A Symbolic Treatment of Randomization

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Outline

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Acknowledgment

This work is done in the context of two projects.

- Compositionality of trace distribution semantics: Ling Cheung, Nancy Lynch, Roberto Segala, Frits Vaandrager.
- Modeling cryptographic protocols in PIOA: Ran Canetti, Ling Cheung, Dilsun Kaynar, Moses Liskov, Nancy Lynch, Olivier Pereira, Roberto Segala, Frits Vaandrager.

I thank everyone involved for the very helpful discussions.

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From the last talk:

- semantic compositionality fails in Segala's Simple Probabilistic Automata Framework;

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This talk: *Symbolic PA*, a new system type designed to weaken the adversary model of Segala's Simple PA.

Disclaimer: This is ongoing work.

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Typically, we do so by specifying an adversary: a function from finite histories to available next transitions.

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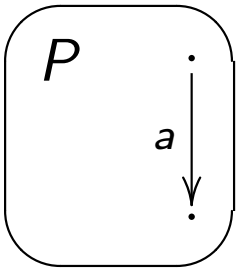
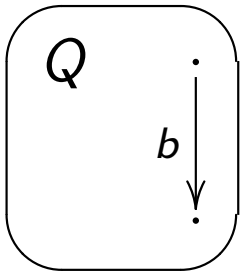
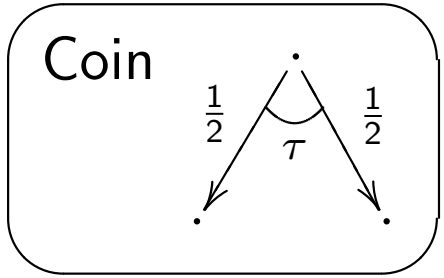
  Example: Segala's Simple PA has a strong adversary model, in that the adversaries have complete knowledge over execution history of all components, including their internal states.
- Different adversary models give rise to different notions of probabilistic executions.

Silly Example I







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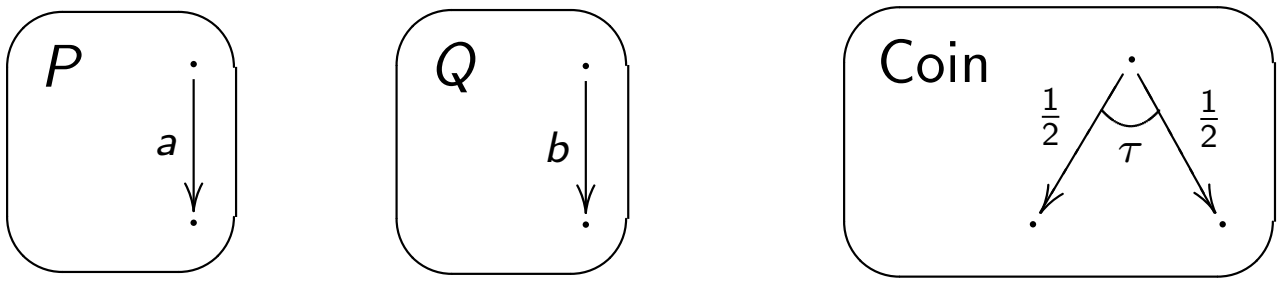
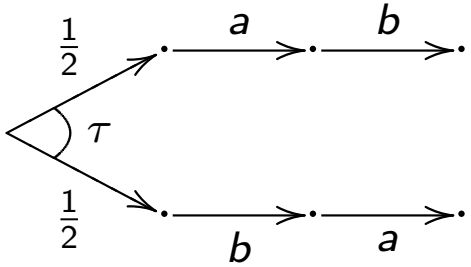


In Simple PA, the following is a "legal" probabilistic execution.



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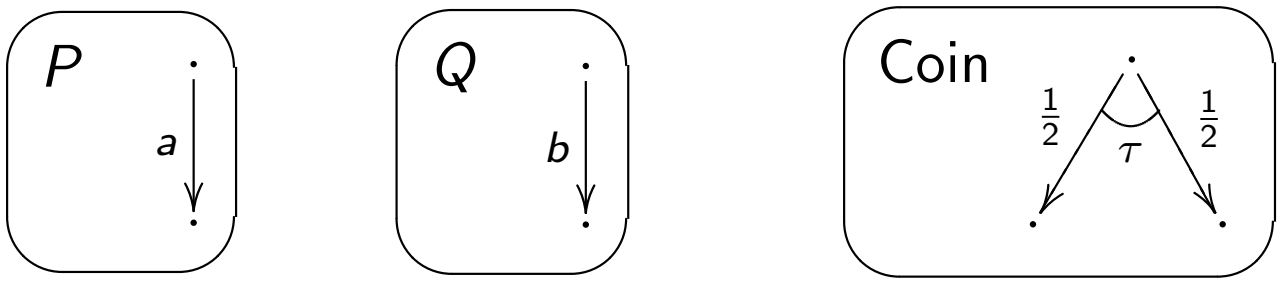

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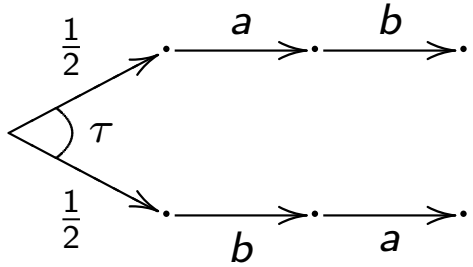
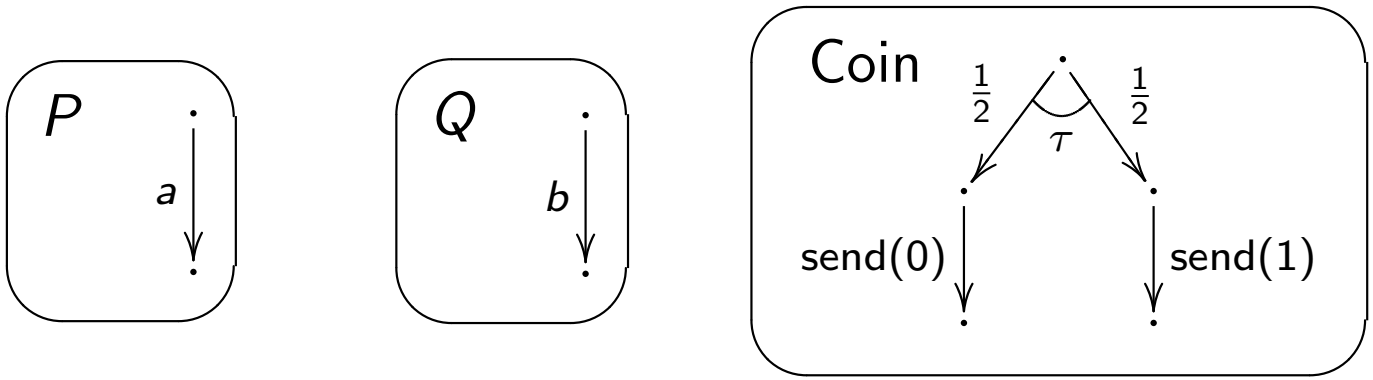


In other words, a hidden random choice in Coin can affect which of P and Q wins the race.

