Switched Probabilistic I/O Automata

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ICTAC 2004, Guiyang, China

Outline

Outline

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To NIII Colloquium Attendees:

Thank you all for coming to my talk!

For this talk . . .

- We need very little probability theory: *discrete distributions*. Examples:
  - fair coin: $\{\langle \text{Head}, \frac{1}{2}\rangle, \langle \text{Tail}, \frac{1}{2}\rangle\}$;
  - fair dice: $\{\langle i, \frac{1}{6}\rangle \mid 1 \leq i \leq 6\}$.

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- Underlying model: nondeterministic automata with asynchronous composition.
  (In our paper: input/output distinction, combination of synchronous and asynchronous compositions, etc.)

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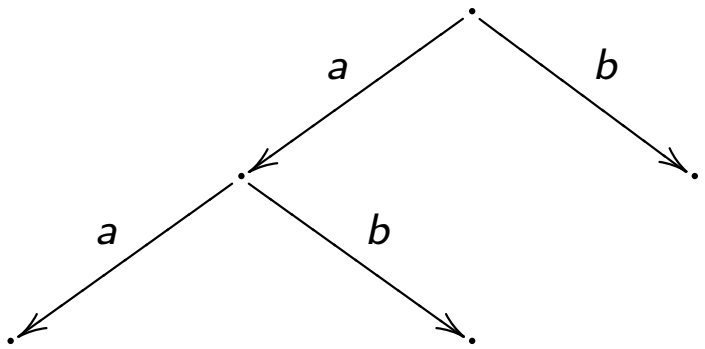
- Underlying model: nondeterministic automata with asynchronous composition.
  (In our paper: input/output distinction, combination of synchronous and asynchronous compositions, etc.)
- Total order semantics: if both actions a and b occur, one must precede the other.

Schedulers and trace distributions

- *History-dependent*, *randomized* schedulers transform nondeterministic choices into probabilistic choices.



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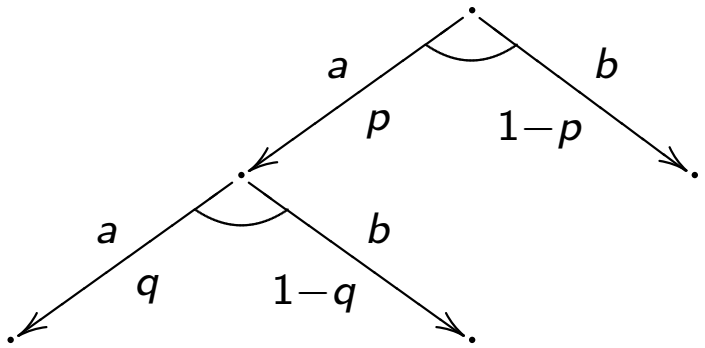

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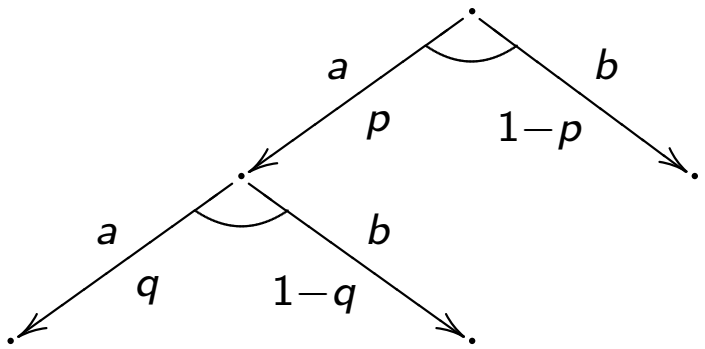
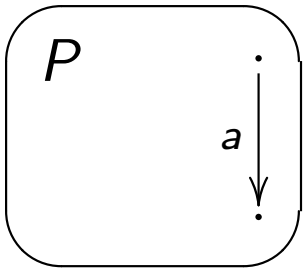
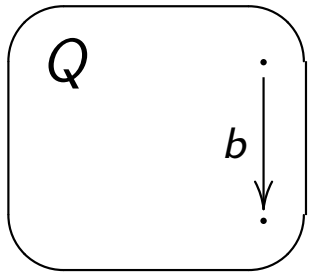


- Each scheduler induces a *trace distribution*: a discrete distributions on finite traces.

$$\{\langle aa,\ pq\rangle, \langle ab,\ p(1-q)\rangle, \langle b,\ 1-p\rangle\}$$

Nondeterministic parallel composition





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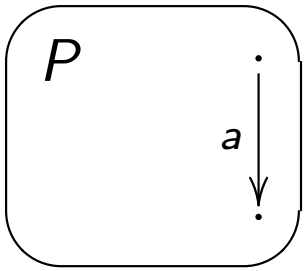
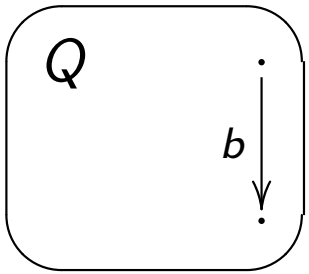
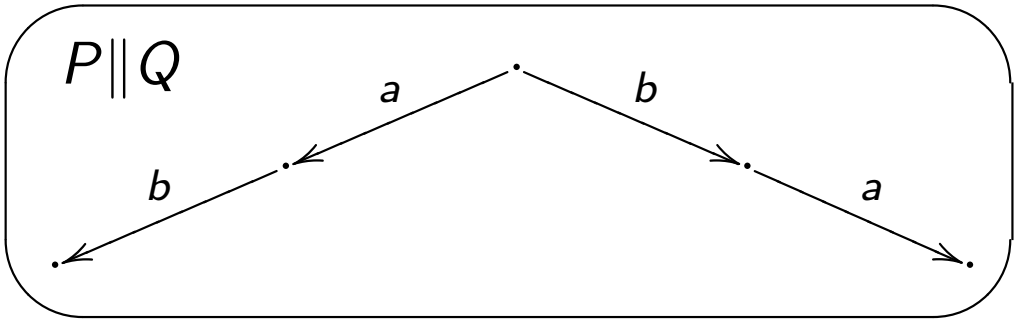
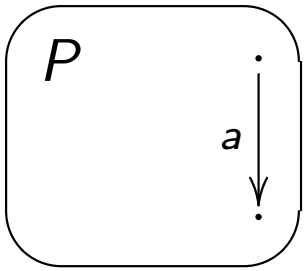


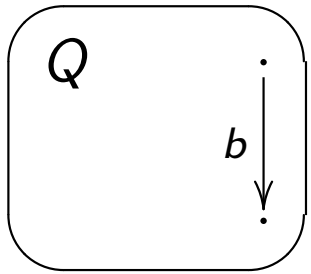


The *interleaving* axiom:



Probabilistic parallel composition





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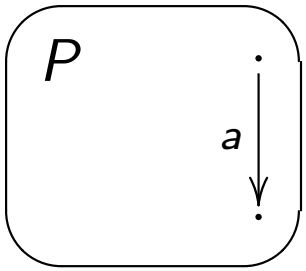
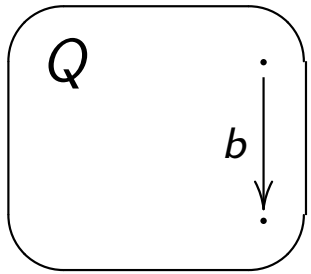
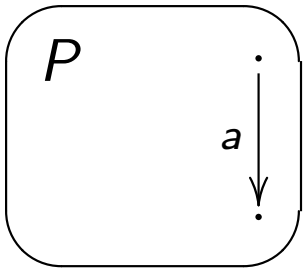
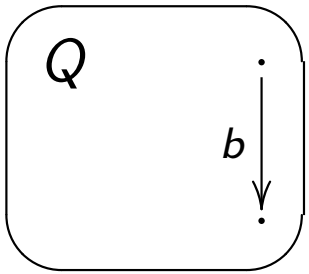




What is a *probabilistic* behavior of $P \| Q$?

Probabilistic parallel composition





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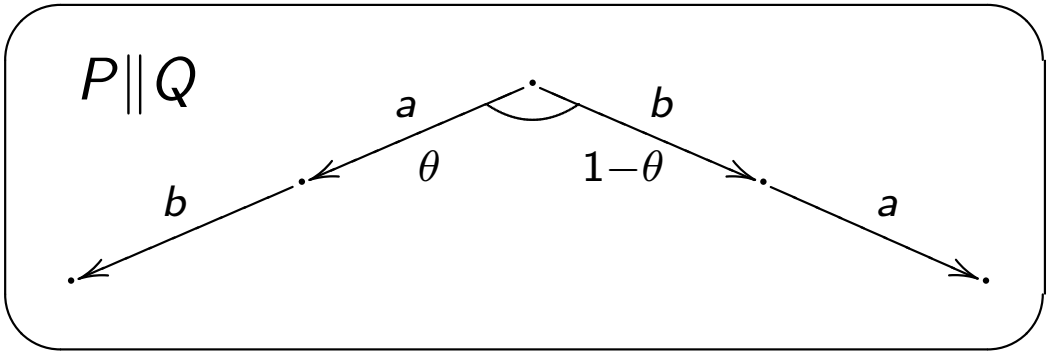
Quick answer: *bias factor* θ.



Probabilistic parallel composition

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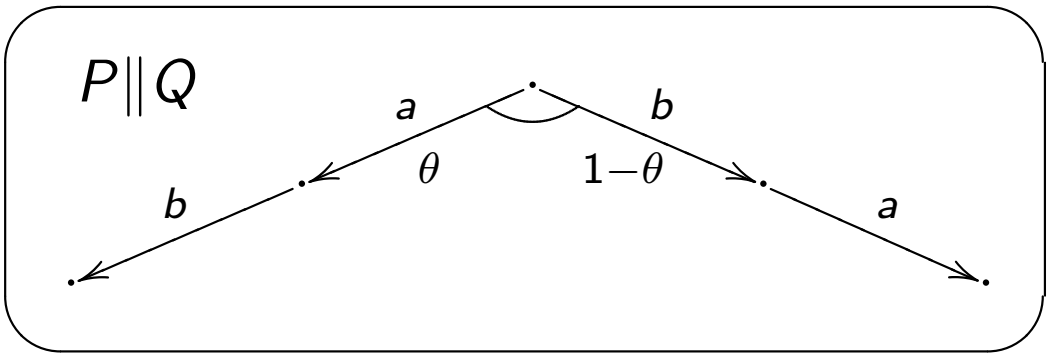
Quick answer: *bias factor* θ.

