

Compositional Security for Task-PIOAs

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Outline

- 1 Introduction
- 2 Time Bounds in Task-PIOA
- 3 Polynomial Composition
- 4 Compositional Security

Analysis of Cryptographic Protocols

Three main targets:

- correctness
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- security games are easier to prove;
- simulation-based security is composable.

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ϕ : *real* protocol

ψ : *ideal* protocol

\approx : indistinguishable (perfectly, statistically, computationally)

Composability: One-Page Proof

Theorem. If $\phi \leq_E \psi$, then $\phi \parallel \eta \leq_E \psi \parallel \eta$.

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Hidden hurdles: associativity, compatibility, ...

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Most importantly, \approx must be preserved under substitutions.

Two Layers of Composability Claims

Hard: Composability in the underlying model of concurrent computation.

Easy: Composability in the security layer.

Stop Being Sloppy ...

A protocol ϕ is a family $\{\phi_1, \phi_2, \dots, \phi_k, \dots\}$, indexed by *security parameter* k .

Description Bounds

$\phi = \{\phi_1, \phi_2, \dots, \phi_k, \dots\}$ is said to have *polynomially bounded description* if there is a polynomial $p(k)$ such that, for all k ,

- every constituent (e.g., state, action, task) of ϕ_k can be
 - encoded with fewer than $p(k)$ bits and
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- all relevant (probabilistic) Turing machines can be encoded with fewer than $p(k)$ bits.

Caution: This is *not* polynomial-time in the traditional sense.

Bounded description $\not\Rightarrow$ bounded runtime.

(Distinctive feature of task-PIOA!)

Computational Implementation

$$\phi \leq_{\text{neg,pt}} \psi \Leftrightarrow \forall p, q_1 \exists q_2, \epsilon \forall k$$

$\forall p(k)$ -bounded environment Env
 $\forall q_1(k)$ -bounded task schedule ρ_1
 $\exists q_2(k)$ -bounded task schedule ρ_2
 $|\mathbf{P}_{\text{acc}}(\phi_k \| Env, \rho_1) - \mathbf{P}_{\text{acc}}(\psi_k \| Env, \rho_2)| \leq \epsilon(k)$

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$$\begin{aligned} \phi \leq_{\text{neg,pt}} \psi &\Leftrightarrow \forall p, q_1 \exists q_2, \epsilon \forall k \\ &\quad \forall p(k)\text{-bounded environment } Env \\ &\quad \forall q_1(k)\text{-bounded task schedule } \rho_1 \\ &\quad \exists q_2(k)\text{-bounded task schedule } \rho_2 \\ &\quad | \mathbf{P}_{\text{acc}}(\phi_k \| Env, \rho_1) - \mathbf{P}_{\text{acc}}(\psi_k \| Env, \rho_2) | \leq \epsilon(k) \end{aligned}$$

Theorem. If $\phi \leq_{\text{neg,pt}} \psi$, then $\phi \| \eta \leq_{\text{neg,pt}} \psi \| \eta$.

Proof. Set $Env' := \eta \| Env$ and use associativity.

Polynomial Composition

What if we compose multiple instances?

(E.g., a parent process that invokes dynamically multiple copies of the same protocol.)

i -th copy of ϕ : $\phi_i = \{(\phi_i)_1, \dots, (\phi_i)_k, \dots\}$

i -th copy of ψ : $\psi_i = \{(\psi_i)_1, \dots, (\psi_i)_k, \dots\}$

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Let b be a polynomial.

$(\hat{\phi})_k := (\phi_1)_k \parallel \dots \parallel (\phi_{b(k)})_k$

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“Theorem”. If $\phi_i \leq_{\text{neg,pt}} \psi_i$ for every i , then $\hat{\phi} \leq_{\text{neg,pt}} \hat{\psi}$.

Naive Solution

Repeated application of the binary composition theorem.

$$(\phi_1)_k \| ((\phi_2)_k \| \dots \| (\phi_{b(k)})_k \| Env)$$

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Naive Solution

Schedule length bounds:

$$\begin{array}{l} \forall q_1 \exists q_2 \\ \quad \forall q_2 \exists q_3 \\ \quad \quad \forall q_3 \exists q_4 \dots \end{array}$$

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Problem!

q_i 's may grow exponentially: $\forall i \ q_{i+1} = 2 \cdot q_i$

Schedule length bound for $\hat{\psi}$ is $\hat{q}(k) = 2^{b(k)} \cdot q_1(k)$.

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Worse yet: error ϵ depends on schedule length bound q_i , so a different ϵ_i at every step!

$\hat{\epsilon}(k) = \sum_{i=1}^{b(k)} \epsilon_i(k)$ still negligible?

