Week 6 (21 March, 2005): A Bit of Background

Bed and Breakfast, Voorjaar 2005
Nijmeegs Instituut voor Informatica en Informatiekunde

April 4, 2005

Examinator: dr. Hanno Wupper
Docenten: drs. Ling Cheung, drs. Martijn Hendriks, dr. Stijn Hoppenbrouwers

In class we will learn about *natural deduction*, which is one of the many proof systems in symbolic logic. To put things in context, we found some nice, simple articles from the *Platonic Realms Interactive Mathematics Encyclopedia*, available at `http://www.mathacademy.com/pr/prime/`.

On Logic

In the broadest sense, the term "logic" refers to the reasoning processes or forms of argument people use to reach valid conclusions from one or more premises (assumptions). More specifically, any particular logic is a system of inference, by which one reasons from premises to conclusions, or by which one validates one's conclusions. An incorrect inference, i.e., one which violates the rules of logic, is called a fallacy, and conclusions reached by means of a fallacy are called invalid. (By contrast, when the premises of an argument are false but the argument is formally valid, then the conclusion is called unsound.)

Ordinary logic falls into two broad types; deductive and inductive. Reasoning from general premises to a specific conclusion is deductive. For example, knowing that heavy clouds and a brisk breeze often signal an oncoming storm, one could conclude deductively that it might rain this afternoon. Reasoning from specific premises to a general conclusion, on the other hand, is inductive. For example, observing that every raven one sees is black, one might conclude inductively that all ravens are black. Deductive conclusions are necessarily true if the premises are true and the logic used formally valid (free of fallacy). Inductive conclusions are never certain, but are only more or less reliable. In mathematics only deductive reasoning is used (but see the entry for mathematical induction).

Mathematicians use symbolic logic, that is, logic that can be reduced to a purely syntactical system which disregards the semantic content of any of the symbols in use. The two primary forms of symbolic logic are the propositional calculus and the predicate calculus. The latter augments the former with existential and universal quantifiers (permitting sentences that begin with "there exists ... such that" and "for all ..."). When quantification is permitted only over elements of the universe of discourse, the logic is called "first-order." When quantification over classes of objects and/or over predicates is permitted, it is called second-order logic.

On Axiom

In formal mathematics, a formula or schema of formulas stipulated as true in the theory under discussion, i.e., assumed to be true at the outset, and so not requiring proof. Axioms are the counterpart in mathematics of suppositions, assumptions, or premises in ordinary syllogistic logic.