Imagine a coin-flipping daemon.



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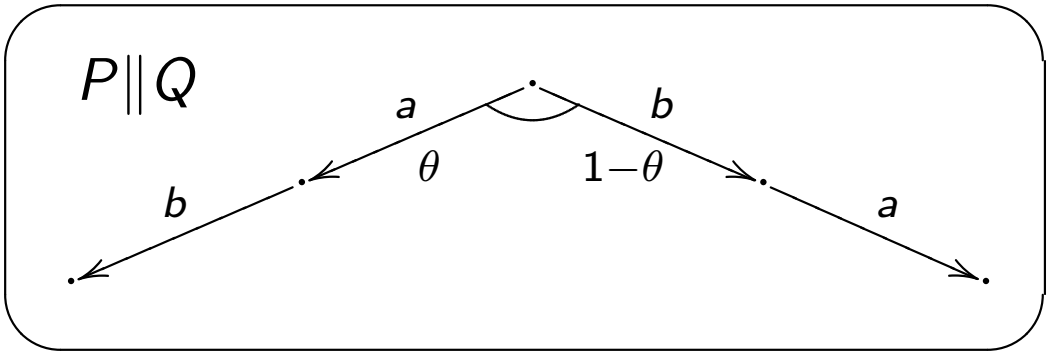

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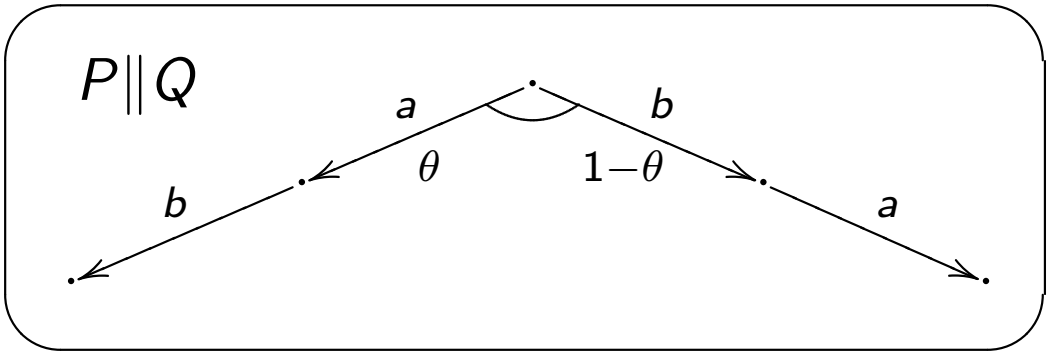

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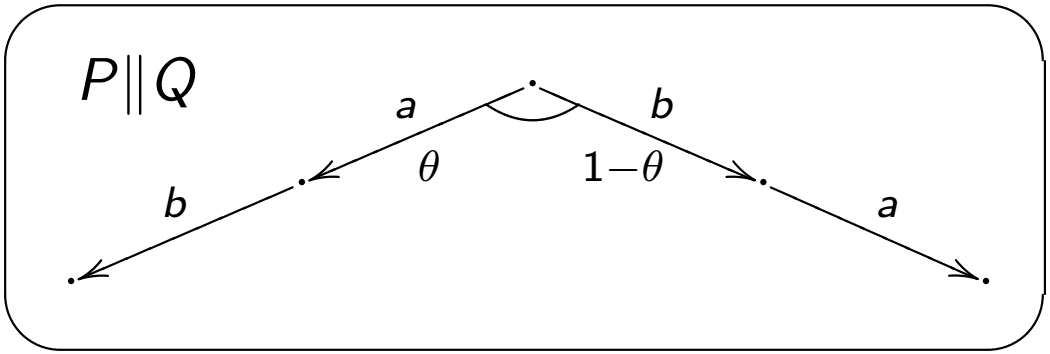

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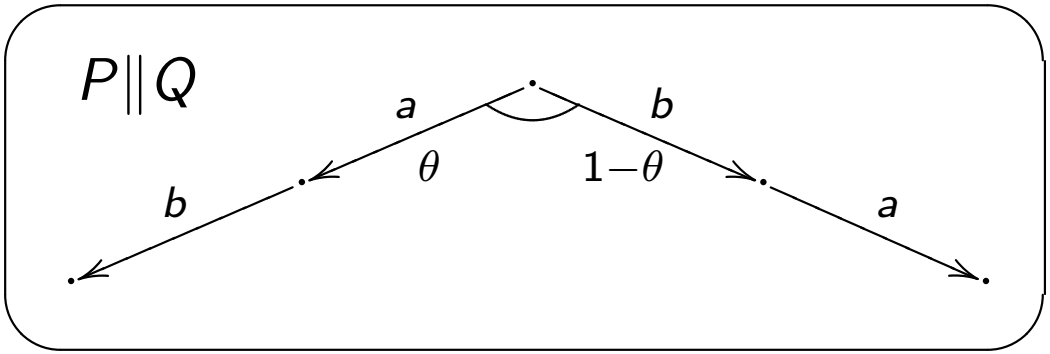

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Variable θ:

- a supply of coins with different biases;
- imaginary daemon chooses a coin based on his knowledge.

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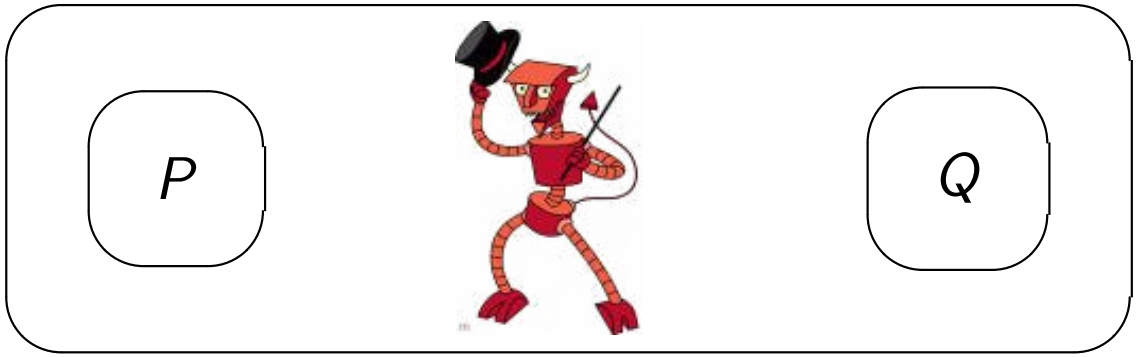
Scenario 1: *context-independent*



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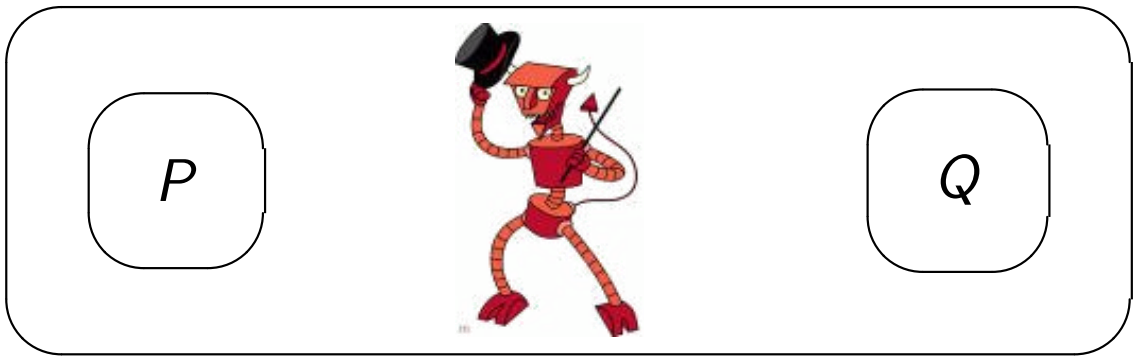
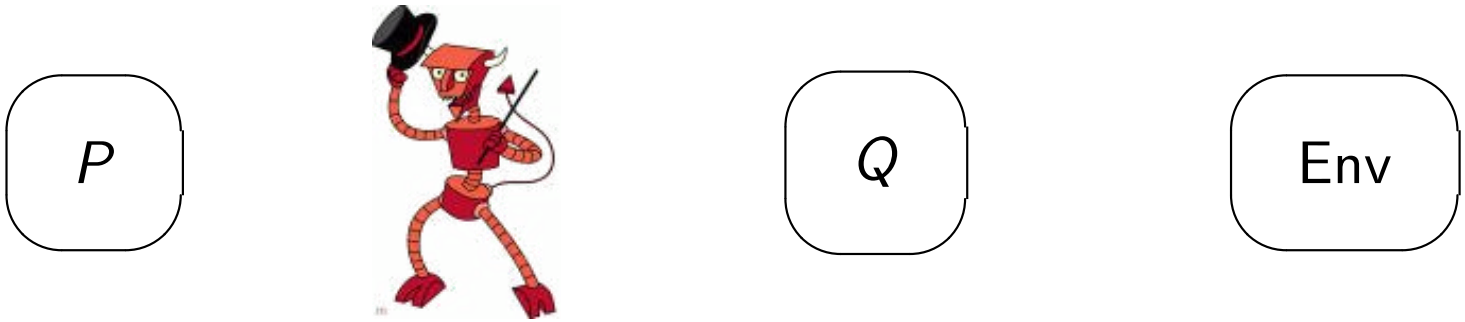


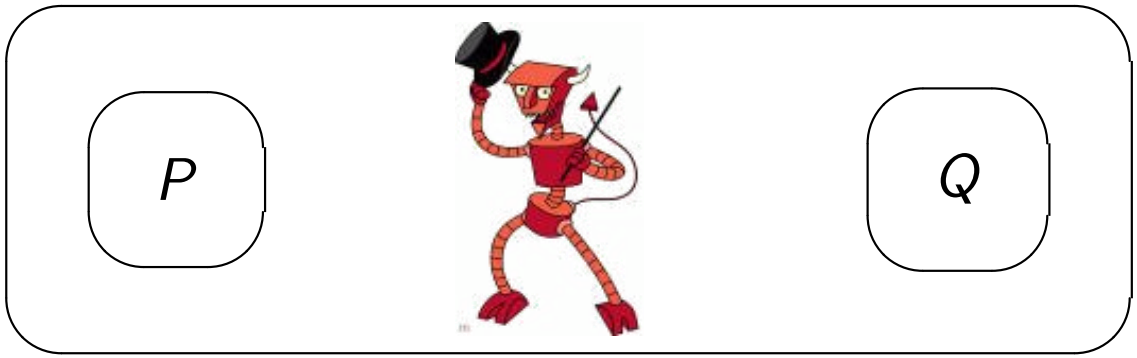
Scenario 2: *context-dependent*



Scenario 1: context-independent composition

How much does the daemon know?