Computational Implementation (Take 2)

$$\begin{aligned} \phi \leq_{\text{neg,pt}}^{\text{strong}} \psi &\Leftrightarrow \forall q_1 \exists q_2 \forall p, q \exists \epsilon \forall k \\ &\forall p(k)\text{-bounded environment } Env \\ &\forall \text{ task schedule } \rho_1 \text{ such that} \\ &\quad \text{proj}_{\phi}(\rho_1) \text{ is } q_1(k)\text{-bounded} \\ &\quad \text{proj}_{Env}(\rho_1) \text{ is } q(k)\text{-bounded} \\ &\exists \text{ task schedule } \rho_2 \text{ such that} \\ &\quad \text{proj}_{\psi}(\rho_2) \text{ is } q_2(k)\text{-bounded} \\ &\quad \text{proj}_{Env}(\rho_1) = \text{proj}_{Env}(\rho_2) \\ &|\mathbf{P}_{\text{acc}}(\phi_k \| Env, \rho_1) - \mathbf{P}_{\text{acc}}(\psi_k \| Env, \rho_2)| \leq \epsilon(k) \end{aligned}$$

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Theorem. If $\phi_i \leq_{\text{neg,pt}}^{\text{strong}} \psi_i$ for every i , then $\hat{\phi} \leq_{\text{neg,pt}}^{\text{strong}} \hat{\psi}$

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Proof. Fix k . Define hybrid automata: $H_k^0, \dots, H_k^i, \dots, H_k^{b(k)}$.

$$H_k^i := (\psi_1)_k \parallel \dots \parallel (\psi_i)_k \parallel (\phi_{i+1})_k \parallel \dots \parallel (\phi_{b(k)})_k$$

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$$\begin{aligned} & | \mathbf{P}_{\text{acc}}((\hat{\phi})_k \parallel \text{Env}, \rho_1) - \mathbf{P}_{\text{acc}}((\hat{\psi})_k \parallel \text{Env}, \rho_{b(k)+1}) | \\ & \leq | \mathbf{P}_{\text{acc}}(H_k^0 \parallel \text{Env}, \rho_1) - \mathbf{P}_{\text{acc}}(H_k^1 \parallel \text{Env}, \rho_2) | \\ & \quad + | \mathbf{P}_{\text{acc}}(H_k^1 \parallel \text{Env}, \rho_2) - \mathbf{P}_{\text{acc}}(H_k^2 \parallel \text{Env}, \rho_3) | \\ & \quad + \dots + | \mathbf{P}_{\text{acc}}(H_k^{b(k)-1} \parallel \text{Env}, \rho_{b(k)}) - \mathbf{P}_{\text{acc}}(H_k^{b(k)} \parallel \text{Env}, \rho_{b(k)+1}) | \\ & < b(k) \cdot \epsilon(k) \end{aligned}$$

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Remark: “ $\forall Env$ ” is encapsulated in $\leq_{\text{neg,pt}}^{\text{strong}}$.

Theorem. If $\phi_i \leq_E \psi_i$ **uniformly** for every i , then $\hat{\phi} \leq_{\text{neg,pt}}^{\text{strong}} \hat{\psi}$

Proof. Dummy adversaries and composition theorem for $\leq_{\text{neg,pt}}^{\text{strong}}$.

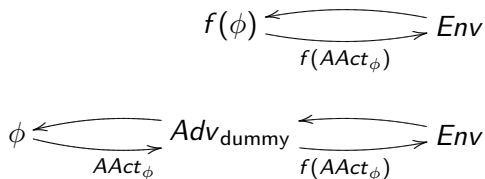
Dummy Adversaries

Dummy adversary: forwarder between protocol and environment.

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Formal property: $f(\phi) \leq_{\text{neg,pt}}^{\text{strong}} \phi \parallel \text{Adv}_{\text{dummy}}$, where f is a renaming function.



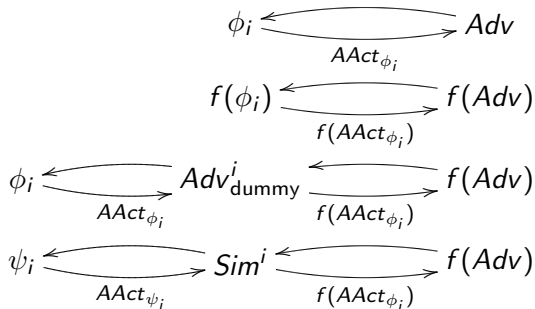
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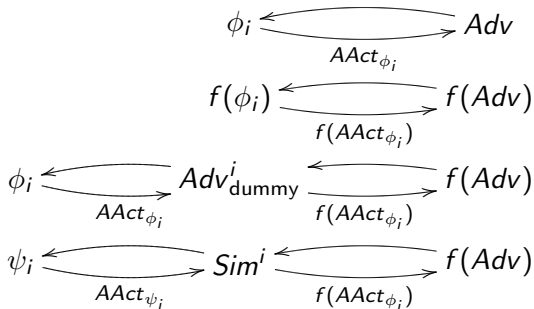
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Step 3. $Sim := (\parallel_i Sim^i) \parallel f(Adv)$.



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- More case studies: statistical ZK, ABE, etc.