Silly Example II

Suppose Coin announces the outcome of its random choice.



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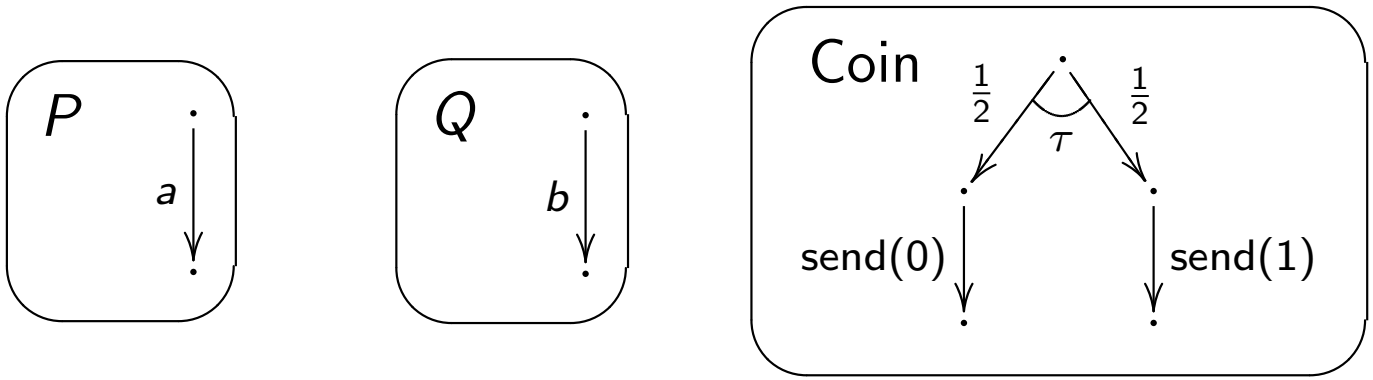

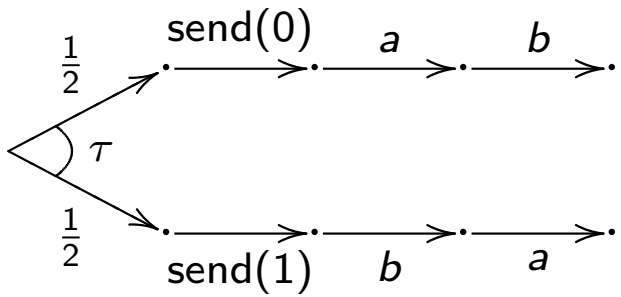

A bit more plausible now?

Perhaps the message from Coin somehow changed the world?

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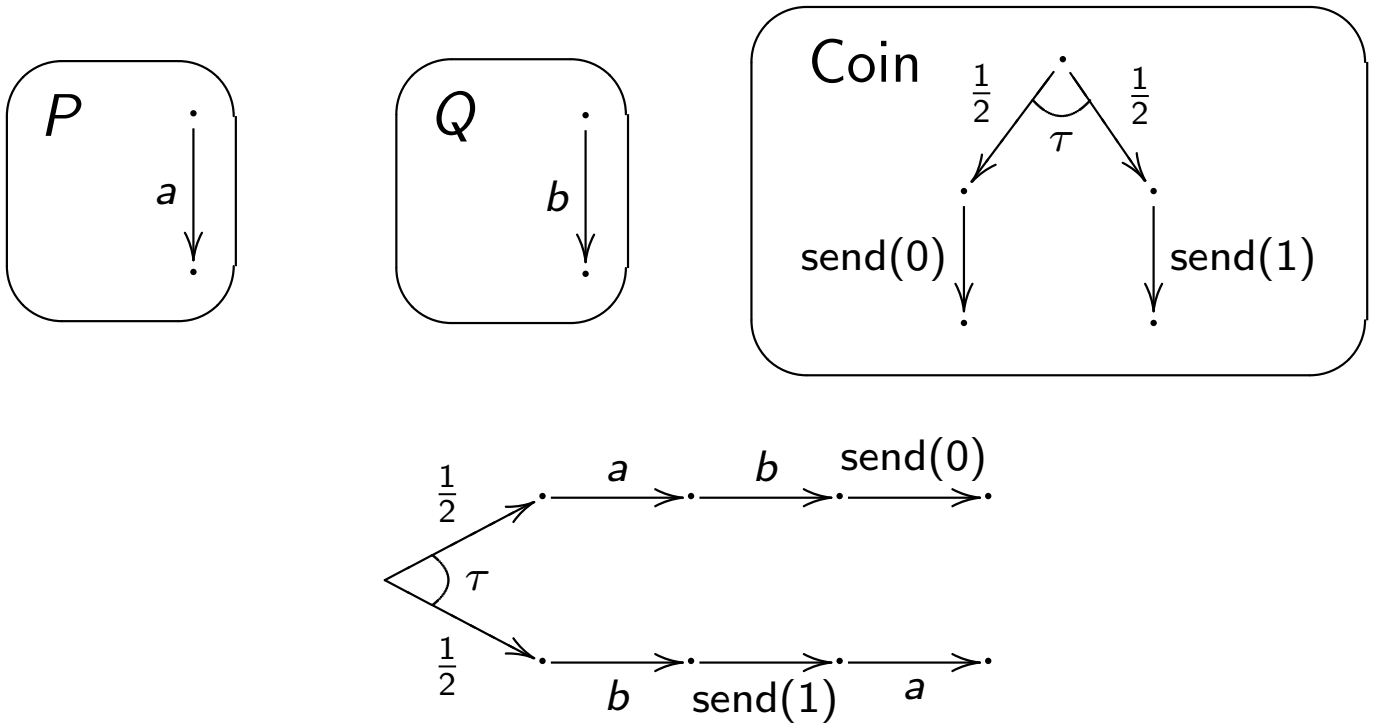

What about this execution?

