \Phi^{(m)}) = E_{\boldsymbol{L}}[\log f(\boldsymbol{Y}, \boldsymbol{L} \mid \Phi) \mid \boldsymbol{Y}, \Phi^{(m)}]$$

Expectation over the missing tissue labels based on the current parameter estimation and the observed data

Maximization step: find

$$\Phi^{(m+1)} = \arg\max_{\Phi} Q(\Phi \mid \Phi^{(m)})$$

#### Expectation step



Statistical classification of the image voxels based on the current parameter estimation



- $f(l_j \mid \boldsymbol{Y}, \Phi^{(m)}) = \frac{f(\boldsymbol{y}_j \mid l_j, \Phi_Y^{(m)}) \cdot \pi_{l_j}^{(m)}}{\sum_k f(\boldsymbol{y}_j \mid l_j = k, \Phi_Y^{(m)}) \cdot \pi_k^{(m)}}$ 
  - Bayes' rule
  - "soft" classification

#### **Maximization step**



#### **EM algorithm summarized**



#### Example





#### **Example**



- Structur - Structur - Stry motter - Dtal interviewed

But how to extract the intra-cranial volume?
But how to initialize automatically ?



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#### **Expected location of tissue types**



gray matter white matterCSFAtlasprobabilityprobabilityprobabilitytemplate

- Average of many binary white matter, gray matter and CSF segmentations after affine normalization
- Expected location of major tissue types in a healthy young population in a standardized coordinate frame
- Source: Montréal Neurological Institute

### **Atlas registration**

- Atlas needs to be brought into spatial correspondence with the image under study before it can be used
- Affine transformations (translation, rotation, scale and skew)
- This can be done fully automatically by maximizing the so-called Mutual Information between the atlas template and the study image



## **Atlas registration**

- Mutual Information measures the statistical dependence between two images [Maes et al., 1997] [Wells et al., 1996a]
- Is assumed maximal when the images are correctly aligned
- Makes very few assumptions about the intensities in the images to be co-registered



Fully-automated registration of the atlas template with the images under study, regardless of the pulse-sequence used.

#### **Atlas registration**



#### Improved spatial model



- The prior probability for tissue type k  $\pi_k$  is provided by the statistical brain atlas
- Depends now on the location in the brain!
- No unknown spatial model parameters  $\Phi_L$  to be estimated

# **Resulting EM algorithm**

#### Expectation step:

$$f(l_j \mid m{Y}, \Phi^{(m)}) = rac{f(m{y}_j \mid l_j, \Phi^{(m)}_Y)(\pi^{(m)}_{l_j})}{\sum_k f(m{y}_j \mid l_j = k, \Phi^{(m)}_Y)(\pi^{(m)}_k)}$$

Classification takes prior knowledge into account

Classification is moderated by the statistical brain atlas [Ashburner and Friston, 1997] This effectively introduces geometrical constraints into the statistical classification

#### Makes the algorithm more robust

Maximization step: remains the same

#### **Fully-automated segmentation**



Atlas initializes EM algorithm

Atlas provides a rough brain mask => no need for brain stripping in a preprocessing step

#### Fully automated, pulse sequence adaptive brain MRI segmentation



update mixture model

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# **MR field inhomogeneity**

- MRI-specific imaging artifact
  - Equipment limitations
  - Patient-induced electrodynamic interactions
- Results in non-uniform tissue intensities
- Also called "bias field"





#### **MR field inhomogeneity**

#### Causes segmentation errors in the automated EM segmentation procedure



#### Improved intensity model



#### Include an explicit model for the bias field in the intensity model [Van Leemput et al., 1999], based on [Wells et al., 1996b]

# Improved intensity model

- Bias field is usually assumed to be multiplicative
- After logarithmic transformation => bias field becomes additive



Old model





Bias field model

Improved model

- Fourth-order polynomial
- Parameters need to be estimated as well

#### **Resulting EM algorithm**



#### **E-step: classification**



#### **M**-step part 1: distribution estimation


### **M-step part 2: bias field estimation**



### **M-step part 2: bias field estimation**



### **Example 1**

### MRI data

White matter without bias field model

White matter with bias field model

Estimated bias field



### Example 2: 2-D multi-slice sequence



MRI data



### Estimated bias field



**Bias-corrected MRI data** 

## Implemented in "EMS" software

Freely available from the website of the Medical Image Computing group, K.U.Leuven, Belgium: bilbo.esat.kuleuven.ac.be



### About ERS

u Versien belgen anverden Malabur een nie gebruike de keren een dit van en dit van en die gebruike het die kere Naarde Bestere Offinisje is die een die bespraak dit Vundige (Vundige Vundige Bester) die bestere Bestere Progra Wit gebruike van Bestere een nie Bestere te bespraak dit Vundige (Vundige Bestere Program Bestere Program Beste

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# Multiple Sclerosis (MS)

- Common disease of young adults
- Primarily affects white matter
- Cause?
  - environmental factors
  - genetic susceptibility
- Relapsing-remitting
  - Relapse, stabilization, (partial) recovery
- Primary progressive

### **MRI in Multiple Sclerosis**

### MRI depicts abnormalities in 95% of patients



Diagnosis accompanied by confirmatory MRI

# **MRI in Multiple Sclerosis**

- Assessing progression
- Monitoring effect of a new drug therapy
  - MS lesion segmentation from MRI
  - More sensitive and more objective marker than neurological disability scales
  - Primary outcome of preliminary clinical trials

Manual analysis????

