Fluidic Topology Optimization with an Anisotropic Mixture Model

Yifei Li
MIT CSAIL

Tao Du
MIT CSAIL

Sangeetha Grama Srinivasan
University of Wisconsin-Madison

Kui Wu
Tencent LightSpeed Studios

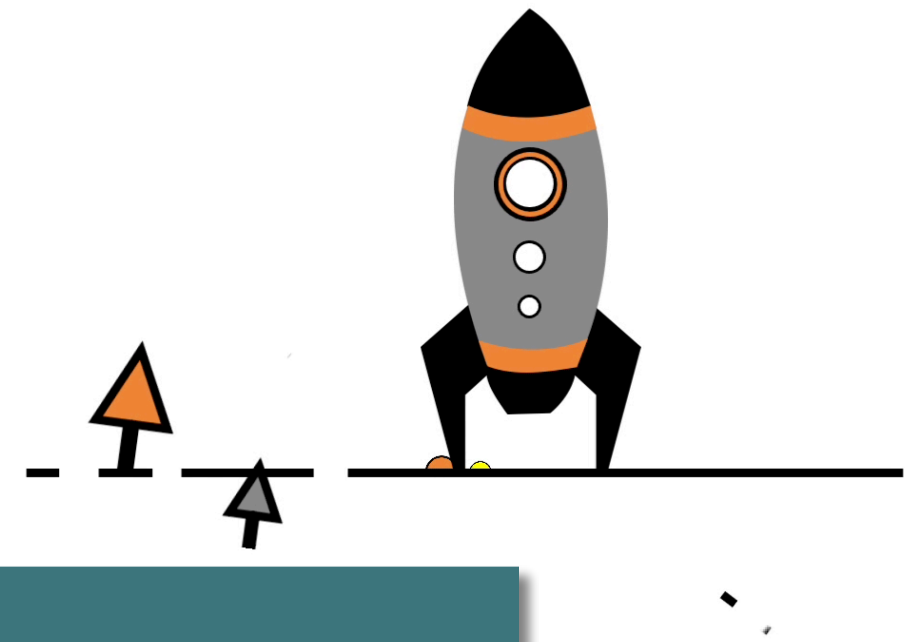
Bo Zhu
Dartmouth College

Eftychios Sifakis
University of Wisconsin-Madison

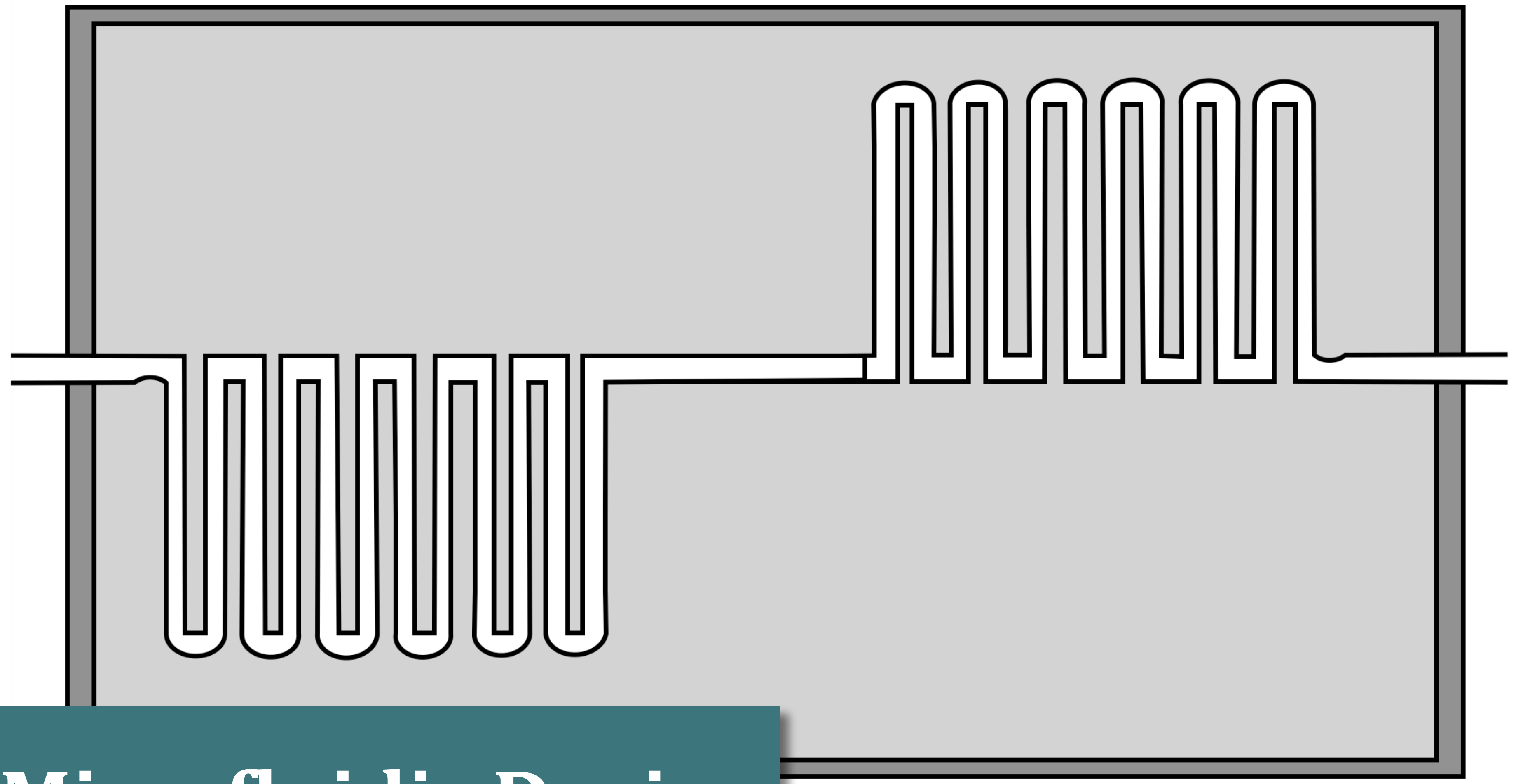
Wojciech Matusik
MIT CSAIL

SIGGRAPH ASIA 2022 DAEGU

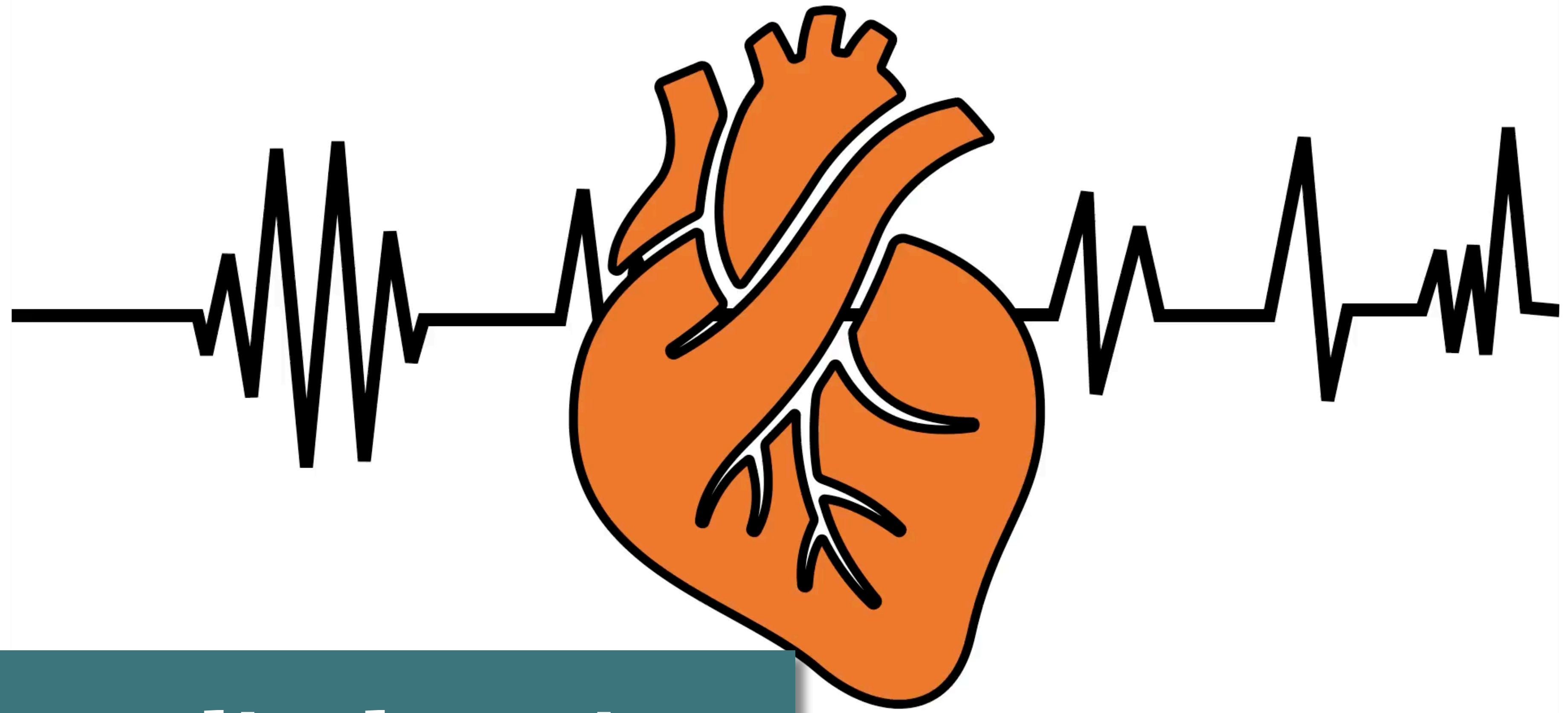
Fluidic devices are everywhere in engineering, medicine, our daily life,...