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In particular, the adversary's knowledge can increase dynamically, as more and more information becomes public.
E.g., the adversary should be able to use the random choice of Coin, but only after the $send(-)$ transition.

Examples of Adversary models

In increasing order of strength.

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- Symbolic PA: unbounded computational capabilities, but limited knowledge of outcomes of coin tosses (i.e., only those that are announced via visible actions).
- Segala's Simple PA: everything.

Randomized Scheduling

All of our adversaries are deterministic. I.e., given any finite execution, the adversary chooses

- either to halt the execution,
- or to extend the execution with one transition,

Why?

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Expressivity: what have we lost?

Basic Ingredients

We have X, a set of symbolic states, together with a transition function

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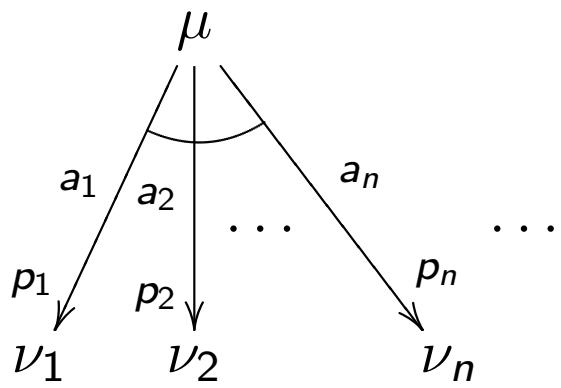
- each bundle is a function

$$f : Act \longrightarrow [0,1] \times X$$

such that $\pi_1(f)$ is a discrete sub-distribution on Act.



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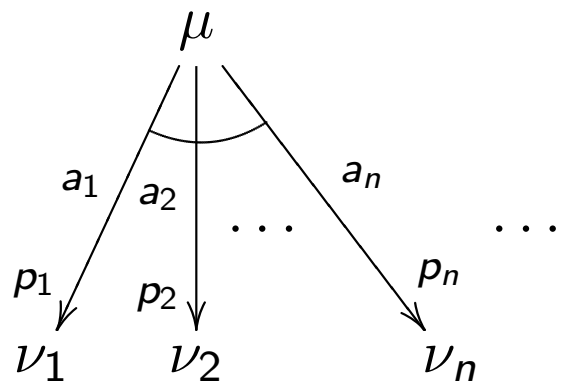

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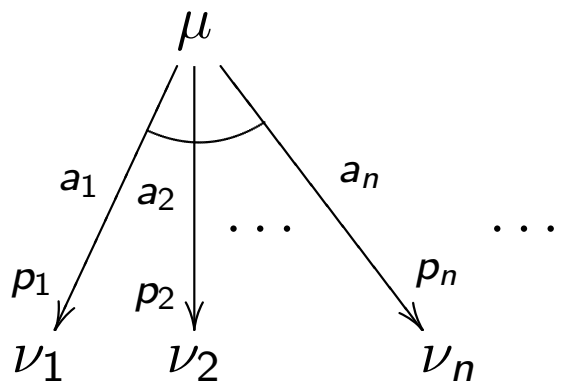

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Why Is It Symbolic?

Think of a symbolic state $\mu \in X$ as a discrete distribution on concrete states.

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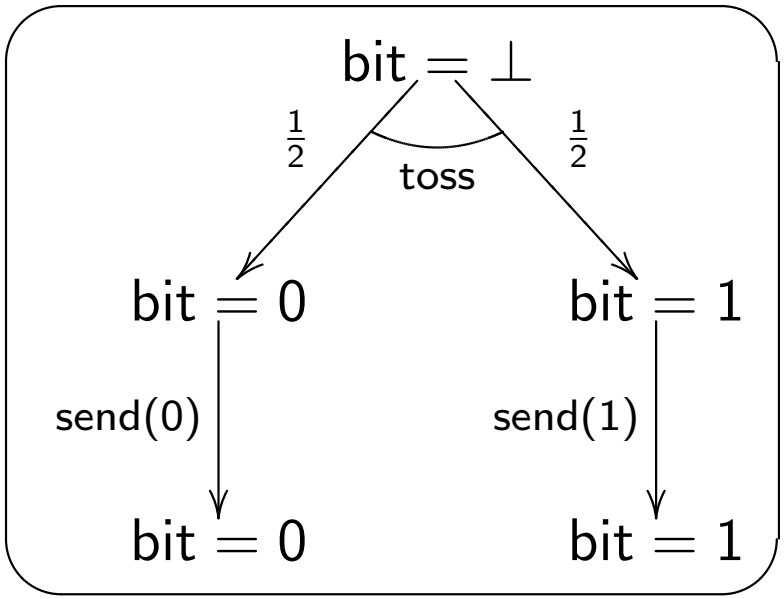
Example: the automaton Coin.

Before (Simple PA):



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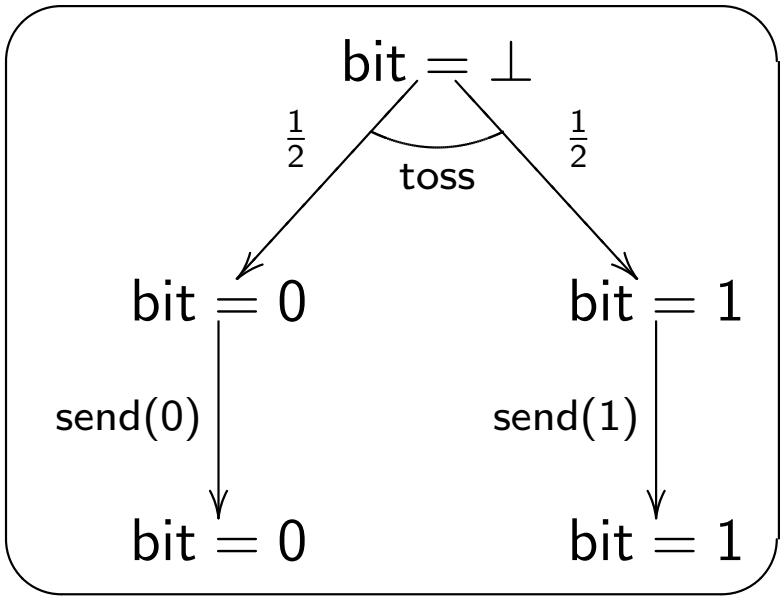