- many hundreds of scans
- inter- and intra-rater variability

Need for automated tools

### Including MS lesion model????



- Widely varying appearance in MRI difficult to model
- Not every individual scan contains lesions

difficult to estimate model parameters

# Lesions as model outliers



Detect lesions as voxels that are not well explained by the model for normal brain MRI



Requires knowledge of the model parameters

But estimation of those model parameters is difficult in the presence of lesions!

### **Parameter estimation?**

- Consider case of one tissue type
- Simulated data with known bias field
- Estimate bias field using Maximum-Likelihood (ML) parameter estimation:

maximize  $\sum \log f(\boldsymbol{y}_j \mid \Phi)$ 





synthetic data

estimated bias field



corrected data

### **Parameter estimation?**

- Consider case of one tissue type
- Simulated data with known bias field
- Estimate bias field using Maximum-Likelihood (ML) parameter estimation:

maximize  $\sum \log f(\boldsymbol{y}_j \mid \Phi)$ 



### **Robust statistics**

- Model outliers should have a reduced weight on the parameter estimation
- $\blacktriangleright \text{M-estimator:} \quad \sum_{j} \log f(\boldsymbol{y}_{j} \mid \Phi) \implies \sum_{j} \log \left( f(\boldsymbol{y}_{j} \mid \Phi) + \lambda \right)$
- Iterative parameter estimation ("W-estimator")

 $\underbrace{ \begin{array}{c} \underline{step 1:} \text{ calculate "typicality" weights} \\ t_j^{(m)} = \frac{f(\boldsymbol{y}_j \mid \Phi^{(m-1)})}{f(\boldsymbol{y}_j \mid \Phi^{(m-1)}) + \lambda} \\ \underline{step 2:} \text{ maximize} \\ \sum_j t_j^{(m)} \log f(\boldsymbol{y}_j \mid \Phi) \end{array} }$ 

### **Robust statistics**



synthetic data





corrected data



1 - "typicality"
= outlier belief



histogram corrected data

"typicality" = weight in parameter estimation

# **Applied to MS lesion segmentation**

- Extension to multiple tissues
- Outlier belief depends on covariances of classes

$$t_j^{(m)} = \frac{f(\boldsymbol{y}_j \mid \Phi^{(m-1)})}{f(\boldsymbol{y}_j \mid \Phi^{(m-1)}) + \lambda}$$

•Statistical meaning? •How to choose  $\lambda$  ?

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# **Applied to MS lesion segmentation**

- Heuristic adaptation that takes the size of the covariance matrices into account
- Re-parameterization to more easily interpretable  $\kappa$



# MS lesions are not the only model outliers...

Partial volume voxels are also model outliers

- On the edge between two or more tissue types
- Mix several tissue types
- Violate model assumptions (cf. later)



## Separating MS lesions from partial volume voxels

- Exploit prior knowledge about MS lesions
  - MS lesions are hyperintense on PD and T2



Constraints on intensity

95% of MS lesions are white matter lesions

**Constraints on location** 



# Markov random field (MRF) model



Tissue type in a voxel is statistically dependent on the tissue type of neighboring voxels





Typical MRF samples, for different MRF parameter sets

# Can be used to confine lesions to locations close to white matter

# Fully-automated MS lesion segmentation



Model parameters are only estimated from normal tissues

- Model adapts itself to each individual scan
- No need for pre- or post-processing
- Only one parameter to be specified:

### significance level ${\cal K}$

[Van Leemput et al., 2001]

# Validation

### Data from clinical trial

- 50 MS patients scanned every month during 1 year
- T1-, T2- and PD-weighted MR images
- European Commission funded research project BIOMORPH
- Automated segmentations compared to expert MS lesion delineations

### Validation





# Expert lesion delineation

# Automated segmentation



 $\tau \cdot \sigma$ 

्र ज

remal

<u>.</u>

abnormal

# Total lesion load (TLL)

- Total lesion volume per scan
- For 10 patients, 2 consecutive time points were analyzed by a human expert
- Expert TLL estimation compared to automated TLL estimation
- Evaluated for different significance levels  $\kappa$

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abhormal

# **Total lesion load (TLL)**



Average automated TLL decreases from 150% to 25% of expert estimates as  $\kappa$  increases

# **Total lesion load (TLL)**



But correlation coefficient is always very high (0.96-0.98)