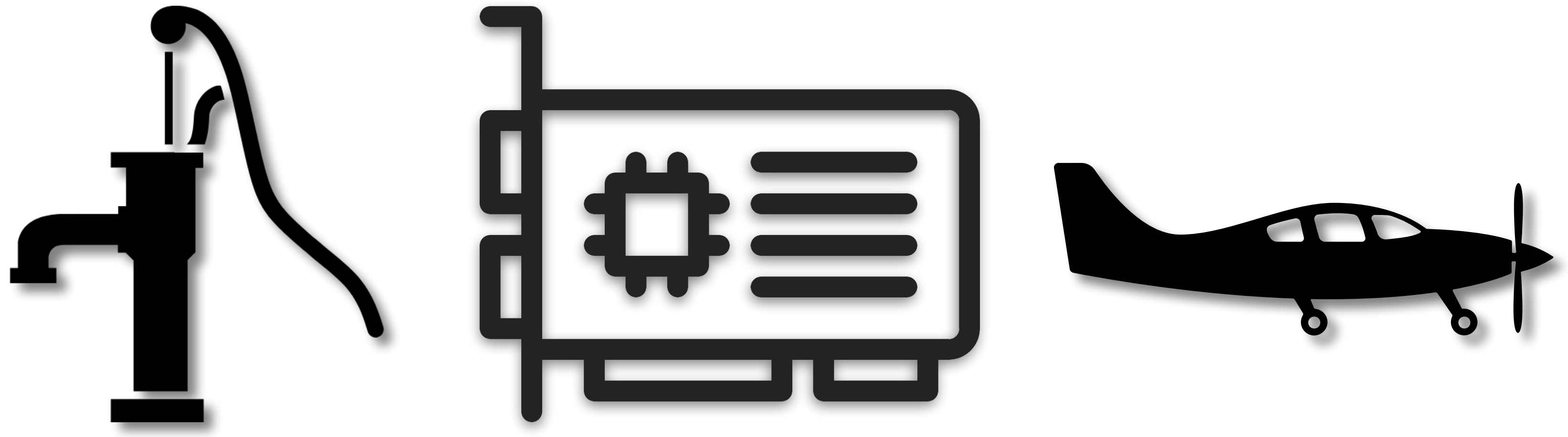
Combustion Engine



Microfluidic Device



Medical Device

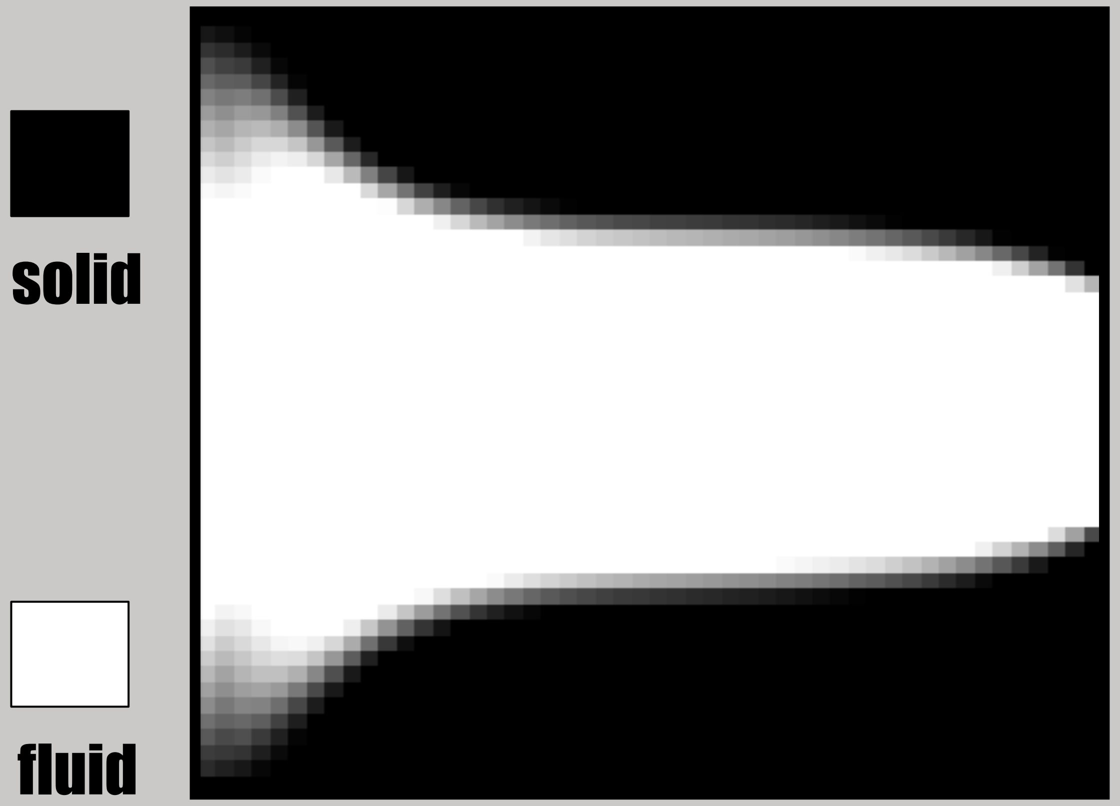


Hydraulic Pump Heat Sink Propeller ...

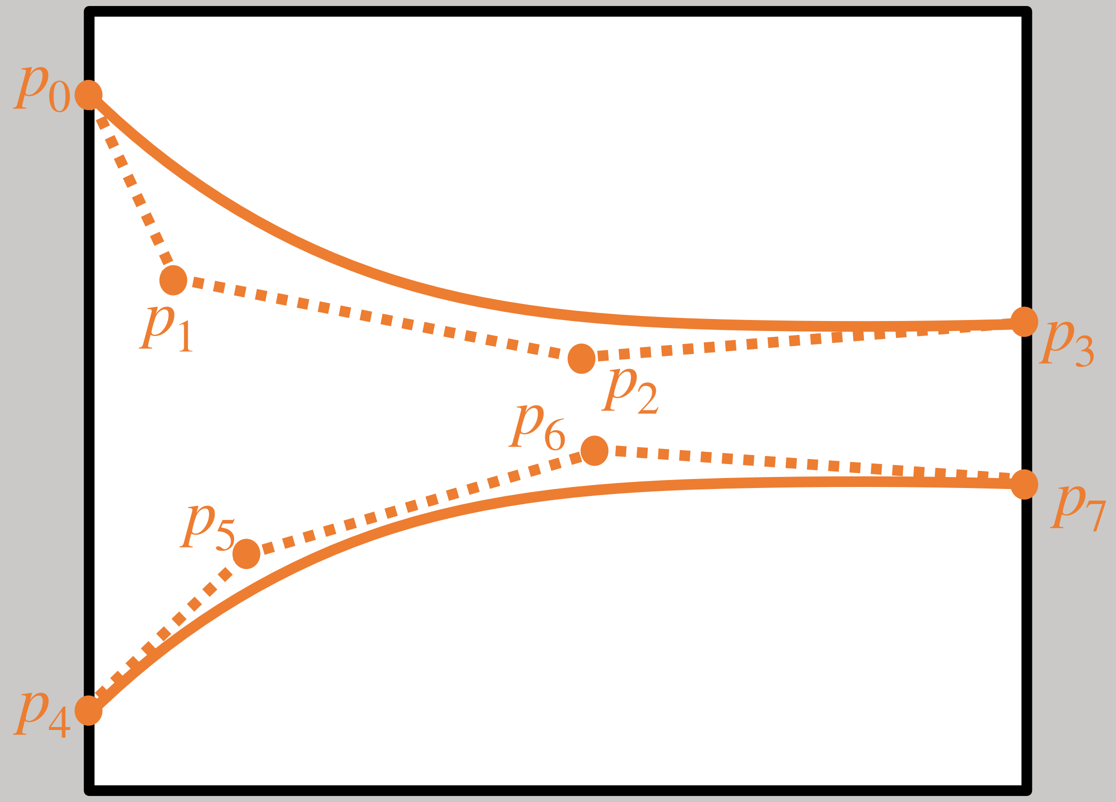
Conventional fluidic device design relies on human expertise