Daemon, P and Q all inside a big black box.



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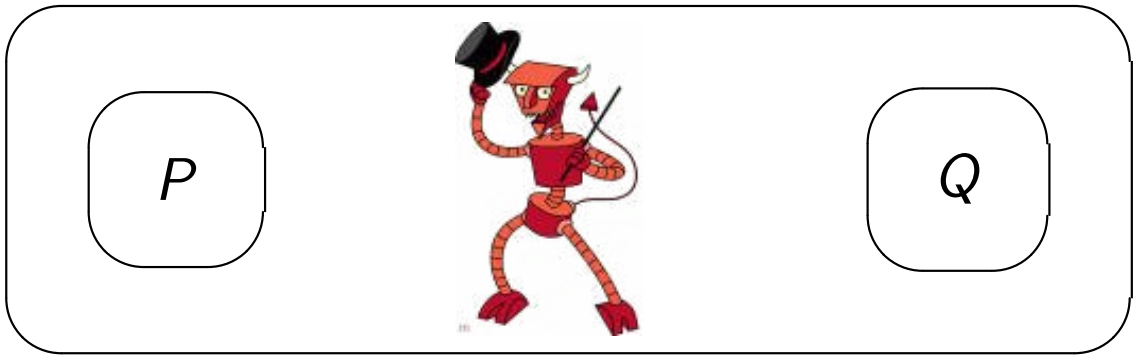

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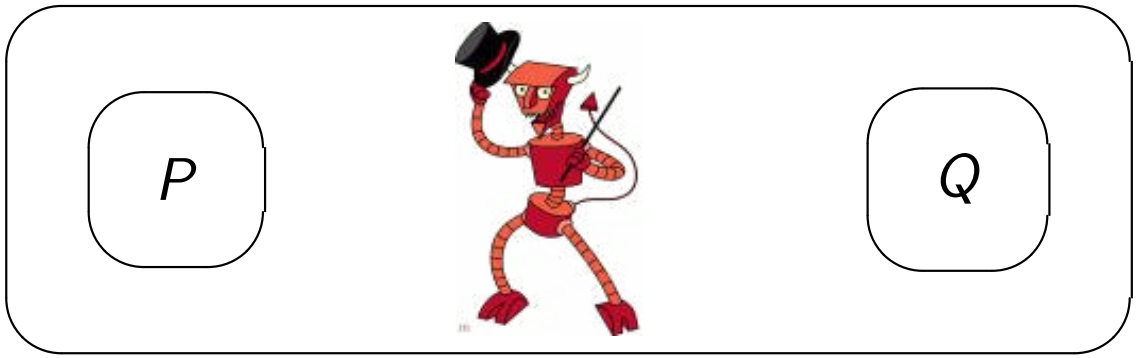

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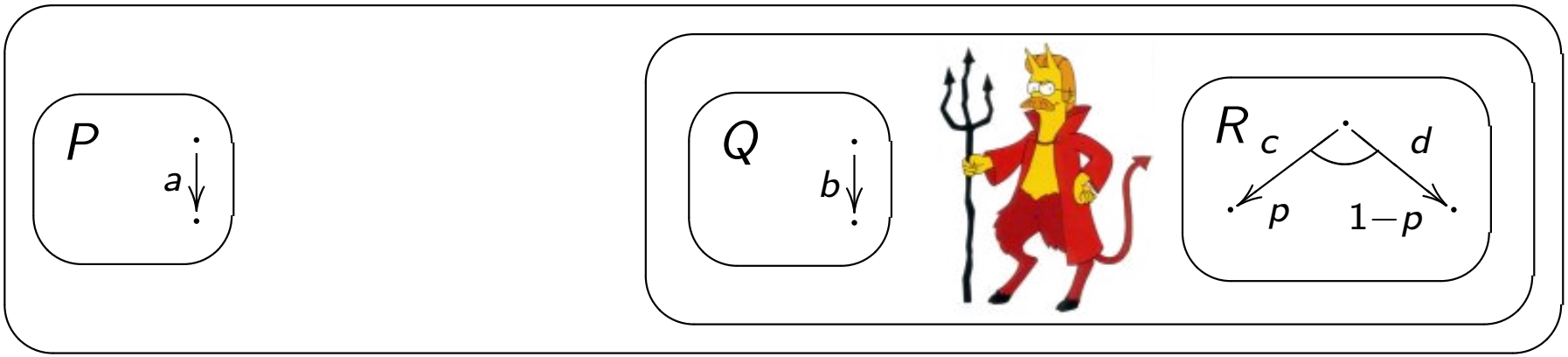
Problem: non-associativity.

Non-associativity: $P \| (Q \| R)$

Context-independent composition



Inner daemon: $\langle R, 1 \rangle$.

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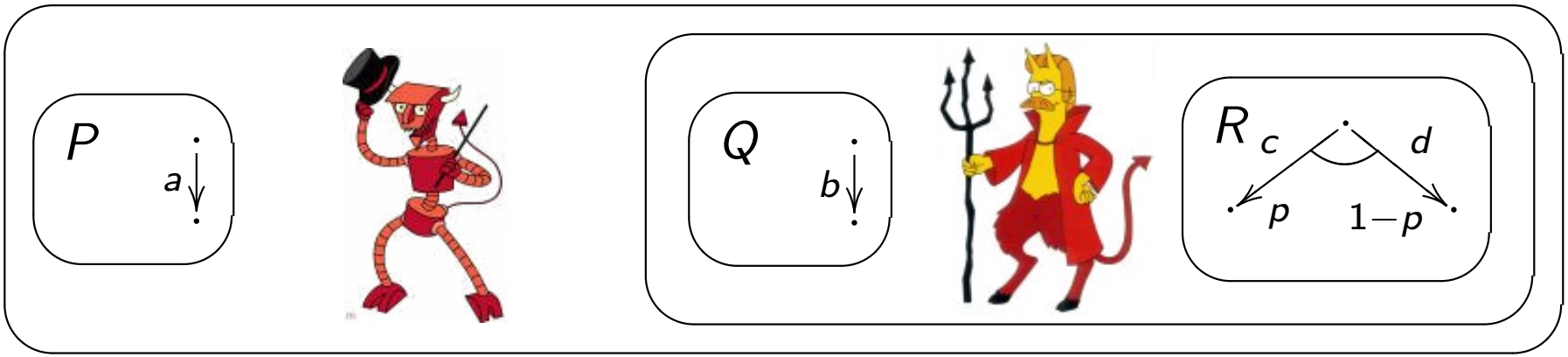

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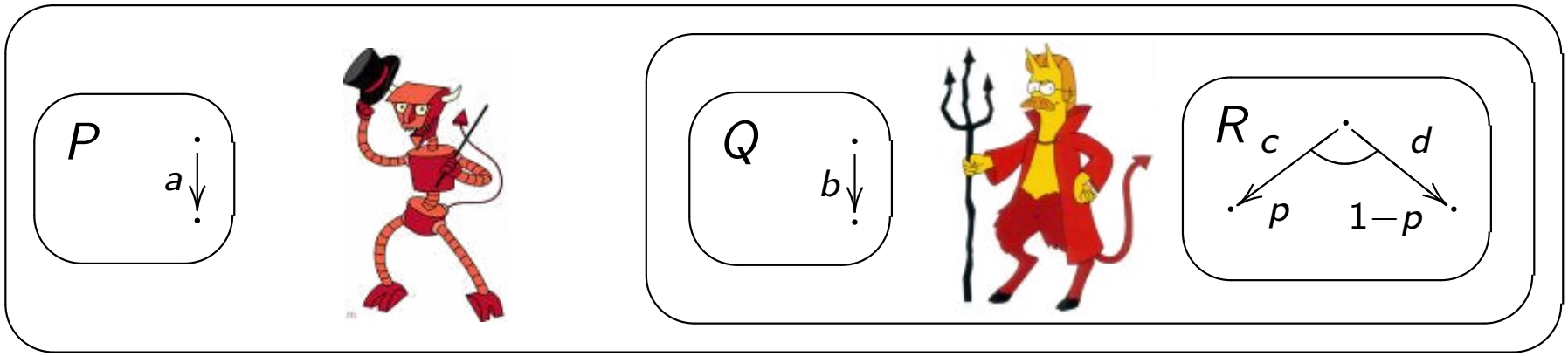

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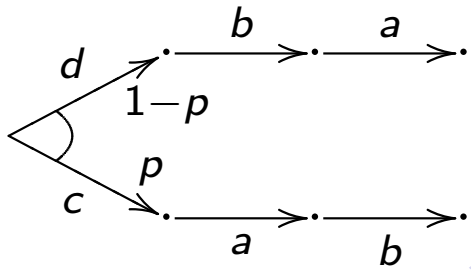
Outer daemon: $\langle Q \| R, 1 \rangle$; if c, then $\langle P, 1 \rangle$, else $\langle Q \| R, 1 \rangle$.

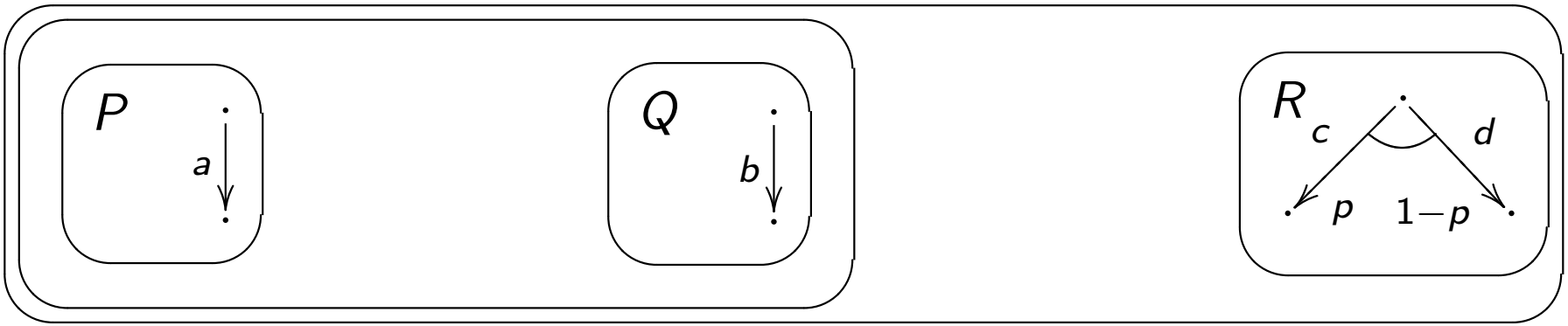
Result: $\{\langle cab, p \rangle, \langle dba, 1 - p \rangle\}$.