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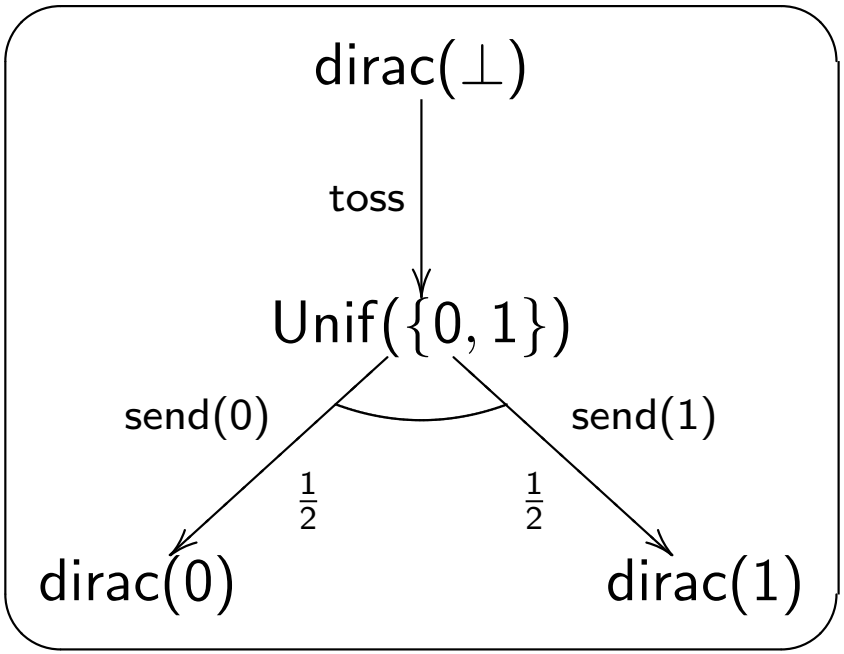

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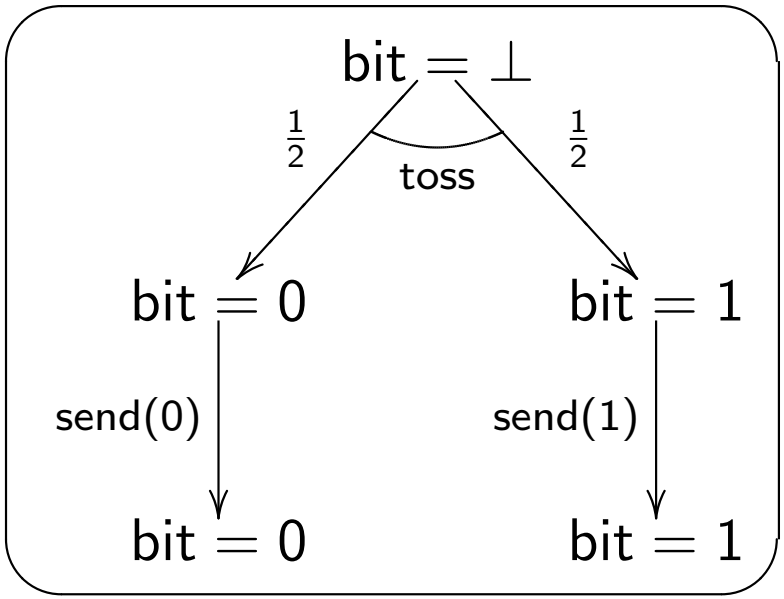

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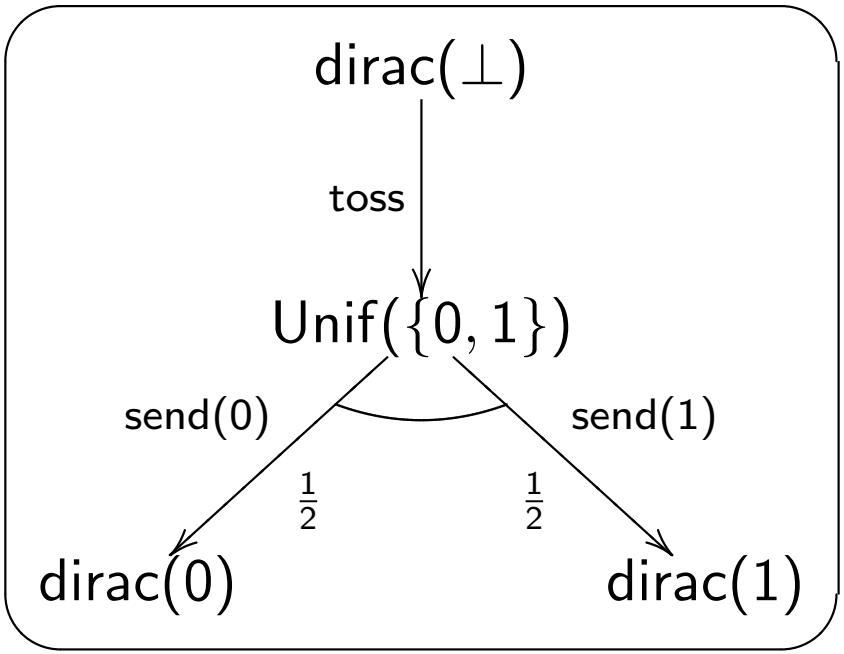

Notice, the adversary learns the value of bit only after observing a send(−) action.

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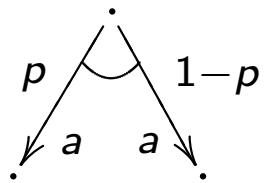

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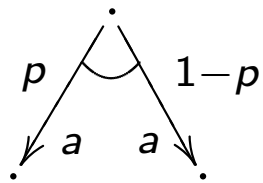

From the Coin example: this restriction is precisely what we need to weaken the adversary model.