Computational Design with Differentiable Simulation

Challenges in Geometry Representation

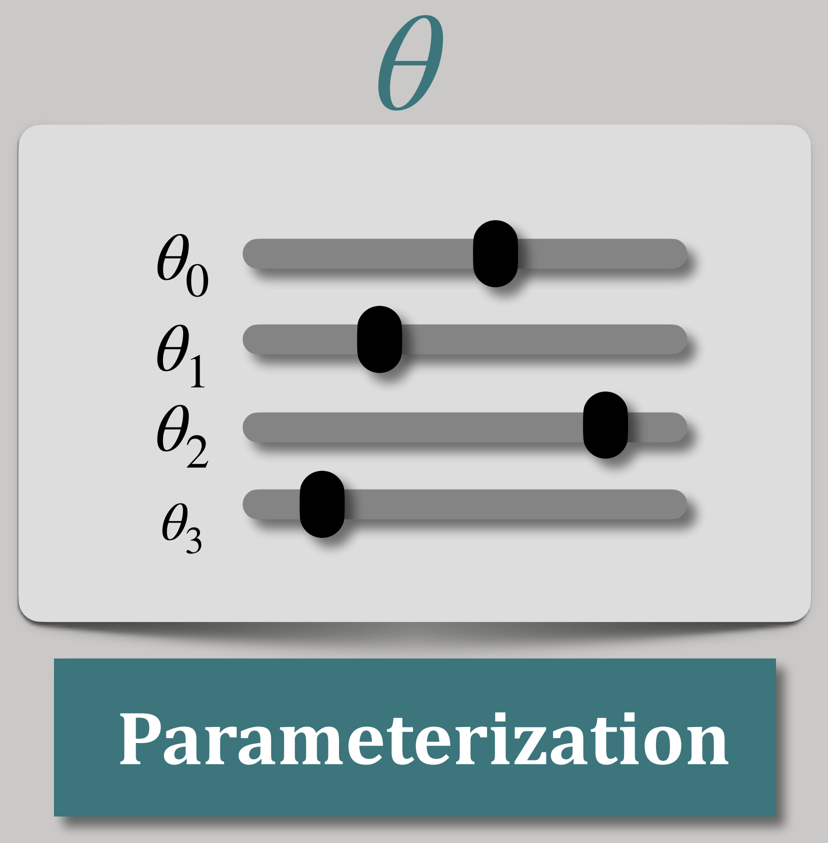


Field-Based



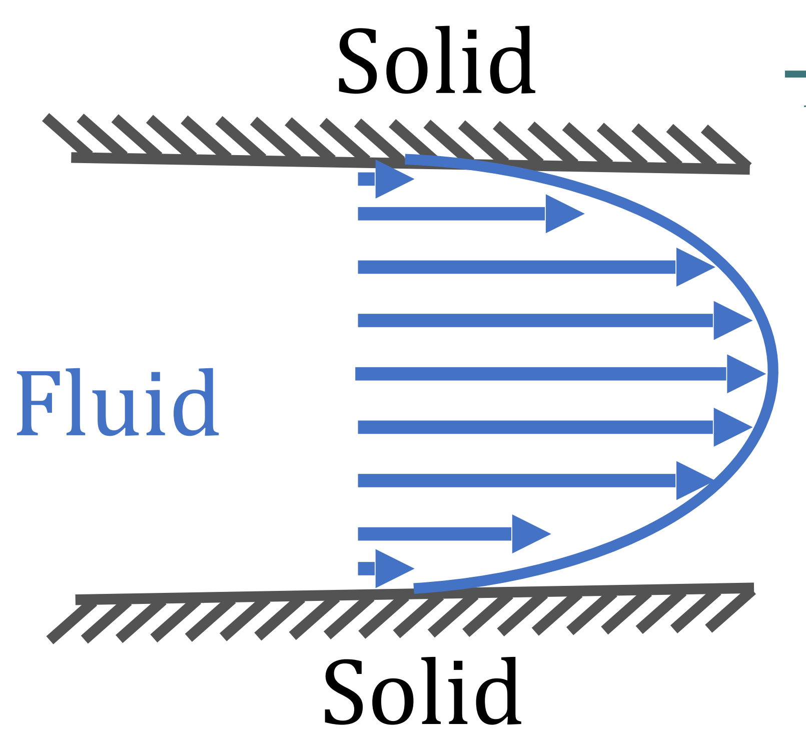
Shape-Based

	Arbitrary Topology	Accurate Boundary
Field-Based	✓	⛔
Shape-Based	⛔	✓



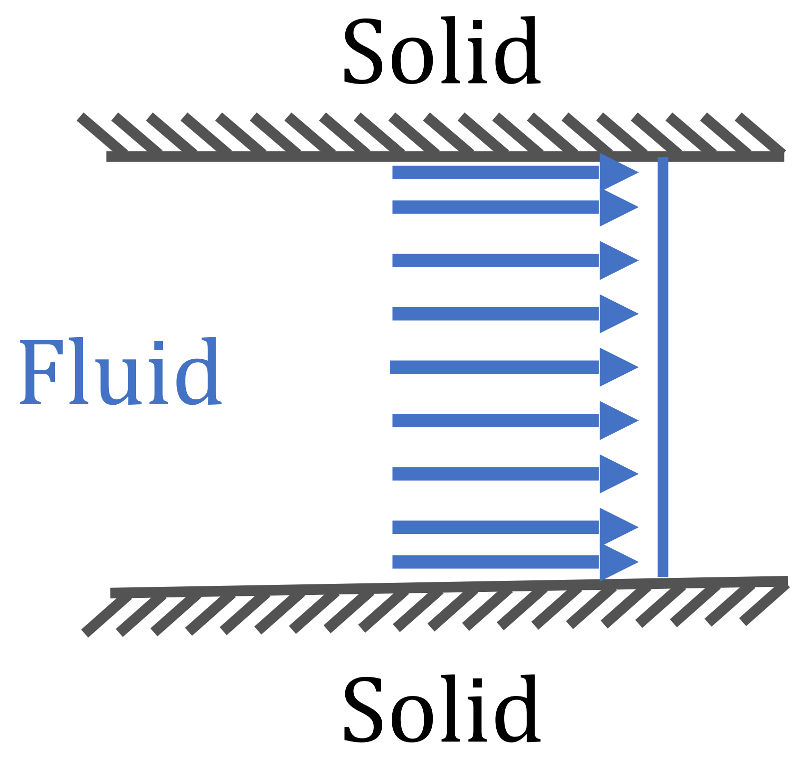
Accurate Simulation at Solid-Fluid Interface Requires Flexible Boundary Condition

No-Slip **model frictional surface**



$\vec{v} = \vec{0}$

Slip **model frictionless boundary**

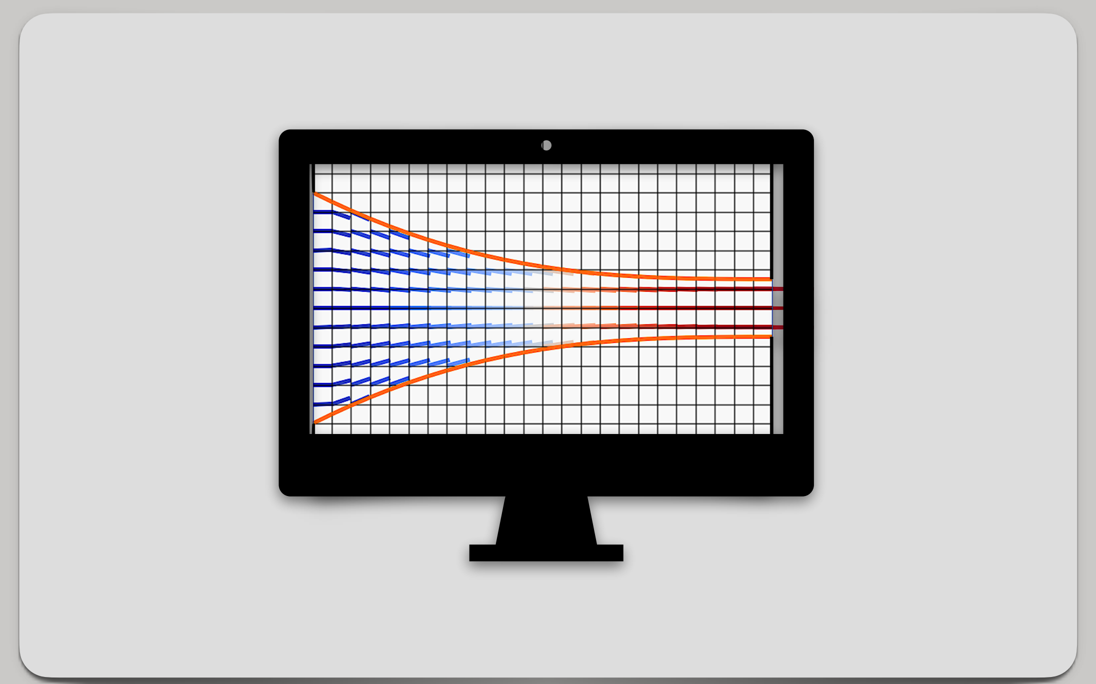


$\vec{v} \perp \vec{n}$

Needed for simulating narrow channels under limited resolution

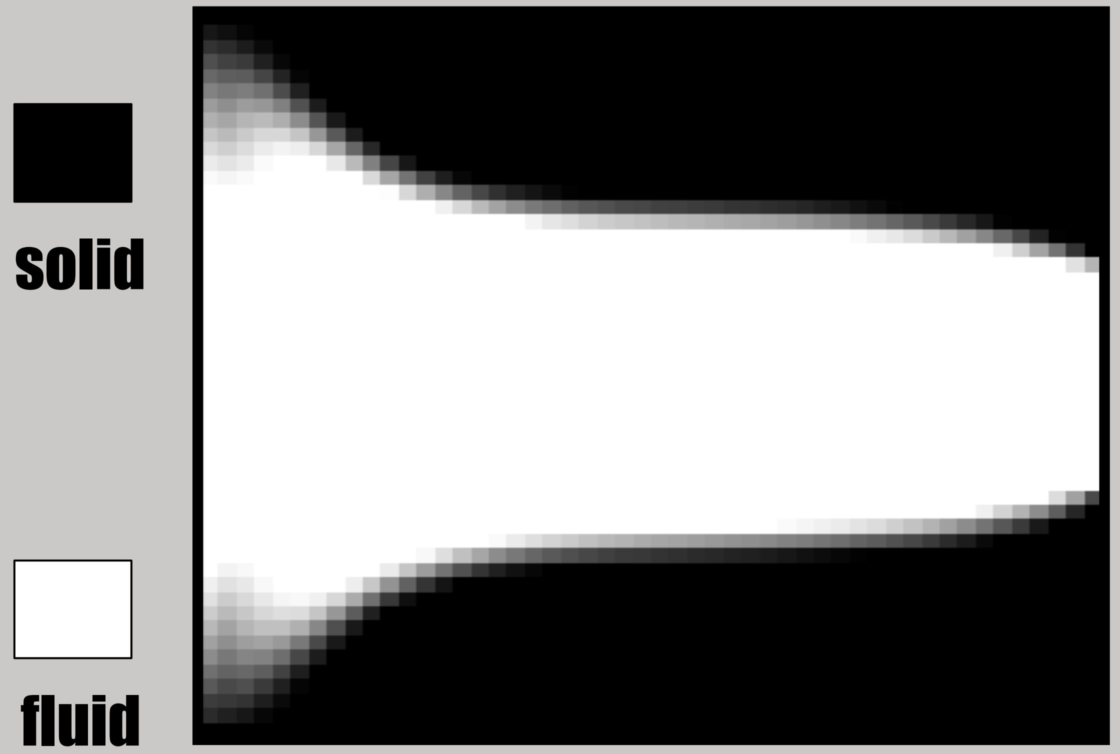
Requires knowledge of local surface normal $\vec{n}$

$\vec{v}$

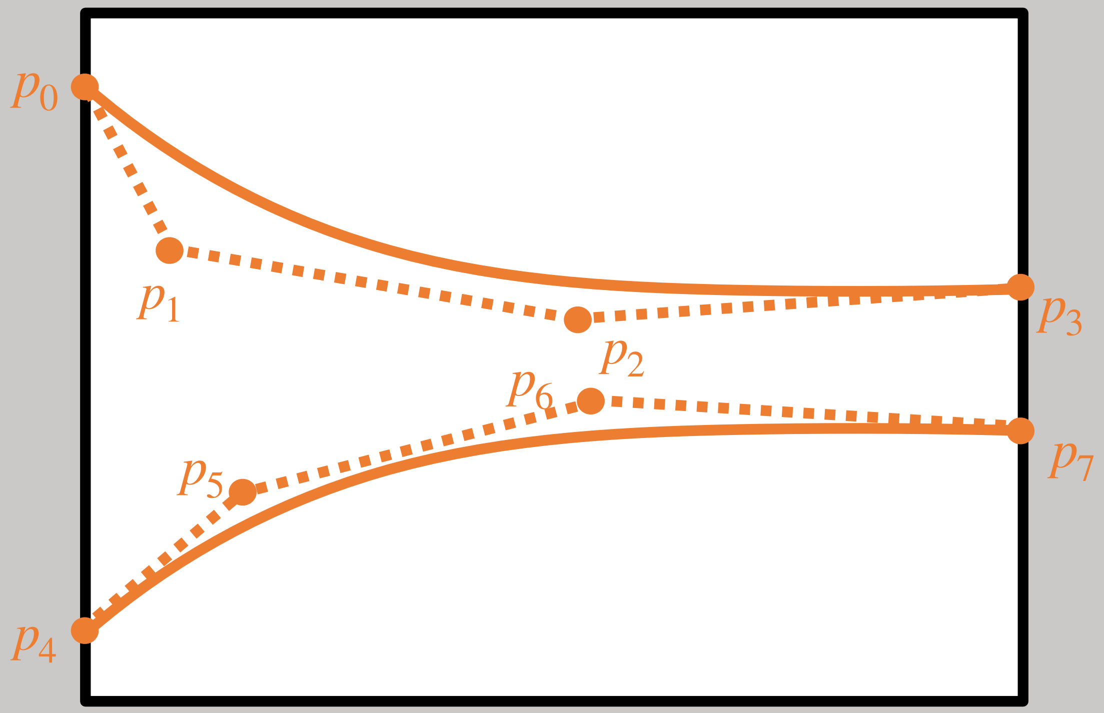


Simulation

Prior Works



Field-Based
[Borrvall et al. 03]