Non-associativity: $(P \| Q) \| R$

Context-independent composition

Claim: $\{\langle cab,\ p\rangle, \langle dba,\ 1-p\rangle\}$ not possible!



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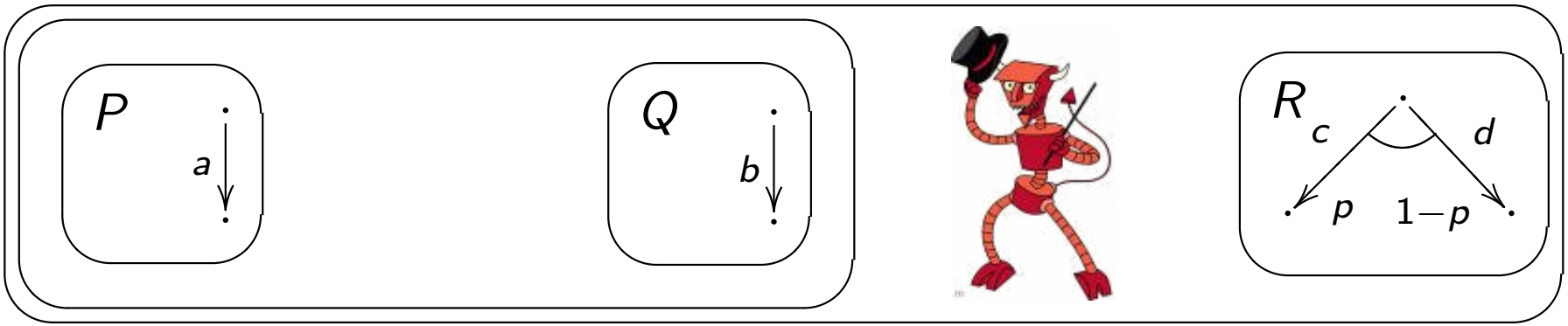

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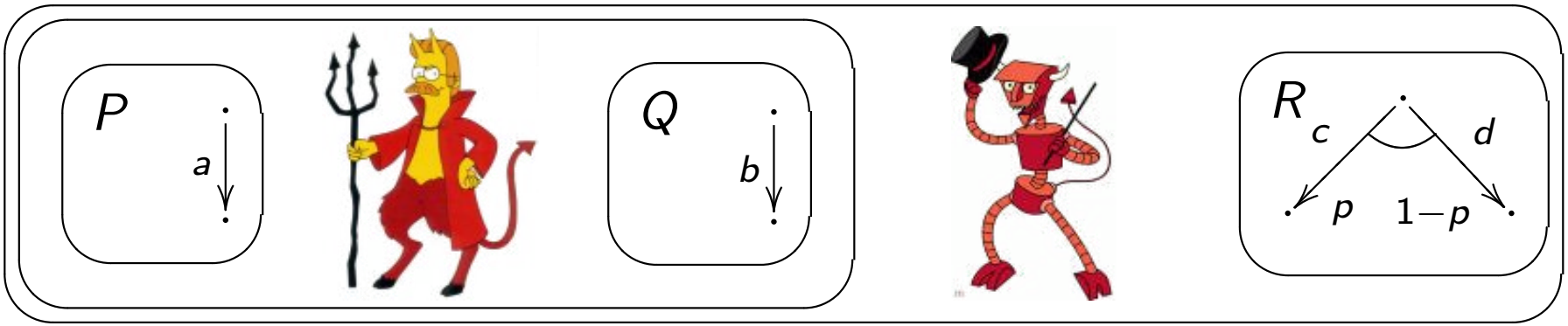

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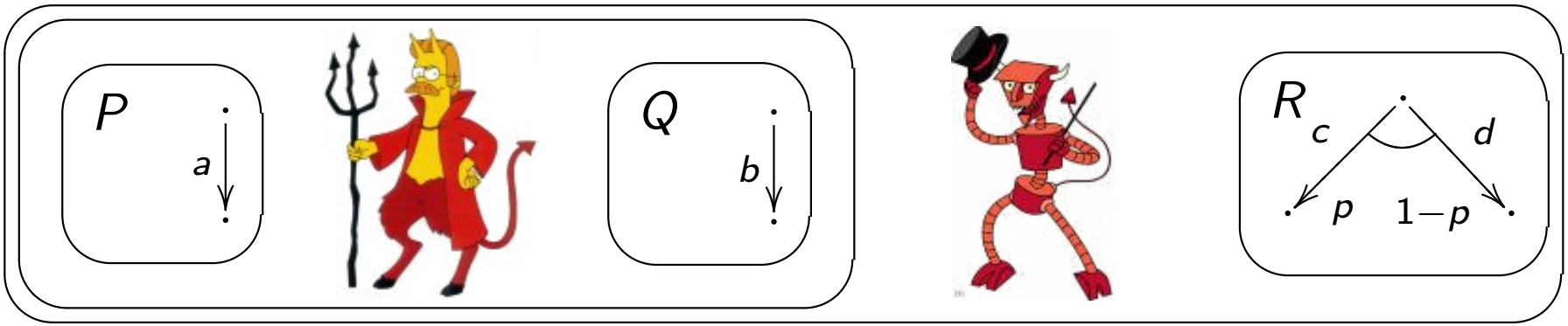

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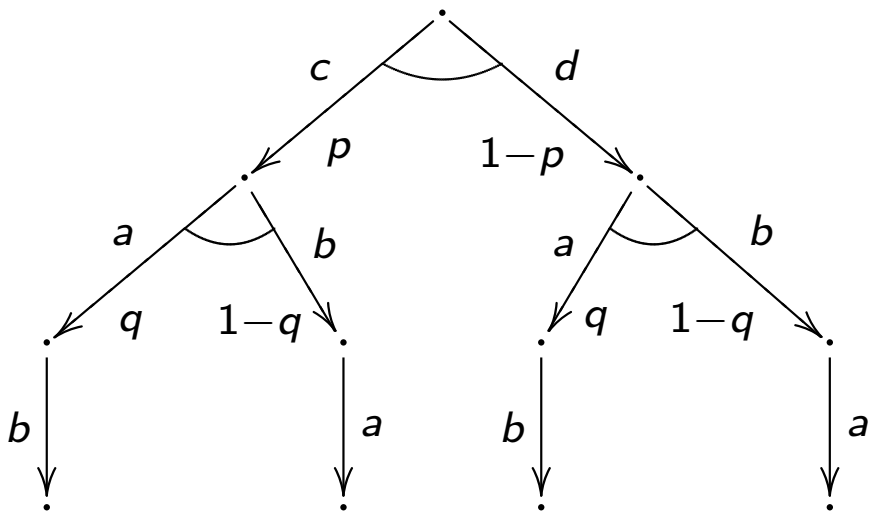

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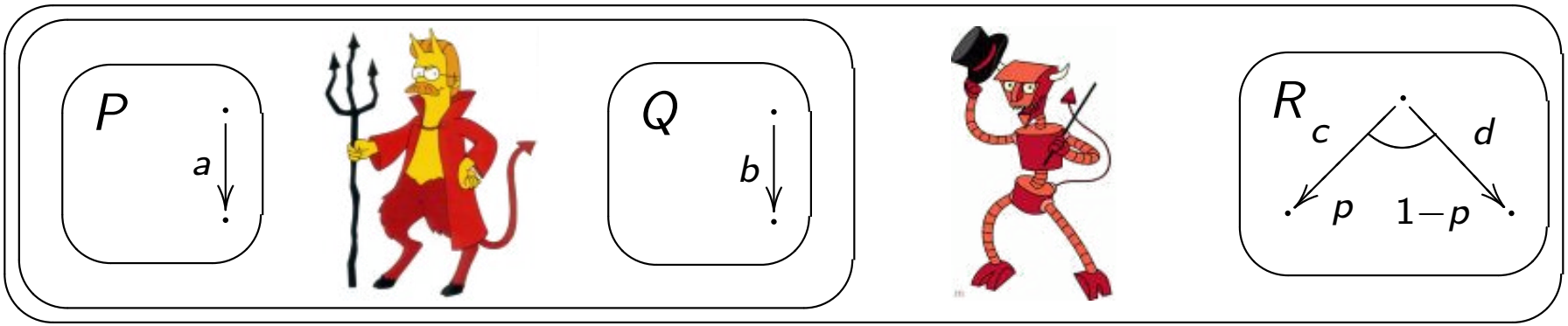
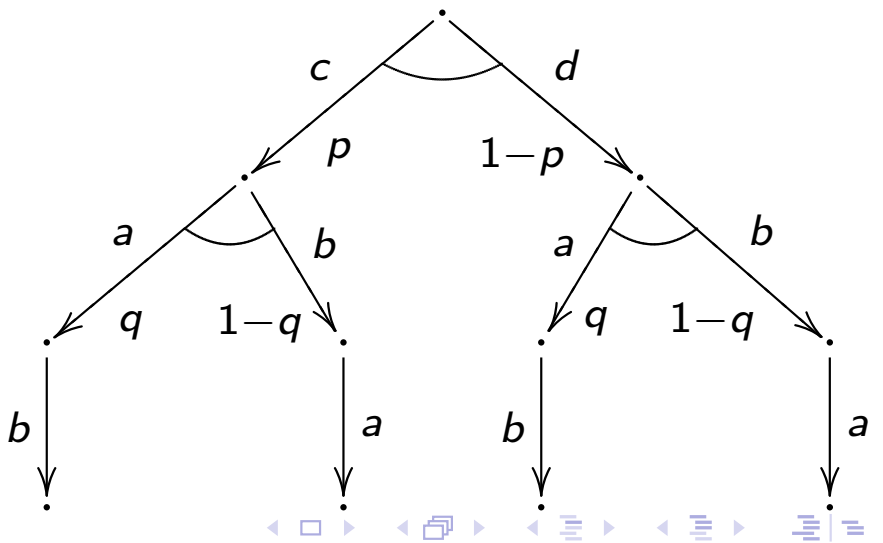