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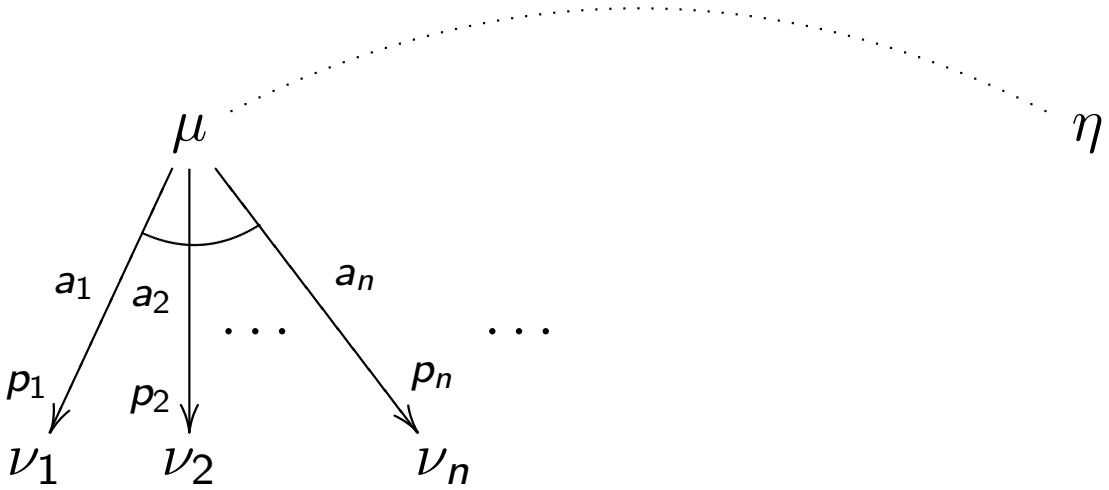

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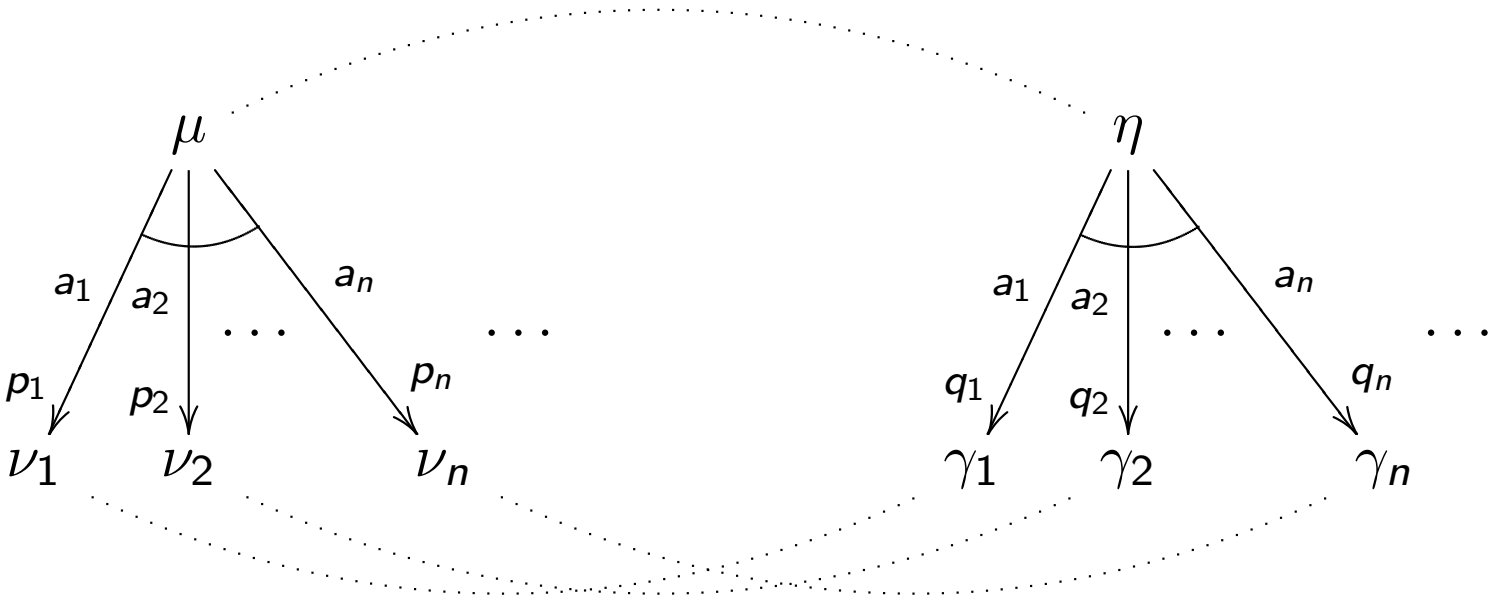

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Proof: Induction on depth of probabilistic executions.

Weak Transition Bundles

Add a special symbol τ to the alphabet Act.

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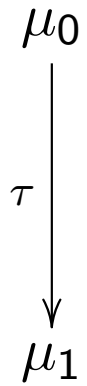
- A hidden transition bundle is a transition bundle f satisfying $\mathrm{Supp}(\pi_1(f)) = \{\tau\}$.



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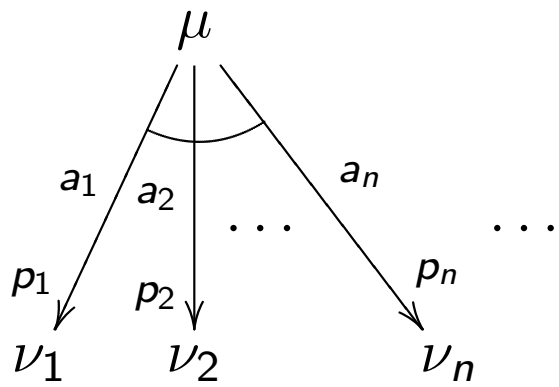

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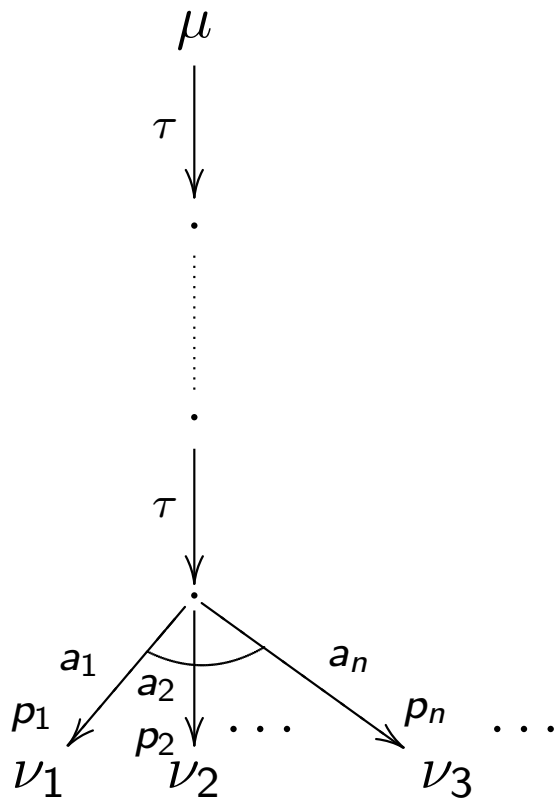