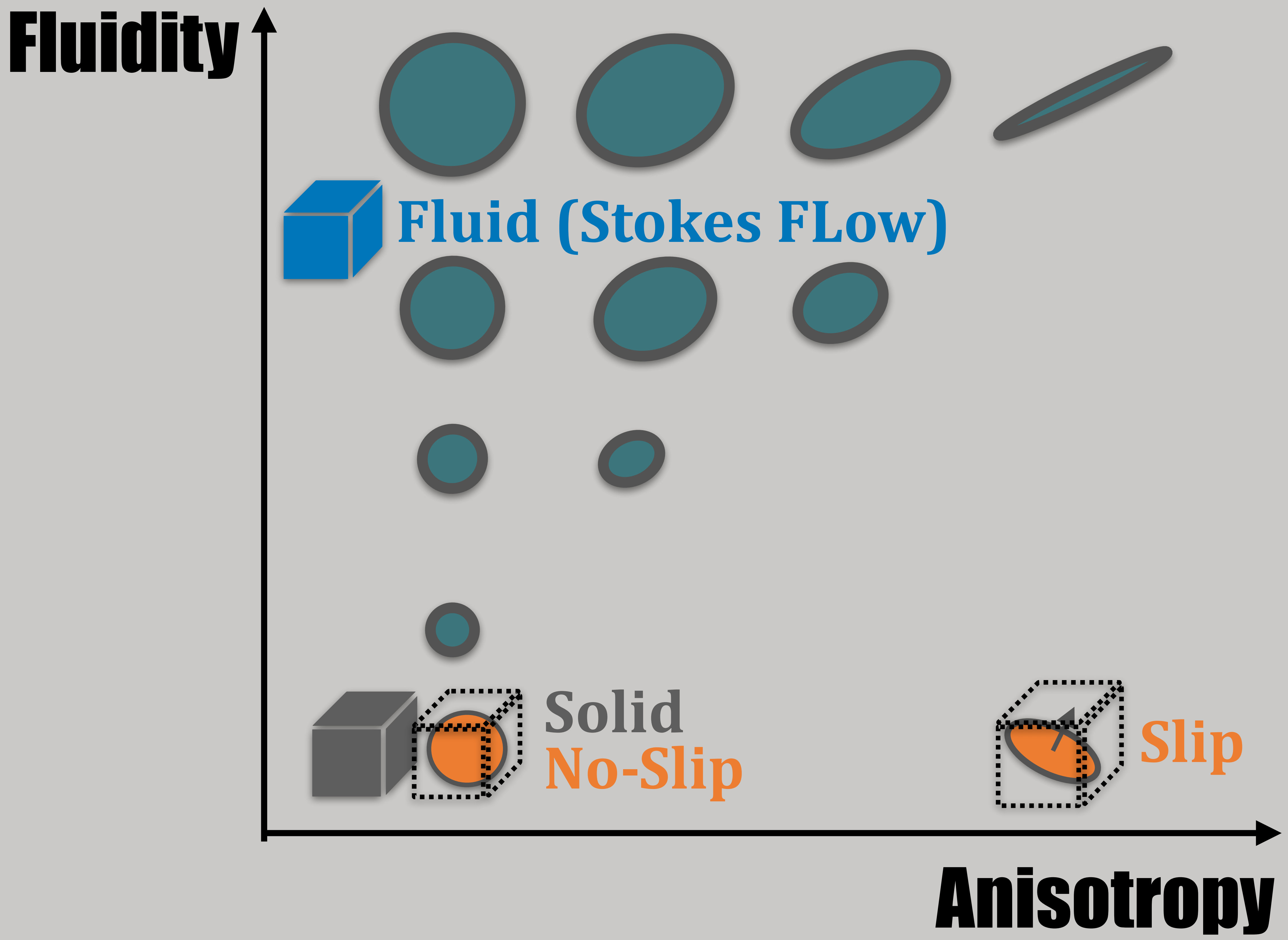
Shape-Based
[Du el al. 20]

	Arbitrary Topology	Accurate Boundary	Flexible Boundary Condition (slip boundary?)
Field-Based [Borrvall et al. 03]	✓	⛔	⛔
Shape-Based [Du el al. 20]	⛔	✓	✓

Topology optimization of fluids in Stokes flow [Borrvall et al. 03]
Functional optimization of fluidic devices with differentiable stokes flow [Du et al. 20]

Contribution

Anisotropic Mixture Model for Continuous & Unified Material Representation

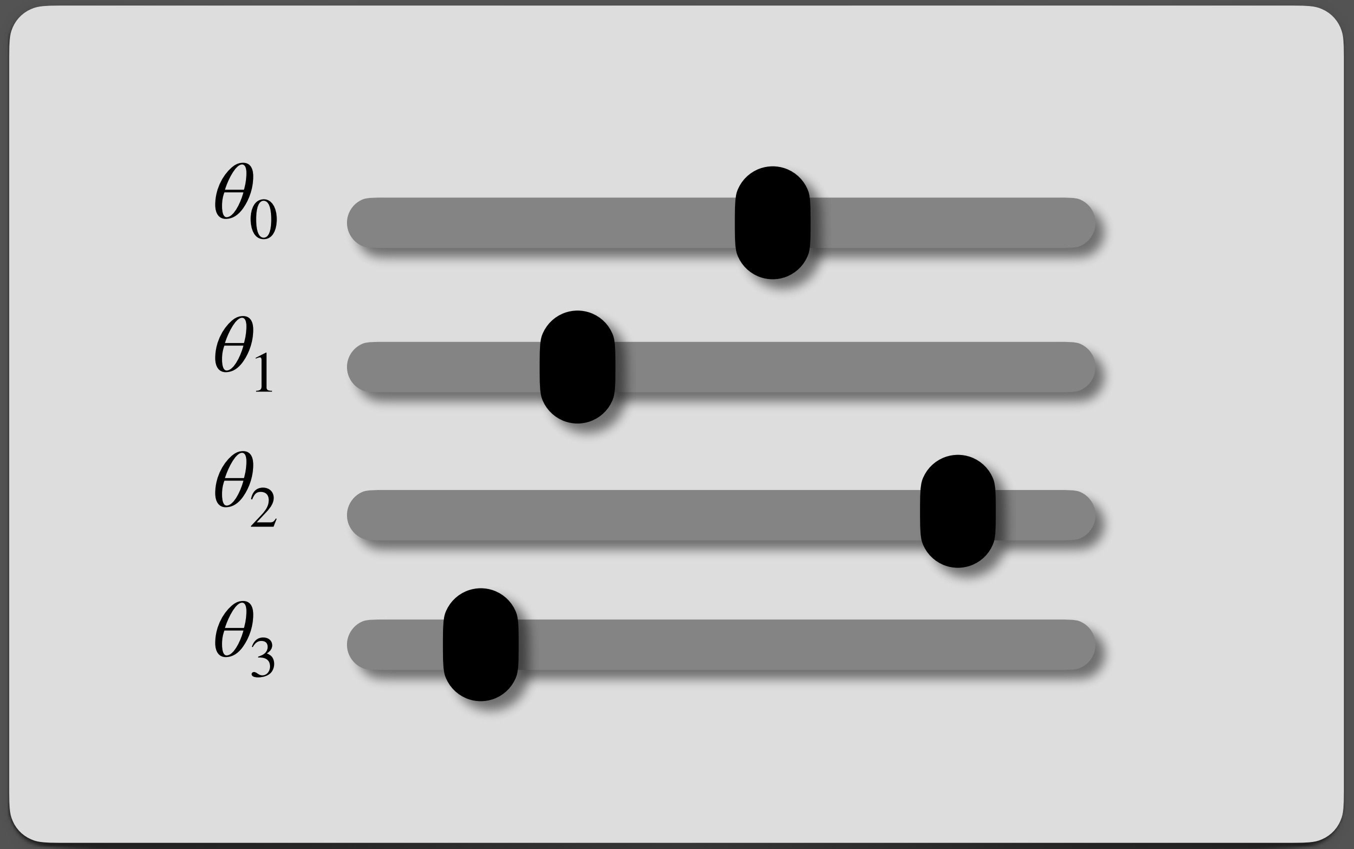


Anisotropic Mixture Model

- Arbitrary Topology ✓
- Sharp Interface ✓
- Flexible Boundary Modeling ✓

Method Outline

Goal: A continuous material model that models solid, fluid and boundary conditions



Anisotropic Mixture Model Parameterizaiton

$$\int \rightarrow \sum$$

Discretization

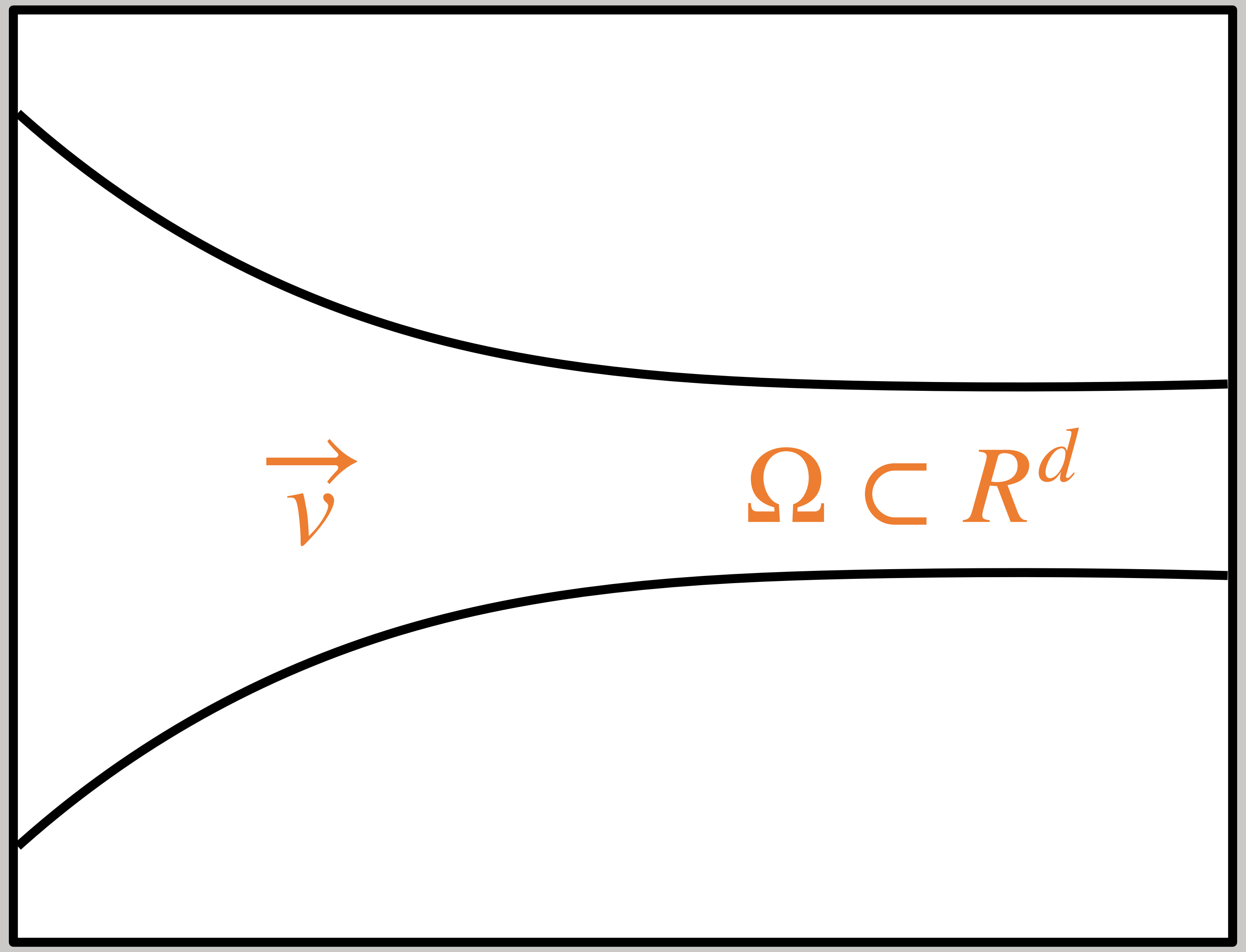
Optimization

Energy Minimization Formulation of Stokes Flow

$$\min_{v} \int_{\Omega} \mu \|\nabla v\|^2 dx + \int_{\Omega} \lambda (\nabla \cdot v)^2 dx$$