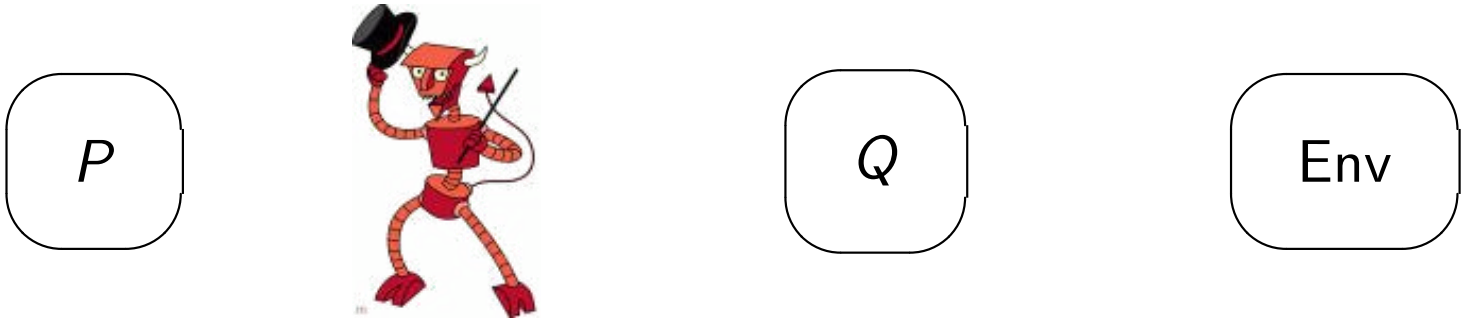
- *Outer* daemon: $\langle R,\ 1\rangle$.
- *Inner* daemon: $\{\langle P,\ q\rangle, \langle Q,\ 1-q\rangle\}$.
- Conclusion: inner daemon doesn't know enough.



Scenario 2: context-dependent composition

How much does the daemon know?

Daemon sees the outside world.



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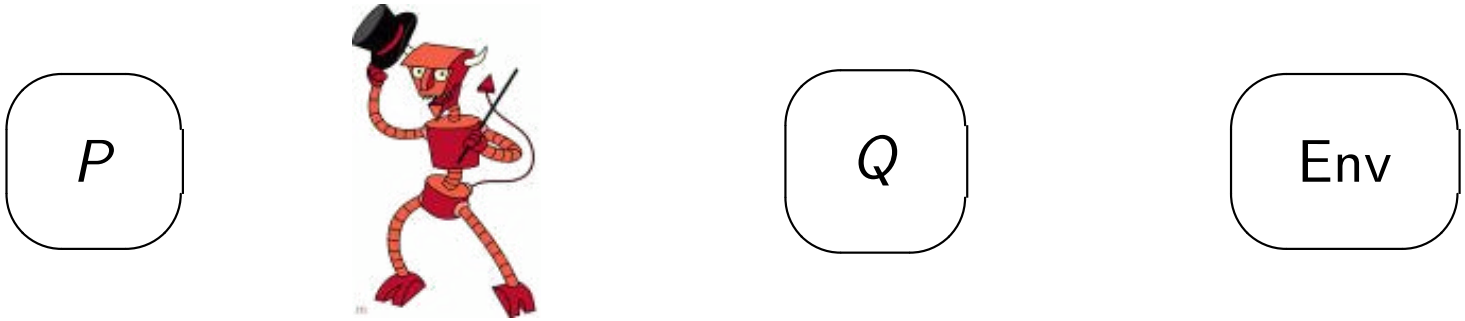

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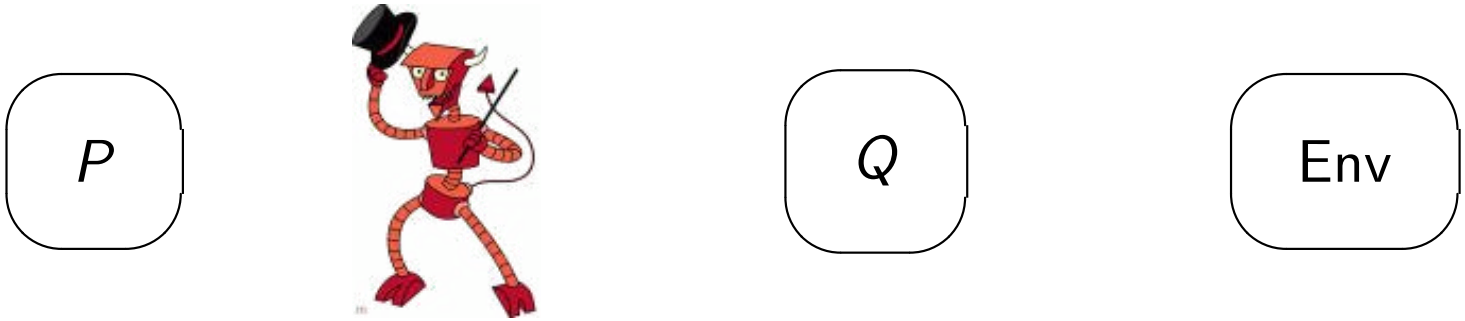


Daemon knows the histories of P, Q and Env.

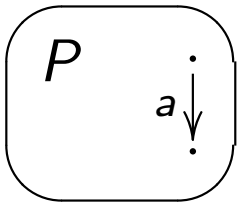
Problem: violation of the interleaving axiom!
I.e., there exists Env such that

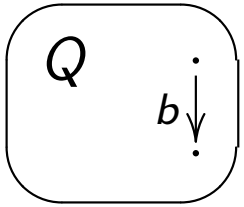
$$(a \| b) \| \text{Env} \not\cong (a.b + b.a) \| \text{Env}\,.$$

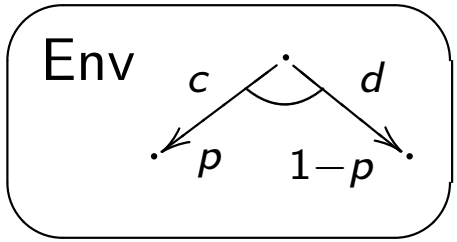
Non-interleaving semantics

Context-dependent composition

$(a \| b) \|$ Env:







Non-interleaving semantics

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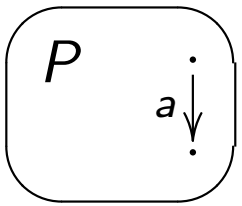
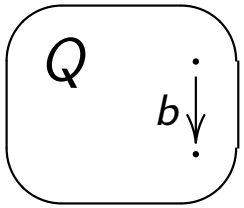
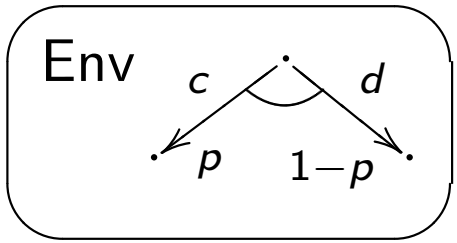
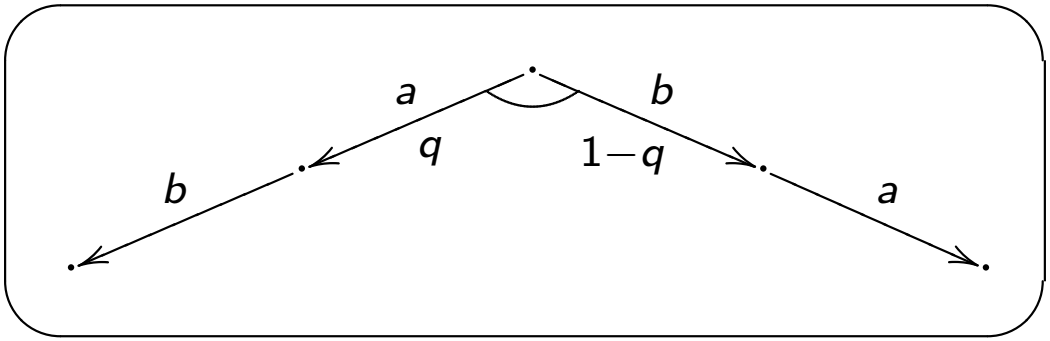
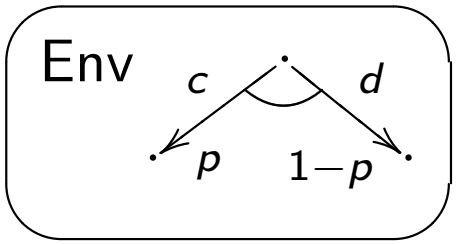






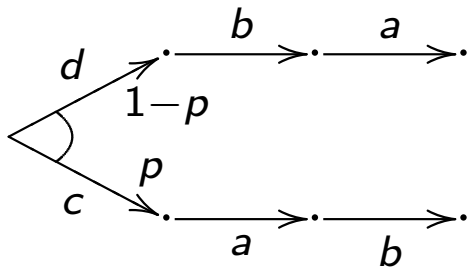
$(a.b + b.a) \|$ Env:





The (same) counterexample

Context-dependent composition



The trace distribution

$$\{\langle cab,\ p\rangle, \langle dba,\ 1-p\rangle\}$$

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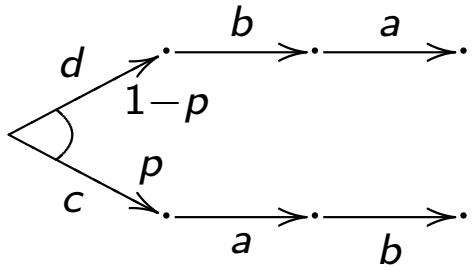

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Conclusion: we have a non-interleaving, but total order semantics.

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In context-dependent composition, the same problem leads to difference between $a \| b$ and $a.b + b.a$.

Two types of nondeterministic choices: global vs. local

Global choice: $a \| b$, resolved by a daemon.