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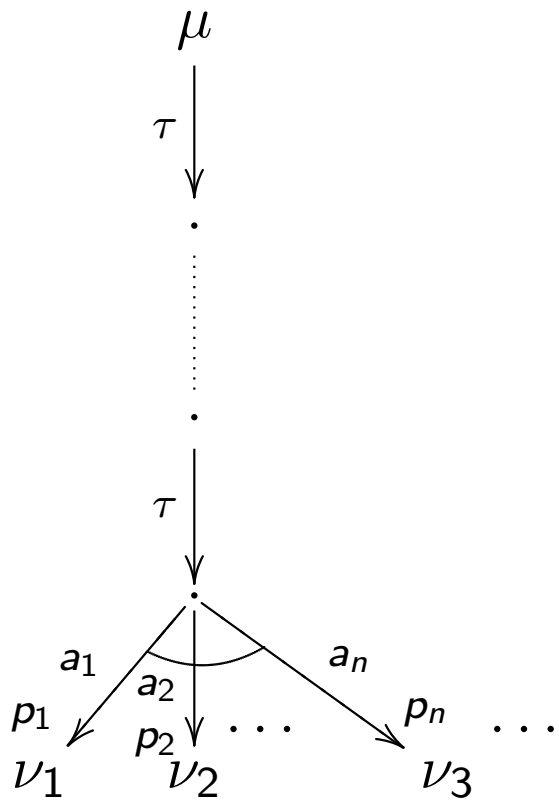
- Why not partially hidden bundles? See appendix . . .



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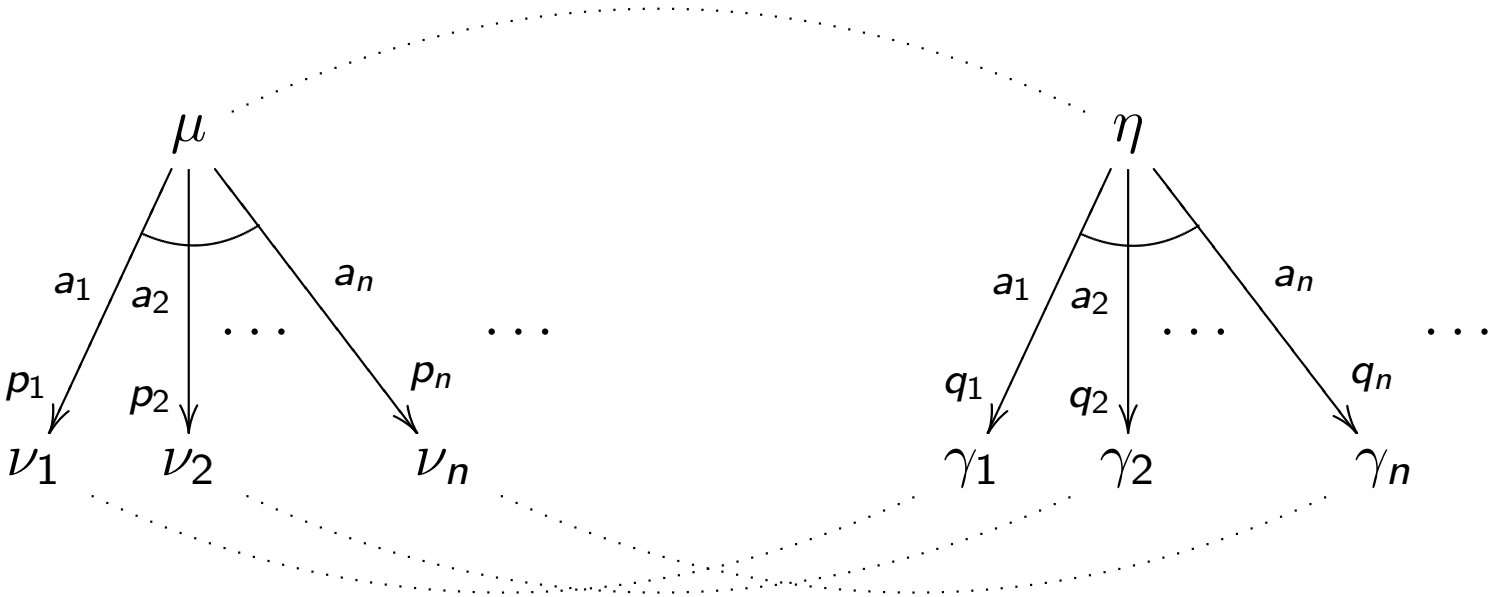

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- apply soundness of strong simulation.

I/O Distinction

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- Input: $\mathrm{Supp}(\pi_1(f)) = \{a\}$ for some $a \in I$.

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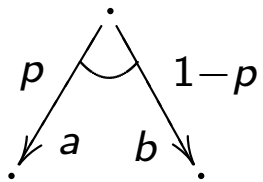

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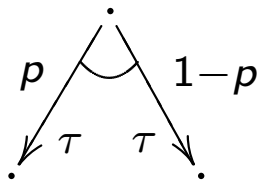

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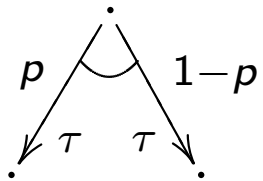

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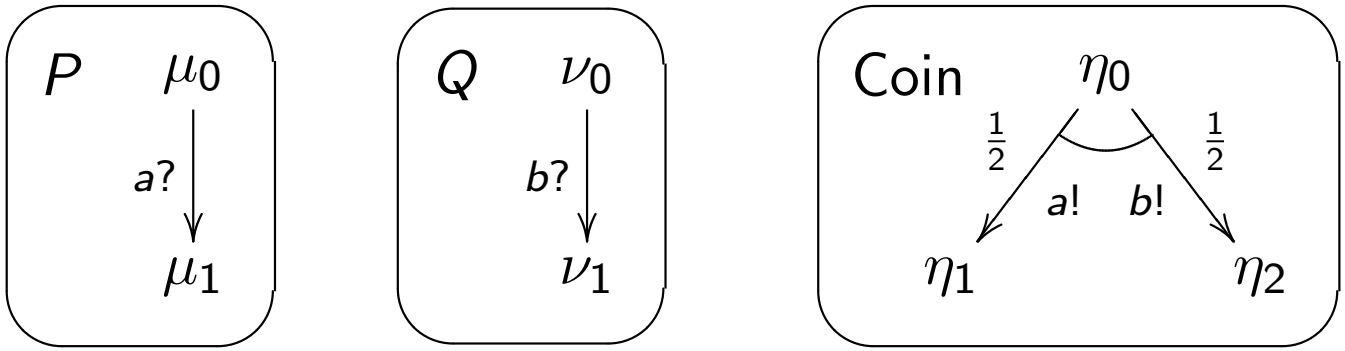