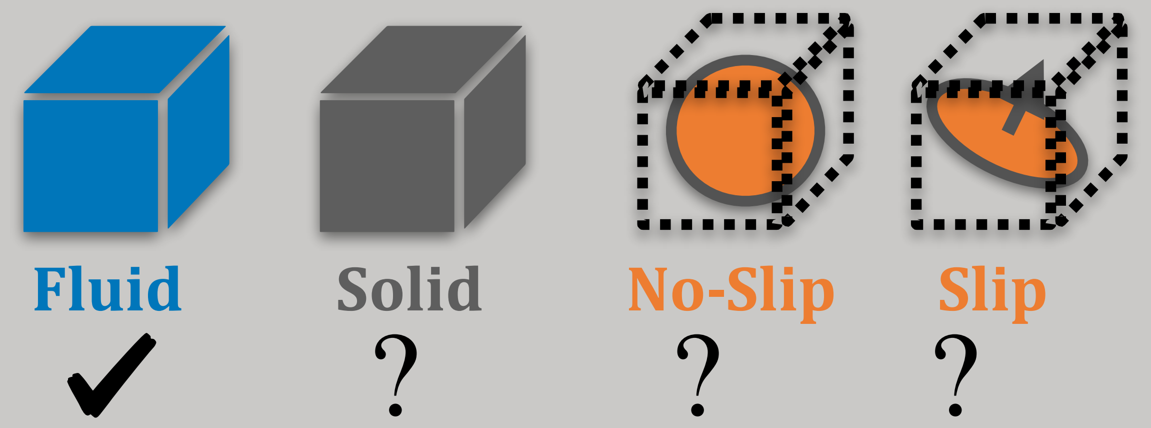
μ Dynamic viscosity λ Incompressibility v velocity

Ω Domain d dimension



No external forces

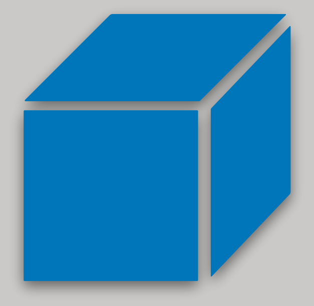
Stokes flow



Energy Minimization Formulation of Stokes Flow

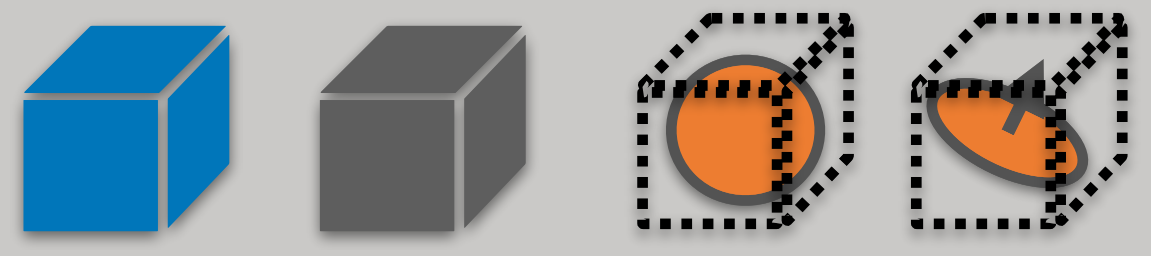
$$\min_{v} \quad \int_{\Omega} \lambda(\nabla \cdot v)^2 dx \; + \; \int_{\Omega} \mu \|\nabla v\|^2 dx$$

incompressibility **viscous shear stress**



Fluid Phase

$\lambda \to \infty$ **perfectly incompressible Stokes flow**

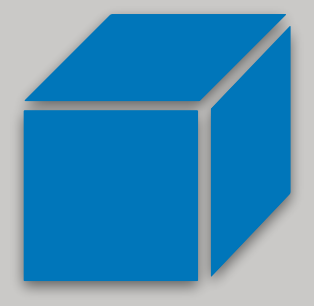


Modeling Solid Phase

Solid as "impermeable fluid"

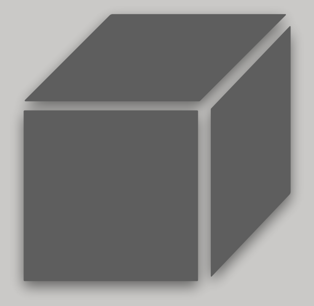
$$\min_{v} \int_{\Omega} \lambda(\nabla \cdot v)^2 dx + \int_{\Omega} \mu \|\nabla v\|^2 dx + \int_{\Omega} k_f \|v\|^2 dx$$

incompressibility **viscous shear stress** **friction**



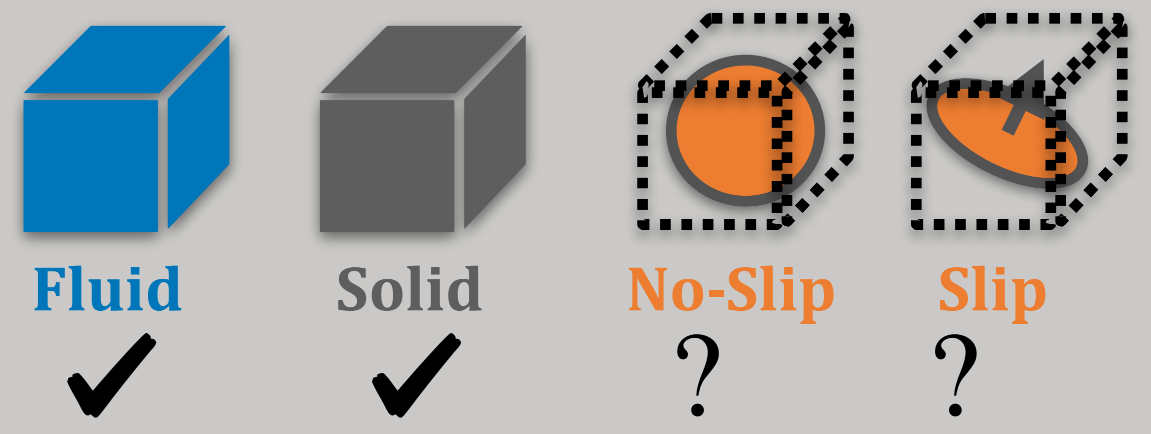
Fluid Phase

$\lambda \to \infty, k_f = 0$



Solid Phase

$k_f \to \infty$ solid as impermeable fluid (v=0)

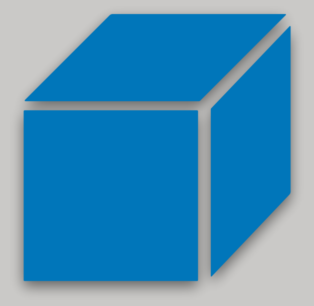


Modeling Solid Phase

Matrix Form Parameterization

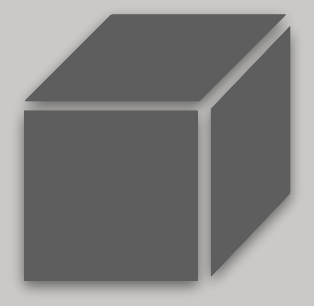
$$\min_{v} \int_{\Omega} \lambda(\nabla \cdot v)^2 dx + \int_{\Omega} \mu \|\nabla v K_m^{\frac{1}{2}}\|^2 dx + \int_{\Omega} \|K_f^{\frac{1}{2}} v\|^2 dx$$

$$K_m \in S_+^d \qquad K_f \in S_+^d$$



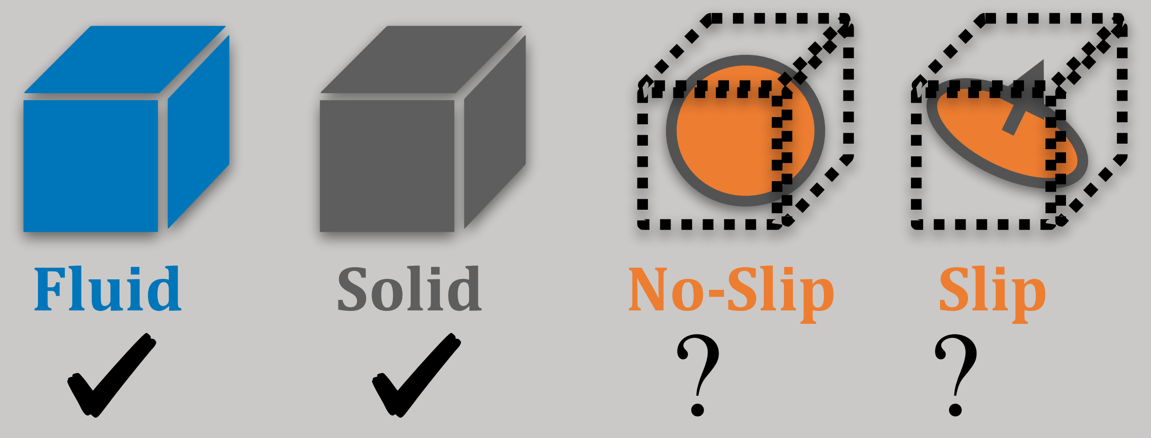
Fluid Phase

$\lambda \to \infty, K_m = I, K_f = 0$



Solid Phase

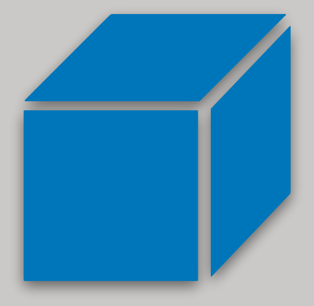
$K_f = \infty \cdot I$



Modeling No-Slip Boundary Condition

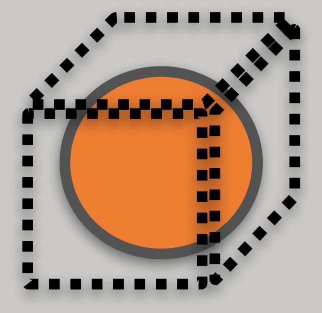
No-Slip shares parametrization with Solid

$$\min_{v} \int_{\Omega} \lambda(\nabla \cdot v)^2 dx + \int_{\Omega} \mu \|\nabla v K_m^{\frac{1}{2}}\|^2 dx + \int_{\Omega} \|K_f^{\frac{1}{2}} v\|^2 dx$$



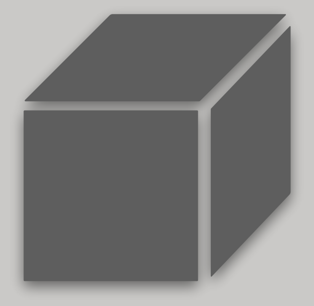
$\lambda \to \infty, K_m = I, K_f = 0$

Fluid Phase



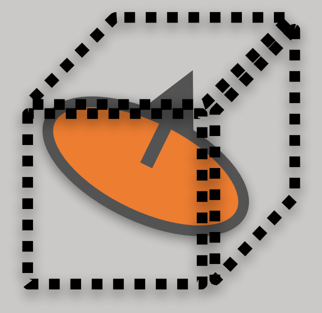
$K_f = \infty \cdot I$

No-Slip ($v = 0$)



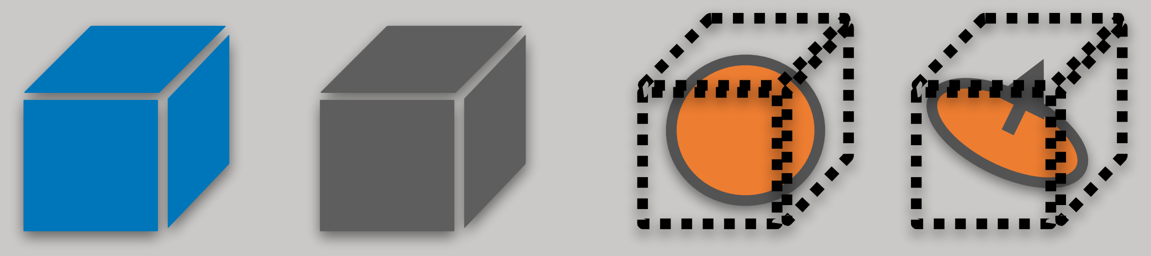
$K_f = \infty \cdot I$

Solid Phase



?