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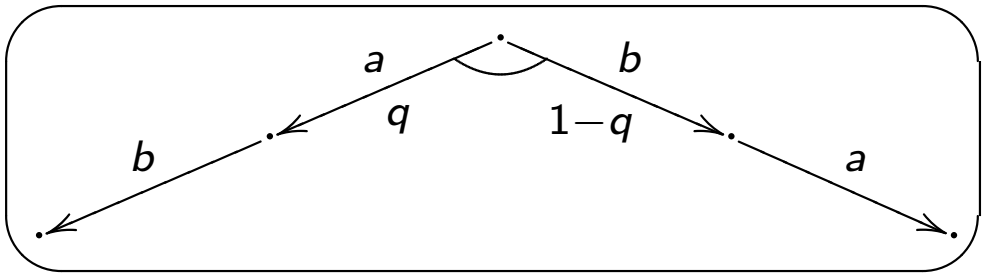
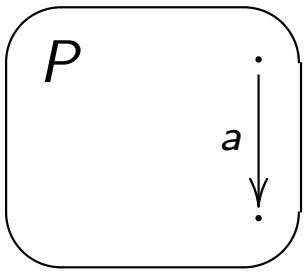
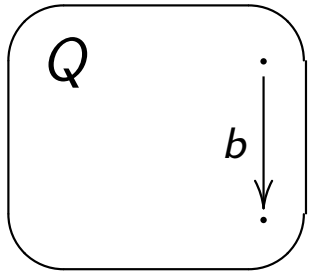


Behavior varies depending on the perspective!

Dissecting the problem, Part I: eliminate global choices.

To better understand the problem, we developed the model of *Switched PIOA*.





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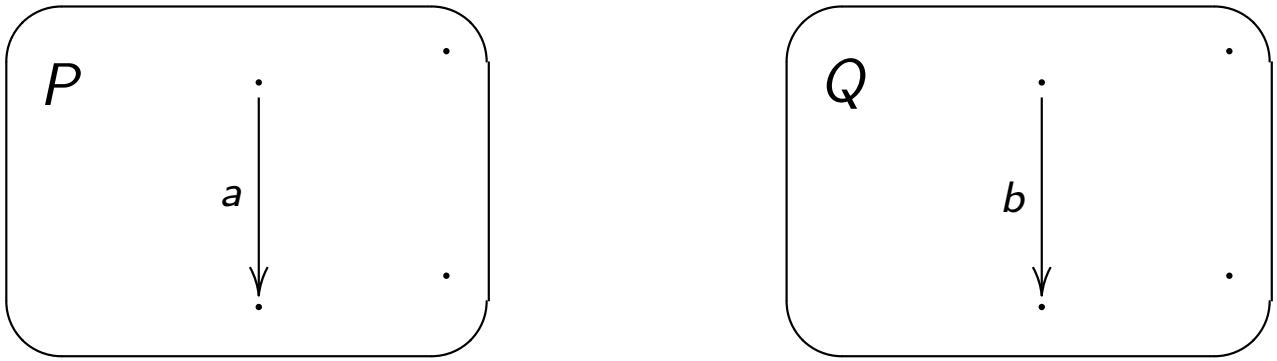

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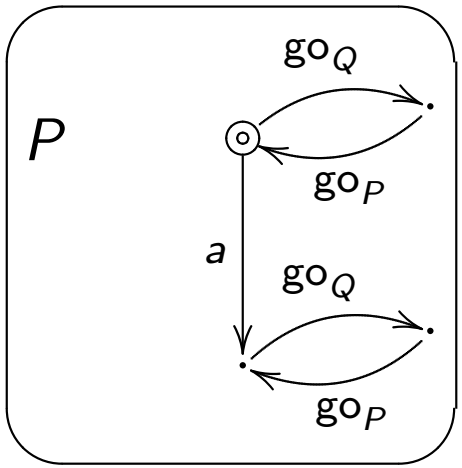

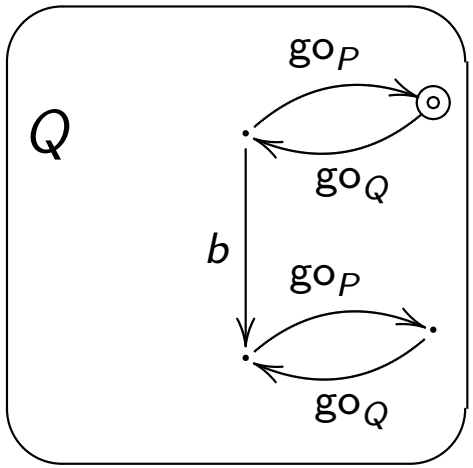

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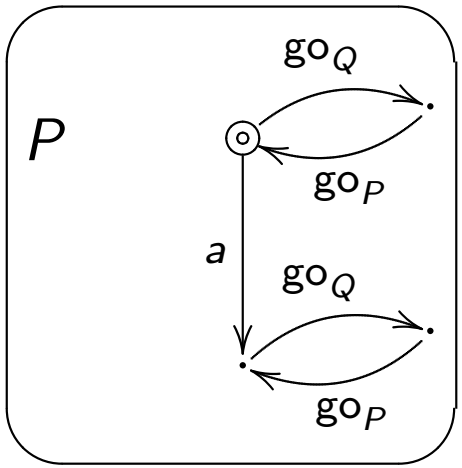

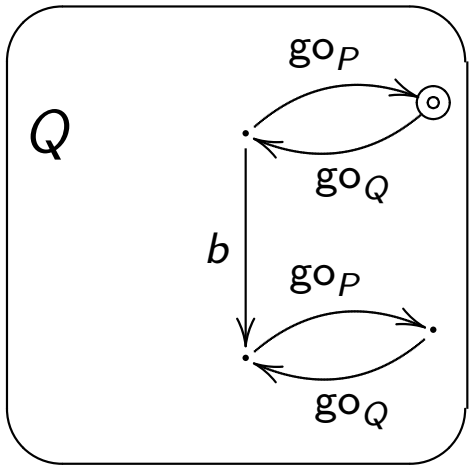

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Every decision is made locally, so no more daemons.

Due to the absence of global choices . . .

Parallel composition in Switched PIOA:

- easy to define;

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Parallel composition in Switched PIOA:

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That's all very nice, but parallel processes don't really exchange control . . .

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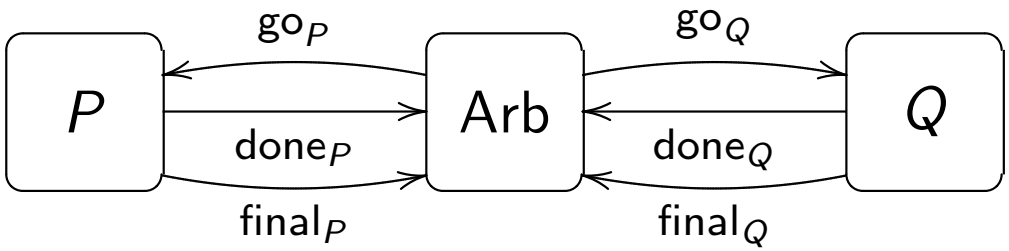

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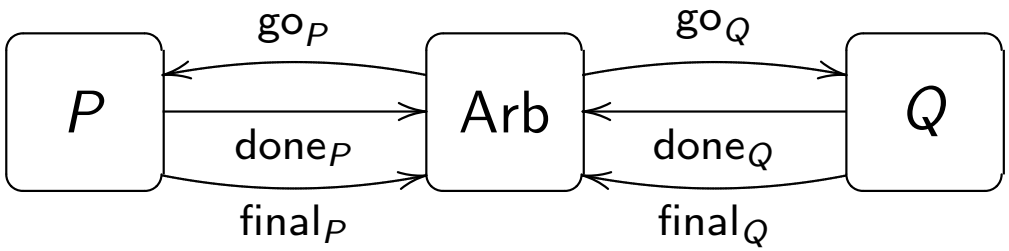

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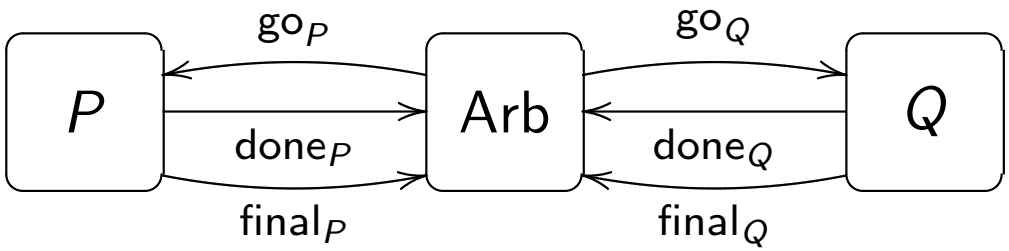

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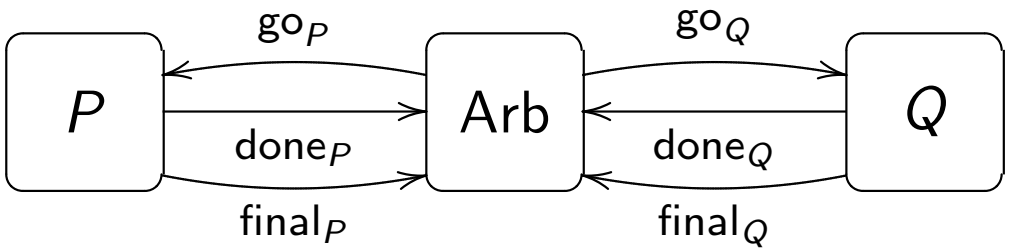

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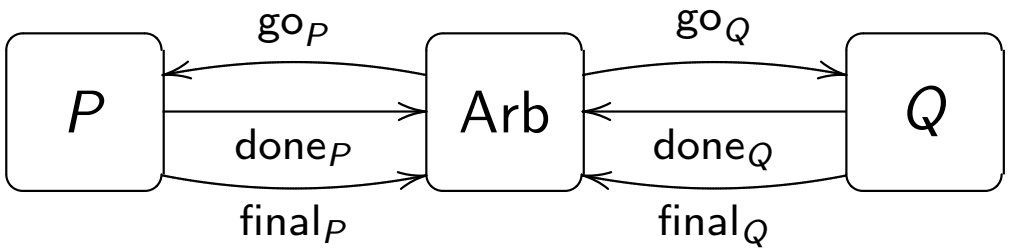