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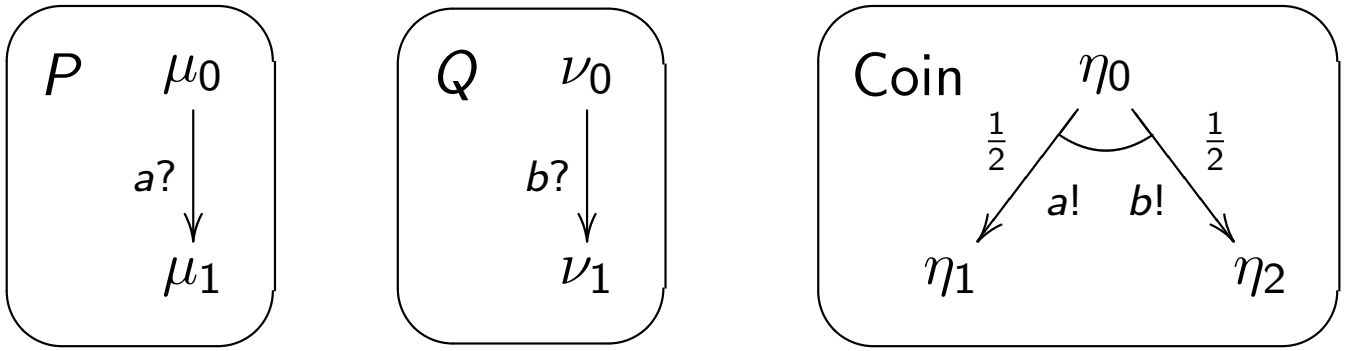
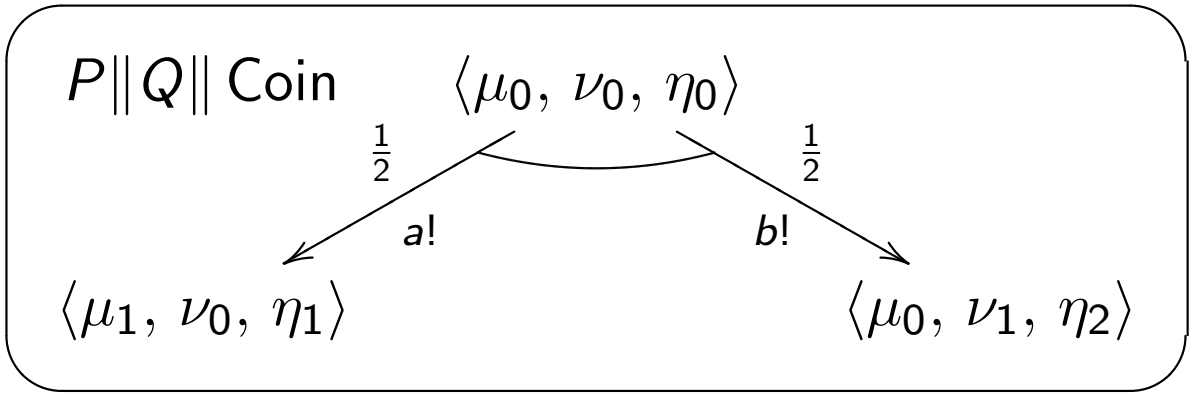


Their composite:



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Questions? Or move on to Oblivious Transfer?

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Idea: Ideal system is "obviously" secure.

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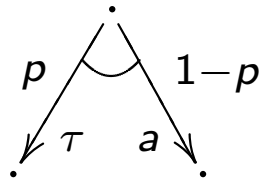
- *Easy* solution: content-oblivious scheduling.
- *Hard* solution: PPT scheduling.

The end . . . really.

Partially Hidden Bundles

Question: Should we allow partially hidden bundles?



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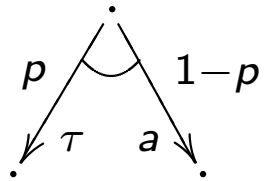

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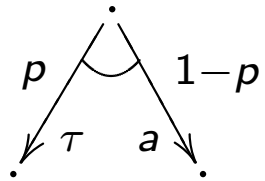

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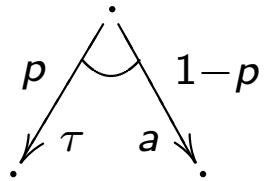

Answer: Not unless we have to.

Pro: More expressive.
Con: No straightforward definition of weak simulation.

Doing Without Partially Hidden Bundles

Claim: Partially hidden bundles are not necessary under a certain finiteness assumption.

Doing Without Partially Hidden Bundles

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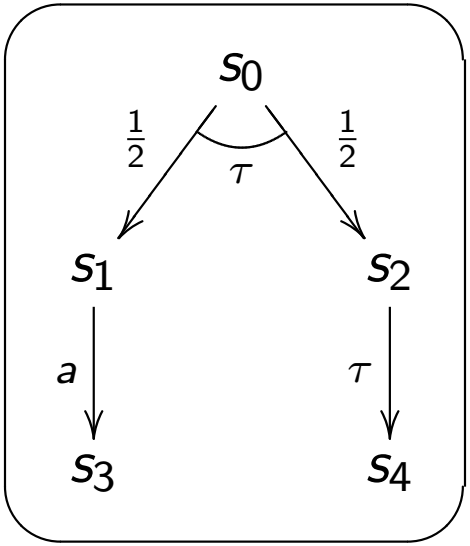

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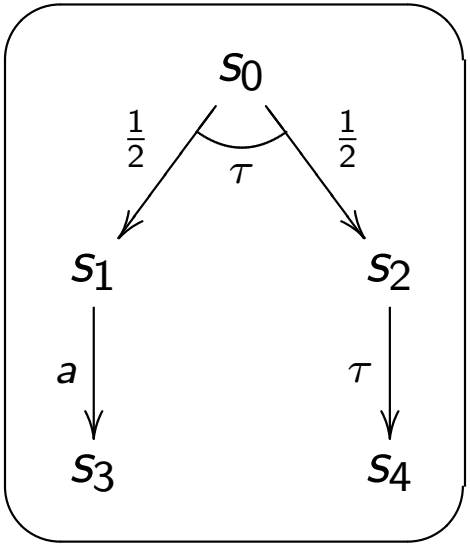