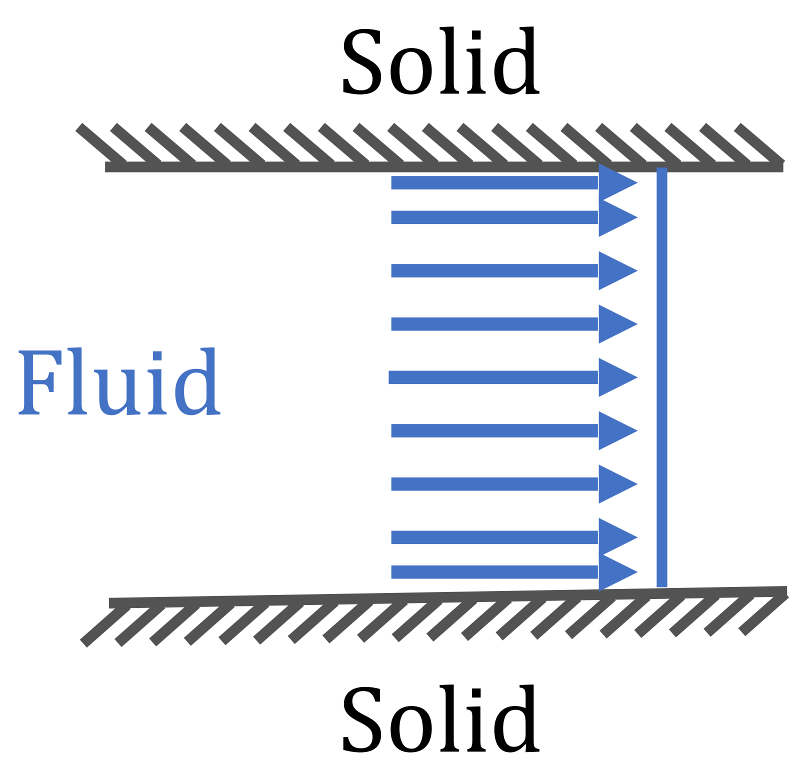
Slip ($v \perp n$)



Modeling Slip Boundary Condition

Anisotropic Modeling of Slip

$$\min_{v} \int_{\Omega} \lambda(\nabla \cdot v)^2 dx + \int_{\Omega} \mu \| \nabla v K_m^{\frac{1}{2}} \|^2 dx + \int_{\Omega} \| K_f^{\frac{1}{2}} v \|^2 dx$$



$\vec{v} \perp \vec{n}$

Anisotropic Behavior

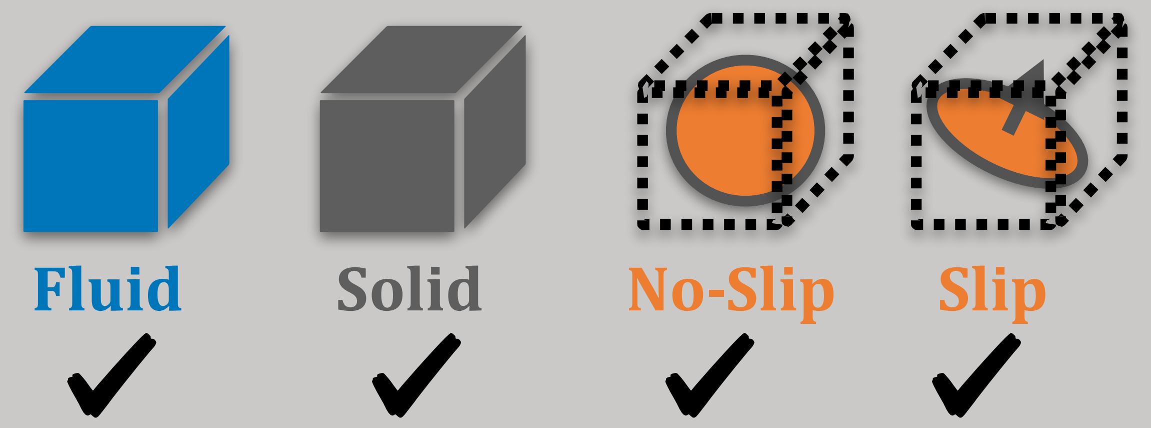
Tangential direction: Stokes flow
Normal Direction: Impermeable flow (solid)

(Sec 4.2 for derivation)

$K_m = I - nn^T$

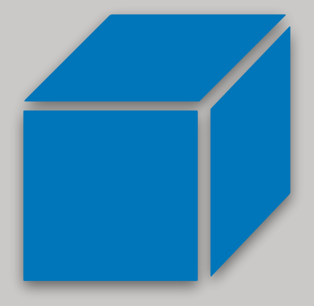
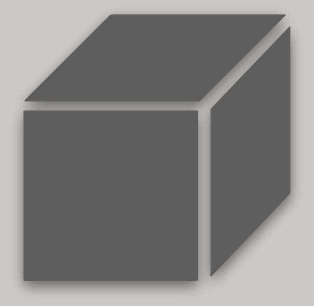
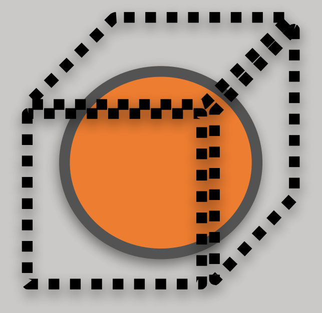
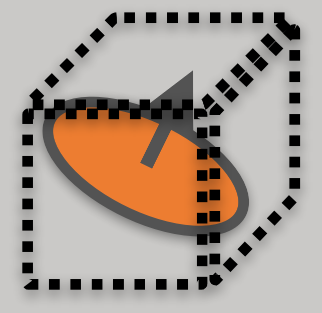
$K_f = k_f nn^T, k_f \to \infty$

$\lambda = 0$



Unified Material Model for All Phases

Over Parameterization of Direct Representation

	Fluid Phase	Solid Phase	No-Slip ($v = 0$)	Slip ($v \perp n$)	
$\mathbf{K}_m$ Shear Stress	I	I	I	$I - nn^T$	n boundary normal
$\mathbf{K}_f$ Friction	0	$k_f I$	$k_f I$	$k_f nn^T$	$k_f \to \infty$ max friction
λ Incompressibility	λ_0	λ_0	λ_0	0	$\lambda_0 \to \infty$ flow incompressibility

Direct Parameterization (d=3)

$\mathbf{K}_m, \mathbf{K}_f \in \mathbf{S}_+^{d\times d}$	$\lambda, k_f \in \mathbb{R}^+$	$n \in \mathbb{R}^d$
12	2	3

Reparameterization

?

Reparameterization Intuition

	Fluid Phase	Solid Phase	No-Slip ($v = 0$)	Slip ($v \perp n$)
$\mathbf{K}_m$ Shear Stress	I	I	I	$I - nn^T$
$\mathbf{K}_f$ Friction	0	$k_f I$	$k_f I$	$k_f nn^T$
λ Incompressibility	λ_0	λ_0	λ_0	0
	fluid (zero friction)	**solid (max friction)**		**$\vec{n}$: boundary normal**
	Isotropic			**Anisotropic**

Low Dimension Parameterization

	Fluid Phase	Solid Phase	No-Slip ($v = 0$)	Slip ($v \perp n$)
$\rho \in [0,1]$ **Fluidity**	1	0	0	1
$\epsilon \in [0,1]$ **Isotropy**	1	1	1	0
$\alpha \in \mathbb{R}^{d-1}$ **Anisotropic orientation**	/	/	/	$\vec{n}(\alpha)$

Reparameterization (d=3)

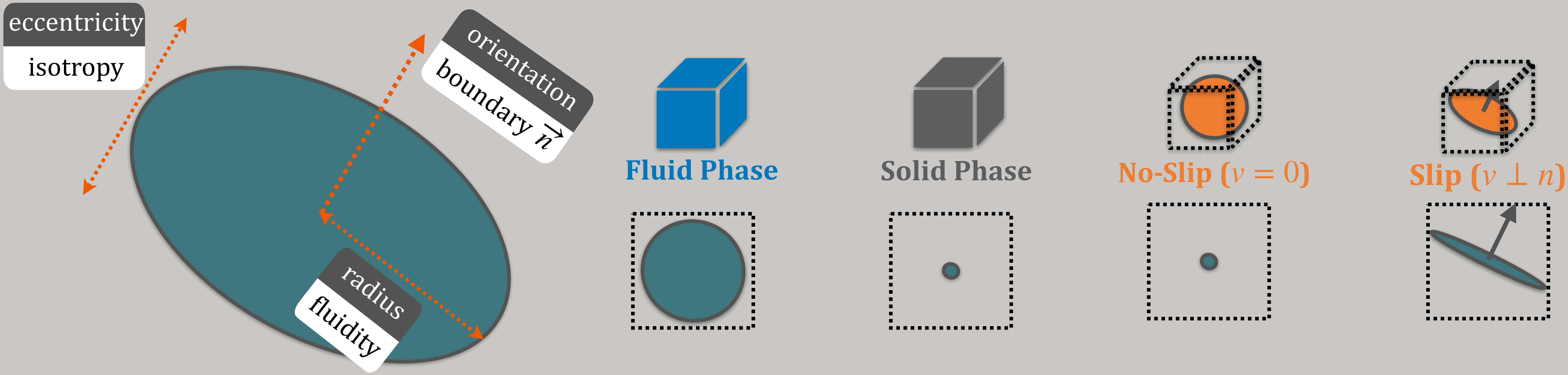
$\rho \in [0,1]$	$\epsilon \in [0,1]$	$\alpha \in \mathbb{R}^{d-1}$
1	1	2

interpolation functions (Sec 5.1) →

Direct Parameterization (d=3)