Exact choice of  $\mathcal{K}$  is unimportant in clinical trials assessing change in lesion volume

# **Total lesion load (TLL)**



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# Partial volume segmentation

- Assumed so far that each voxel belongs to one single tissue type
- In reality, many voxels in brain MR images are a mixture of several tissue types at the same time
  - Complex shape of the tissue interfaces in the brain
  - Limited spatial resolution of MRI
- "Partial volume (PV) effect"

# **Partial volume segmentation**

- Consistently misplacing the tissue borders in a 1 mm isotropic brain MRI with a single pixel in each slice results in large volume errors [Niessen et al., 1999]:
  - ~ 30% for white matter
  - ~ 40% for gray matter
  - ~ 60% for CSF
- Partial volume voxels make lesion segmentation by outlier detection more difficult

→ Need to explicitly model the partial volume effect Unifying Statistical Classification and Geometrical Models – MICCAI 2003

### Improved image model



### **Before downsampling...**



### ... after downsampling (3x)



### **Expectation-Maximization algorithm**



## **Expectation-Maximization algorithm**

Expectation step: find the function  $Q(\Phi \mid \Phi^{(m)}) = E_{\boldsymbol{L},\boldsymbol{Y}} \big[ \log f(\boldsymbol{L},\boldsymbol{Y} \mid \Phi) \mid \boldsymbol{\tilde{Y}}, \Phi^{(m)} \big]$ Involves a partial volume image classification: Not only probability for pure tissues But also probability for e.g. 22% of tissue 1 and 78% of tissue 2 Maximization step: find  $\Phi^{(m+1)} = \arg\max_{\Phi} Q(\Phi \mid \Phi^{(m)})$ 



Unifying framework for PV segmentation literature [Van Leemput et al., 2003]
Koen Van Leemput, Helsinki University Central Hospital

## **Spatial model 1**

spatial

model



intensity model



- Mixing combination in a voxel is independent of the mixing combinations in other voxels
- All non-pure mixing combinations are equally probable downsample
- Often used model, first proposed by [Santago and Gage, 1993 & 1995]





T1, 1x1x1 mm<sup>3</sup>



Parameter initialization





White matter fraction



White-gray matter PV voxels



Koen Van Leemput, Helsinki University Central Hospital

## **Spatial model 2**





intensity model



- Markov random field model
- Clustered regions of the same tissue type before downsampling
- Homogeneous regions of pure tissues bordered by partial volume voxels after downsampling





T1, 1x1x1 mm<sup>3</sup>



Parameter initialization





White matter fraction



White-gray matter PV voxels





T2, 1,18 x 1,18 x 3 mm<sup>3</sup>

EM parameter estimation?

In order to solve this, better spatial models are needed to describe the spatial distribution of tissues in the brain

- MRF model tends to minimize the boundary length between tissues
- This discourages classifications from accurately following the complex shape of the tissue interfaces
- MRF over-smooths the classifications in cases where the intensity information doesn't prevent it

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## **Expectation-Maximization algorithm**

- Image classification performed simultaneously with model parameter estimation:
  - Intuitive algorithm that interleaves classification with model parameter estimation
  - Allows to integrate prior geometrical knowledge into the classification in a natural fashion
- After automated initialization with a statistical atlas, the classifier re-trains itself on each individual scan
  - Segments images of arbitrary pulse-sequences without user intervention

#### Intensity model

- Each tissue has a typical intensity and tissuespecific intensity variations
- MR bias fields can be explicitly modeled
- Lesions can be detected as model outliers
  - This allows to explicitly exclude lesions from model parameter estimations (e.g. bias field correction)
- The partial volume effect can be explicitly modeled

The intensity model is already quite complete

- Affine atlas registration provides only a rough brain mask => misclassifications of non-brain tissues as brain tissue
- Affine atlas registration does not allow to segment brain MR images with large shape differences (e.g. dramatically enlarged ventricles)
- White matter/gray matter/CSF atlas does not allow further parcellation of the brain
- More sophisticated models are needed for robust partial volume segmentation => atlas-based?

Deformable registration [Maes et al., 1999] [Marroquin et al., 2002][Pohl et al., 2002][D'Agostino et al., 2003]

## Future research directions?

Deformable atlas registrations are performed by minimizing a registration metric between an MRI template associated with the atlas and the image to be segmented



gray matter white matter CSF **MRI** probability probability probability template

- Subject image
  But presence of bias fields or lesions may hinder registration
- Many deformable registration algorithms require similar intensities in the two images
- Mutual Information based deformable registration is still difficult Unifying Statistical Classification and Geometrical Models – MICCAI 2003

## **Future research directions?**

- Deformable registration is performed to help the segmentation, but the segmentation could in turn help the registration
- Deformation field as model parameters in the image model, to be estimated by the EM-algorithm?
- Simultaneous registration and segmentation would eliminate the need for an atlas template





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intensity

model

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