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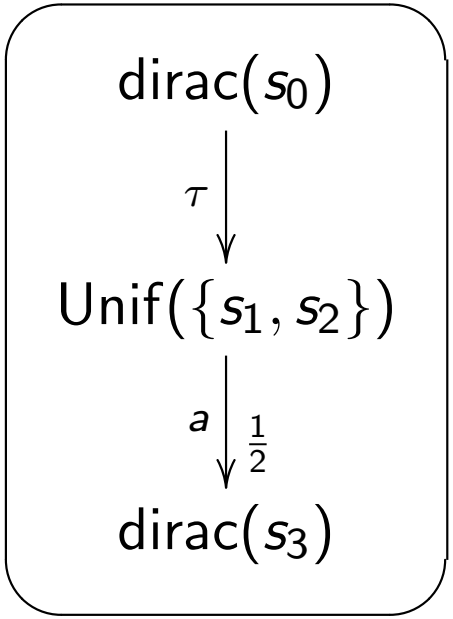

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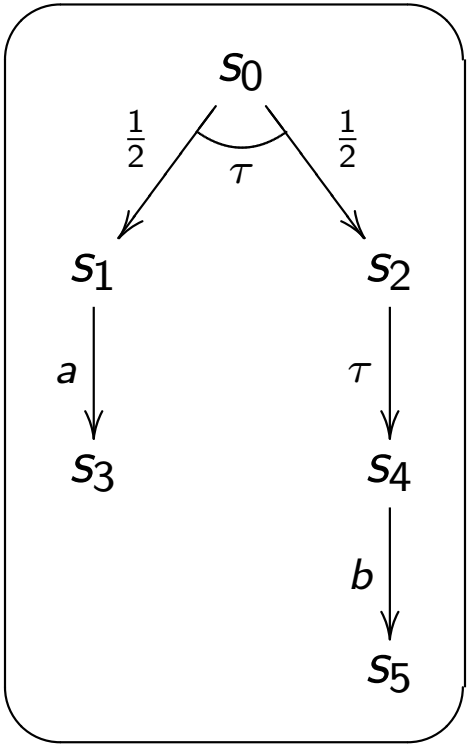

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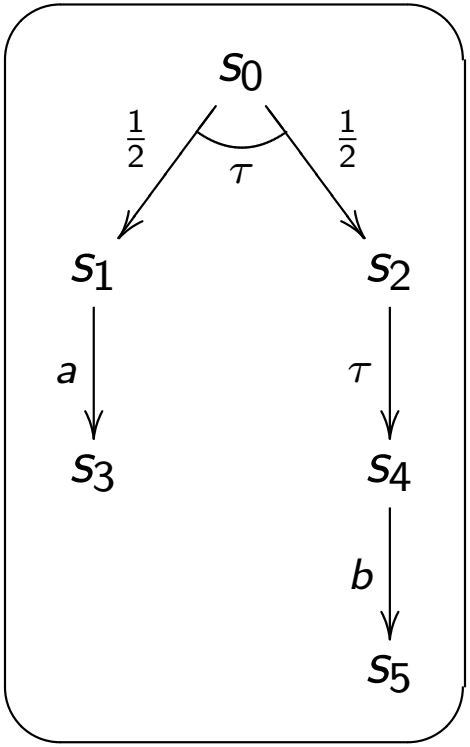

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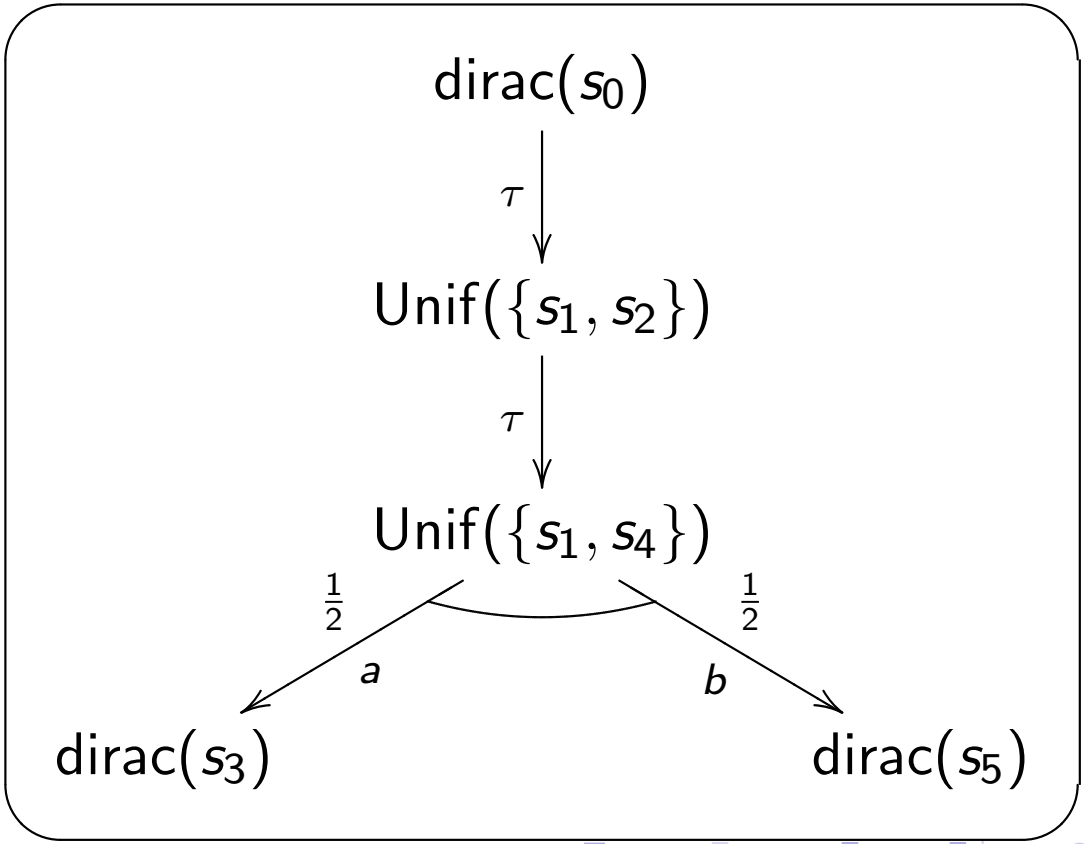

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Notice, this restriction applies only to probabilistic branching, and not to non-deterministic branching.