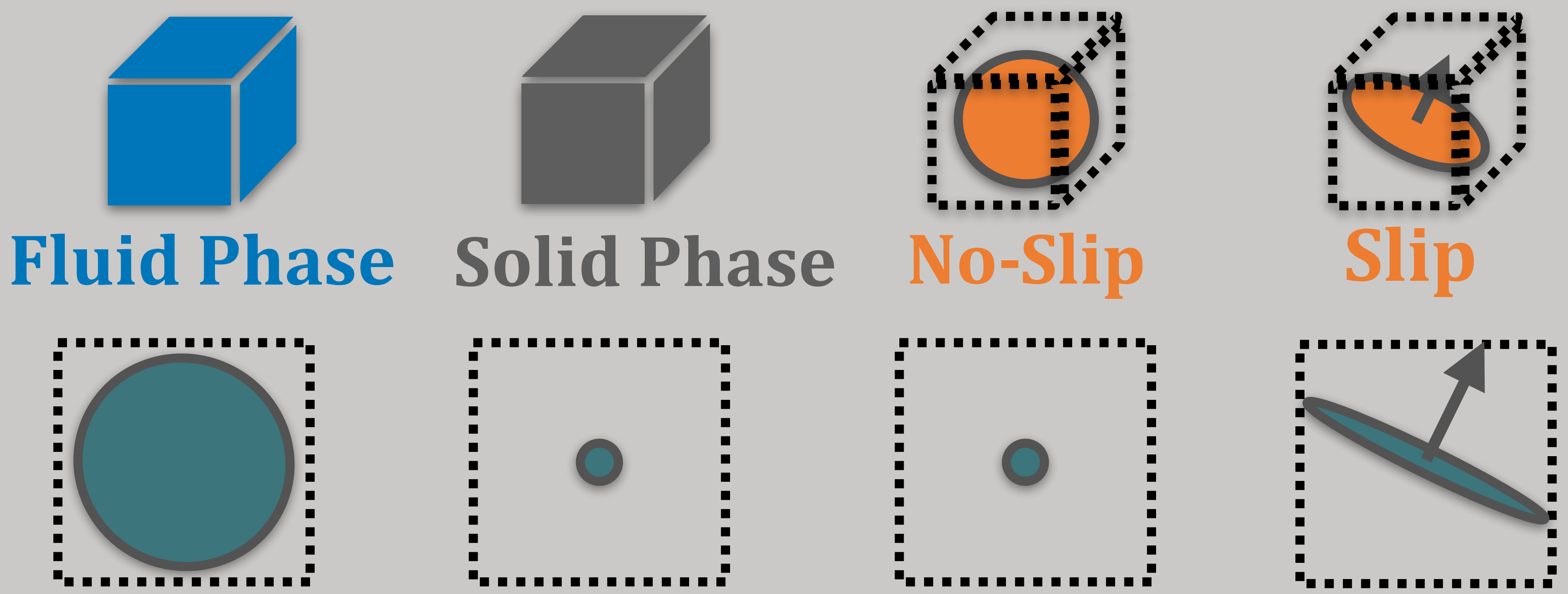
$\mathbf{K}_m, \mathbf{K}_f \in \mathbf{S}_+^{d\times d}$	$\lambda, k_f \in \mathbb{R}^+$	$n \in \mathbb{R}^d$
12	2	3