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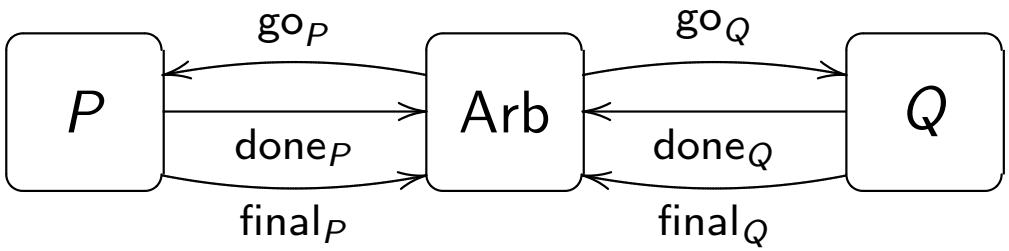

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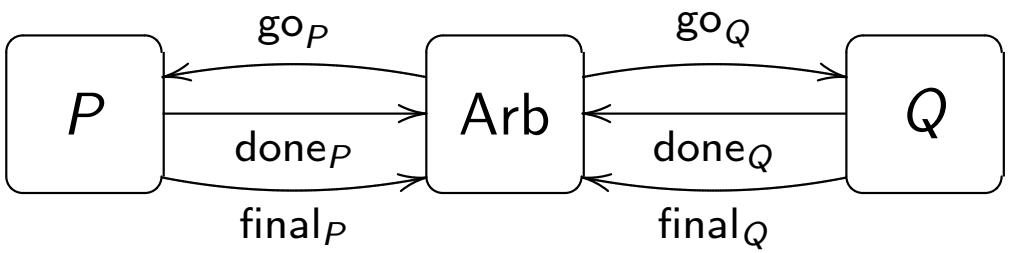
Implementing context-independent composition



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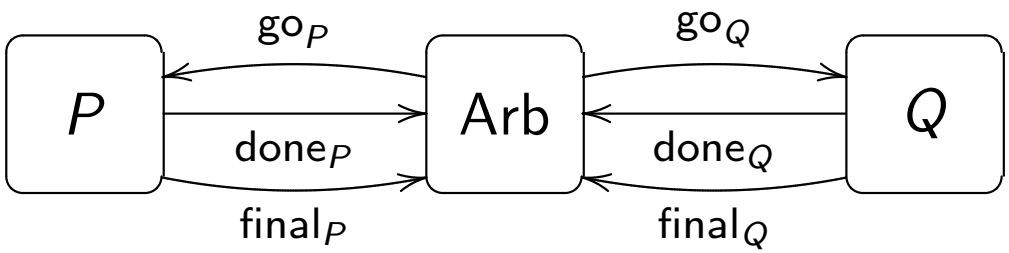

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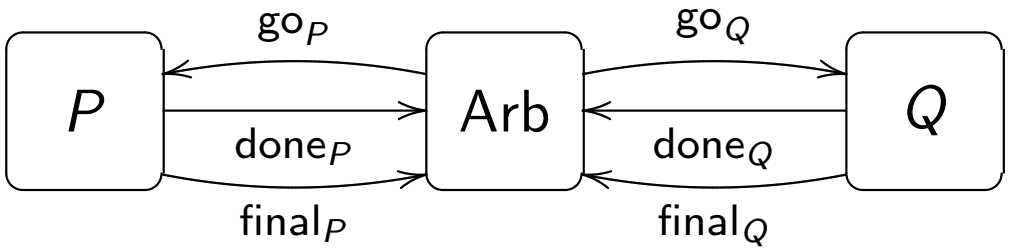

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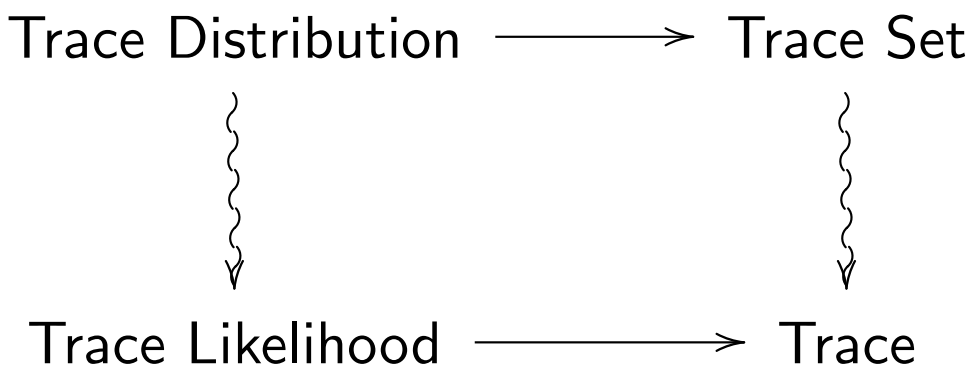

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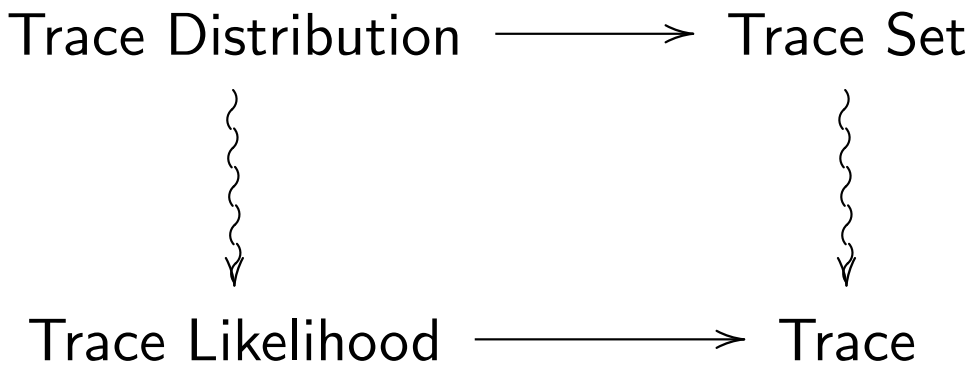

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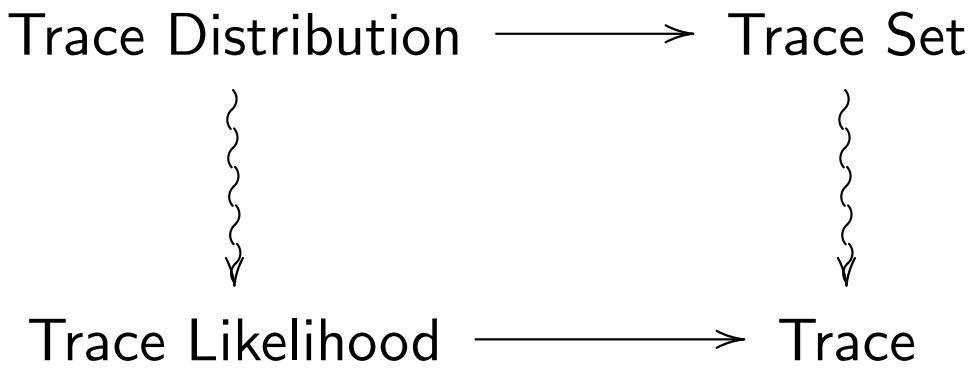

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– End –

5 Appendix

- Trace Set Semantics
- Trace Likelihood Semantics
- Getting Stuck

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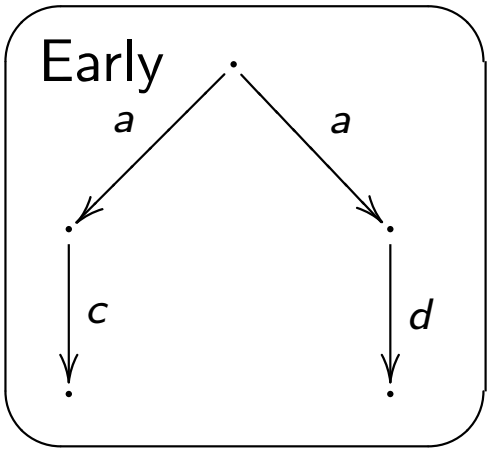
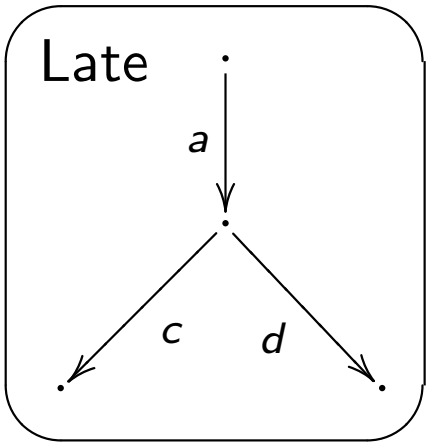
That is, schedulers return sets of possible transitions, rather than discrete distributions over possible transitions.

Trace Set Semantics: Example





Equivalent in semantics: trace, trace set, trace distribution.

Not equivalent in semantics: bisimulation.

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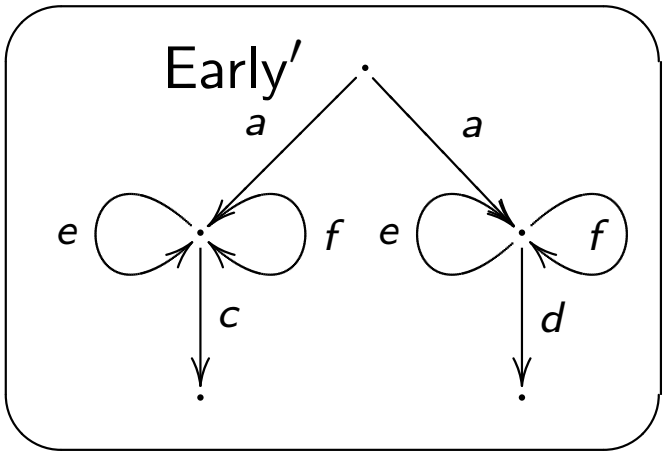

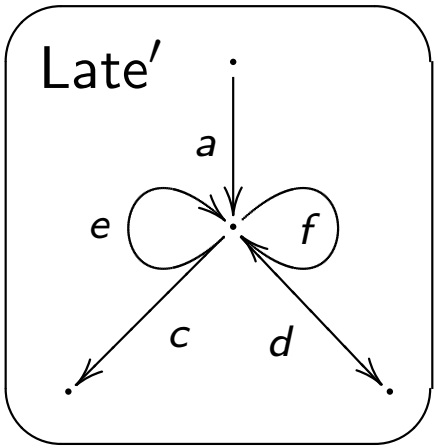

Add input e,f-loops.

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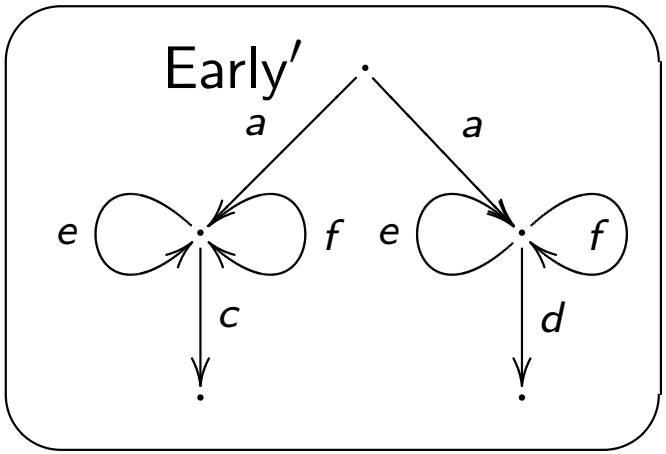

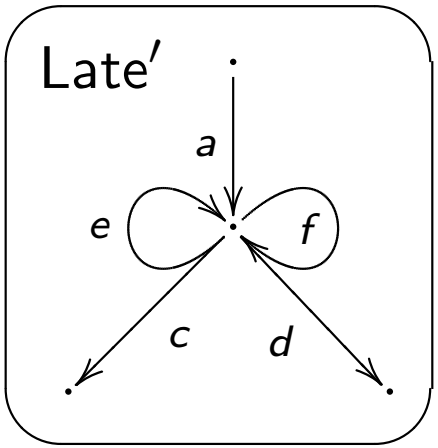

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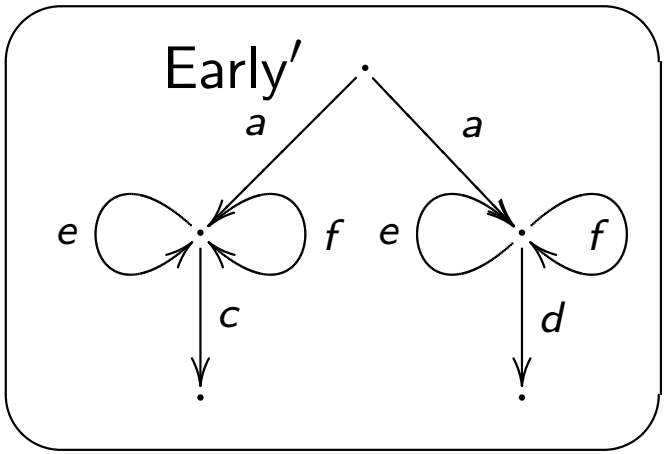

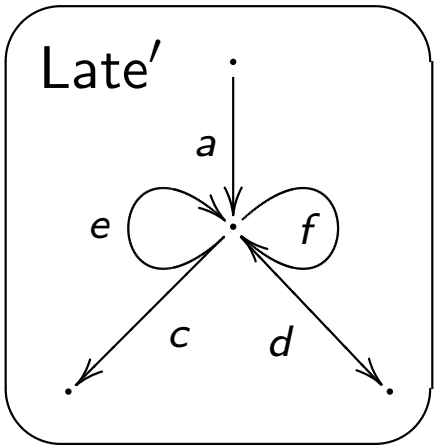

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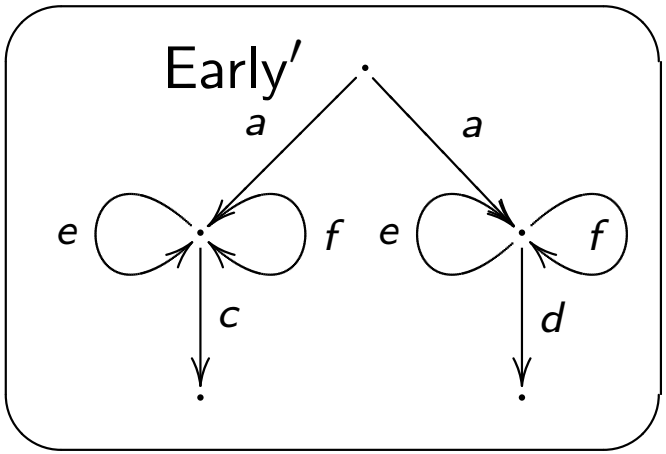

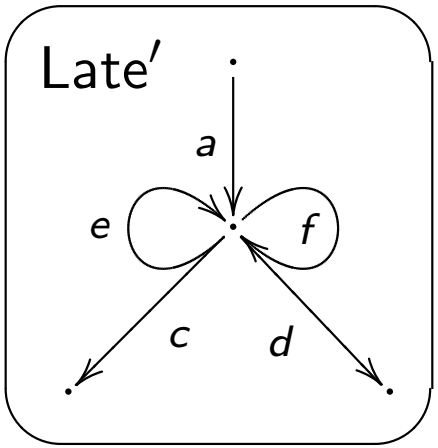

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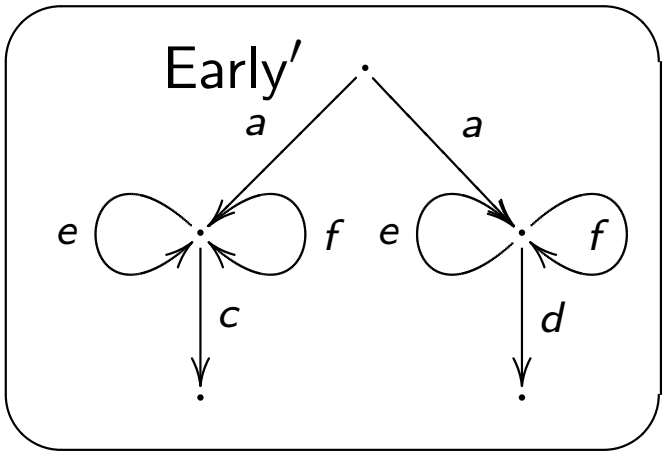

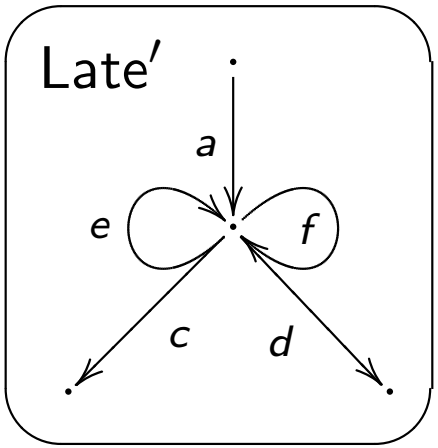

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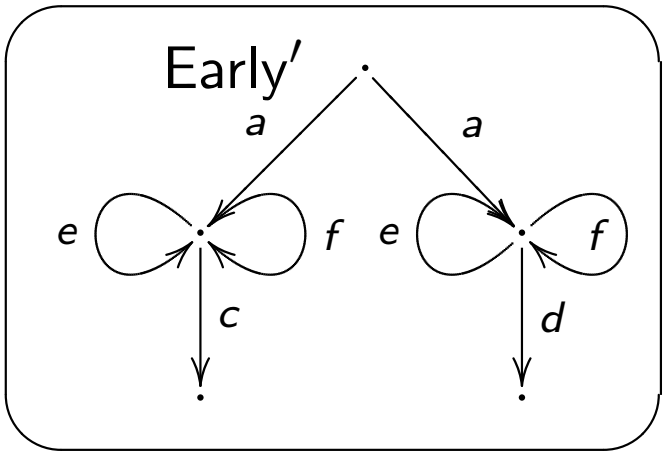

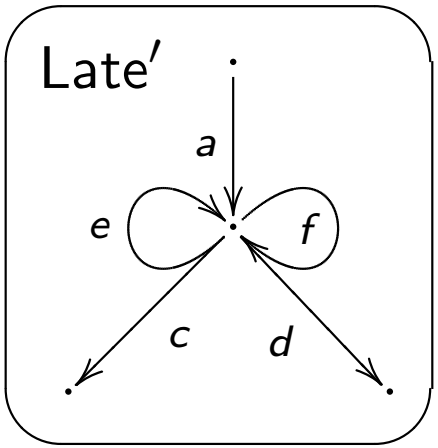

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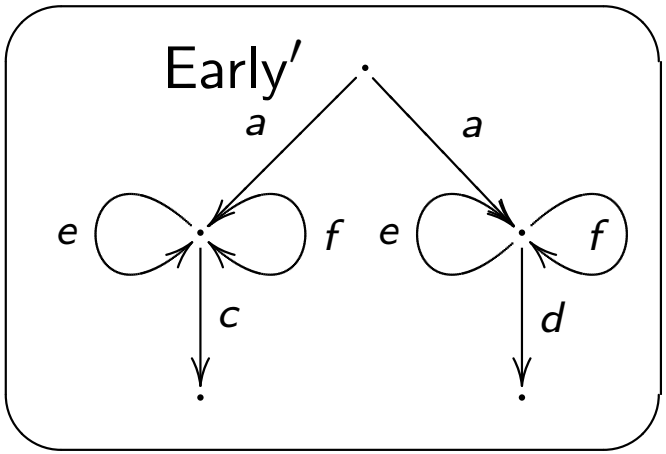

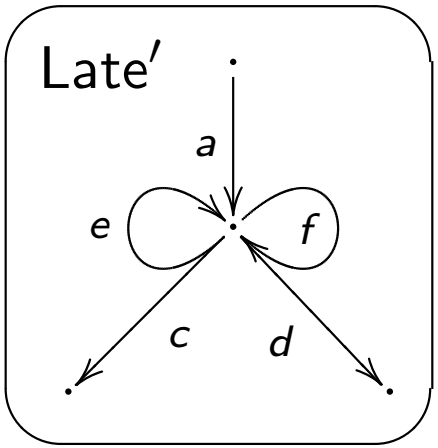

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Important: Early′ cannot choose between inputs e and f.

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Examples of "undesirable" properties of trace distribution semantics can be reproduced in trace set semantics.

Key: each trace distribution contains a collection of traces, rather than a single trace.

In some cases, this allows us to observe branching structure.

An Alternative: Trace Likelihood Semantics

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Frequentist probabilities: prediction about a large number of experiments, not about a single experiment.

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