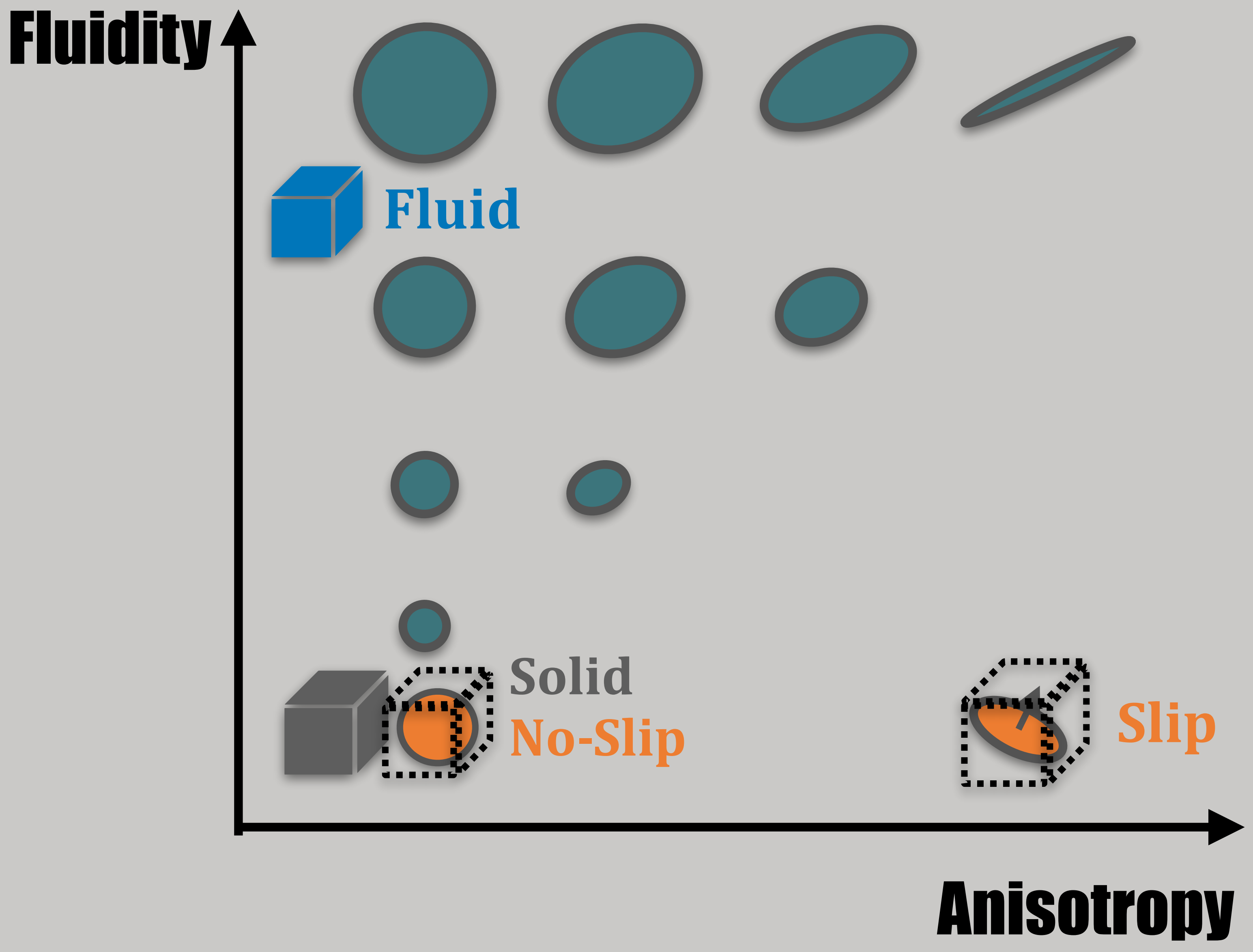
Parameter Visualization as Ellipses

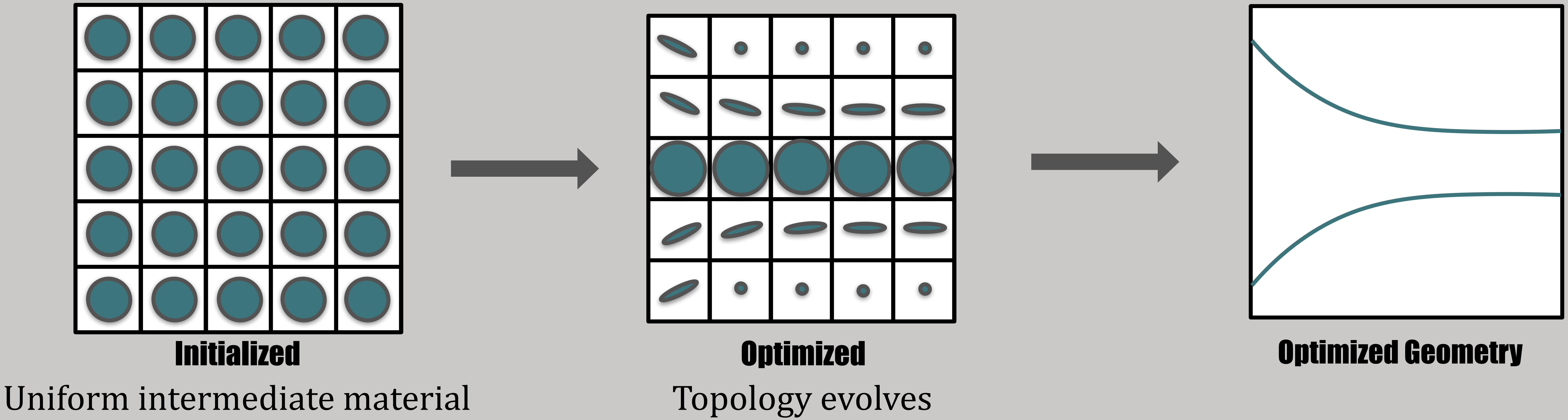


Continuous Representation

Differentiability in Phase Transitions

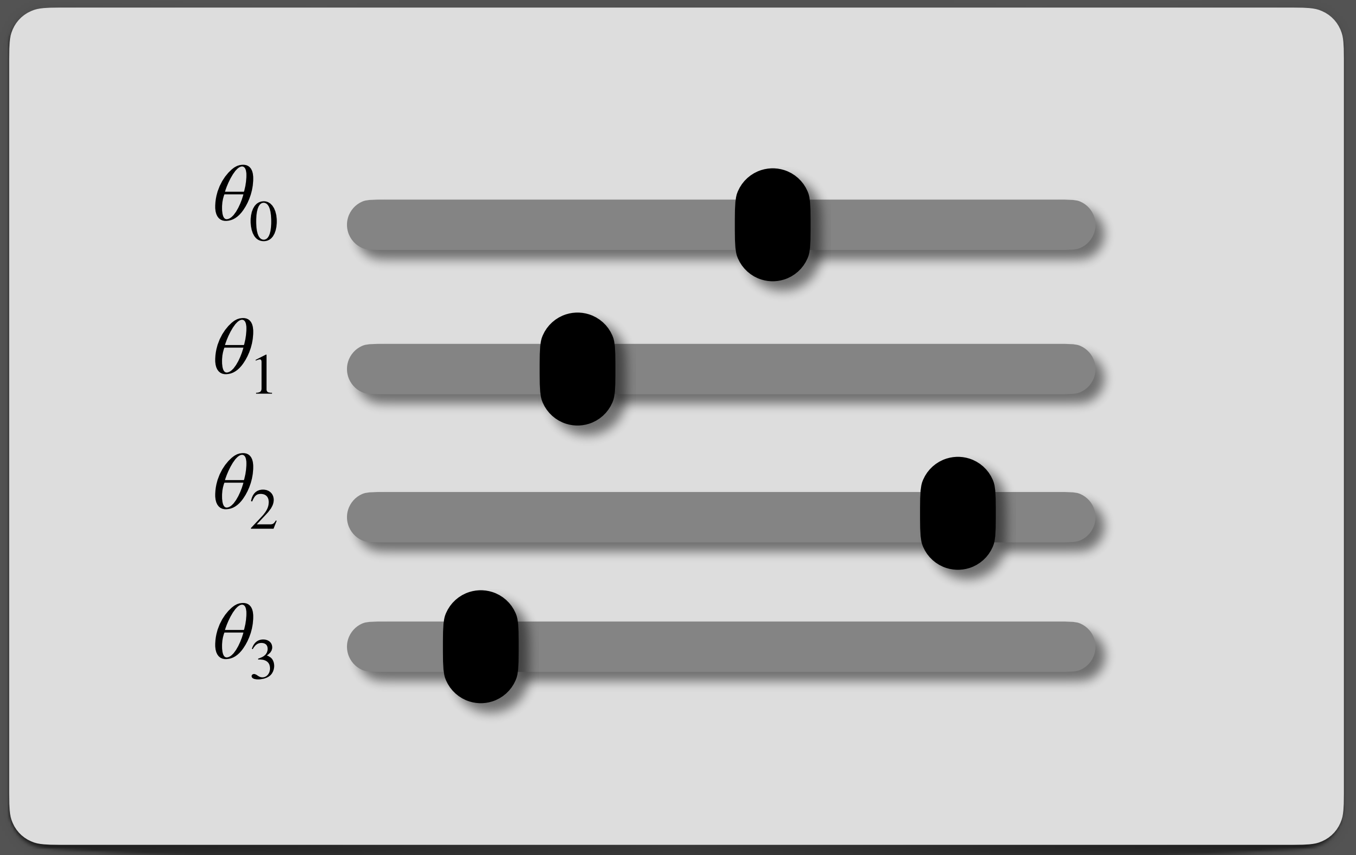






Method Outline

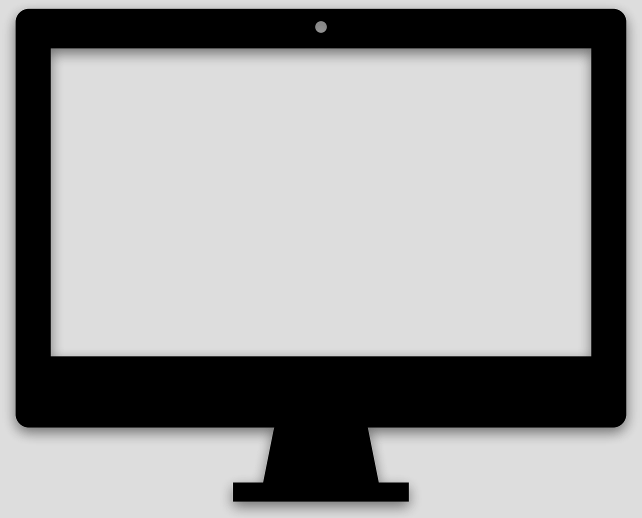
Goal: Develop a physical model that jointly models solid, fluid and boundary



Anisotropic Mixture Model Parameterizaiton

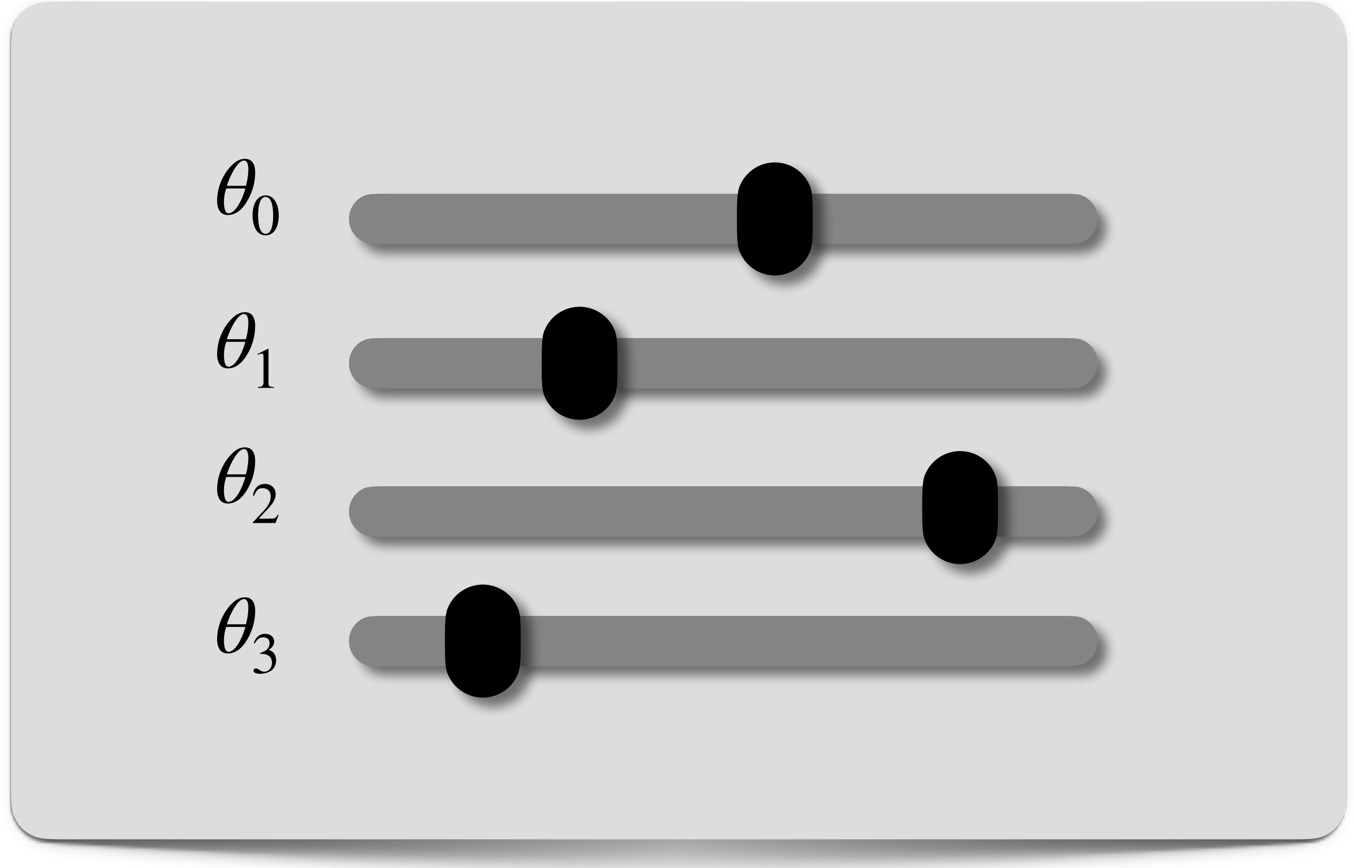
$$\int \to \Sigma$$

Discretization



Optimization

Discretization



Anisotropic Mixture Model Parameterizaiton

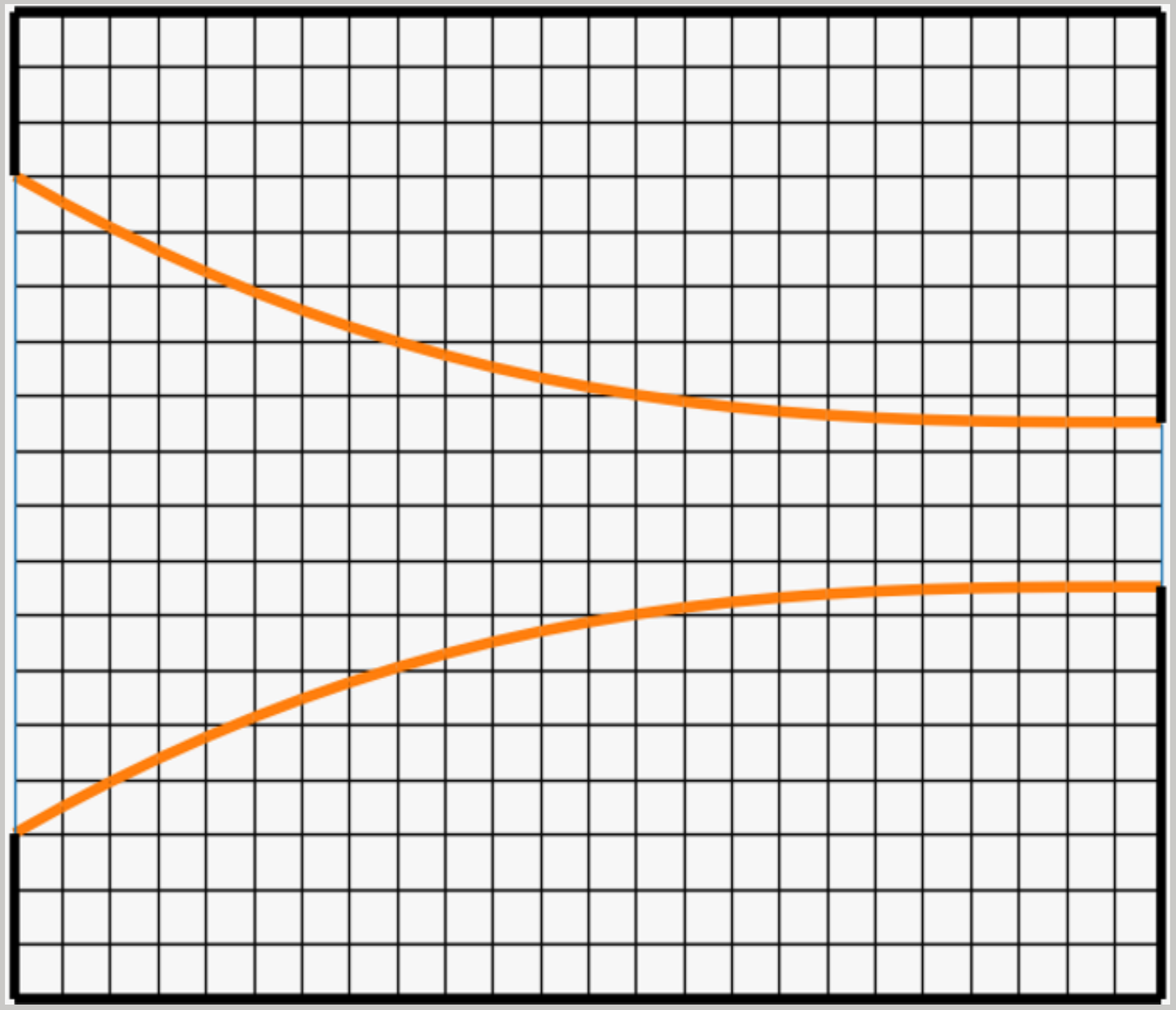
$$\int \to \sum$$

Discretization

Optimization

Discretization Schemes

Domain	Eulerian grid
Values	$\vec{v}$ at grid nodes
Parameter	(ρ, ϵ, α) at cells
Subcell	Gaussian quadrature



Discretization: Energy

Discretization via variational form

$$\min_v \int_\Omega \lambda(\nabla \cdot v)^2 dx + \int_\Omega \mu \|\nabla v K_m^{\frac{1}{2}}\|^2 dx + \int_\Omega \|K_f^{\frac{1}{2}} v\|^2 dx$$

Quadratic energy form (Sec 5.2)

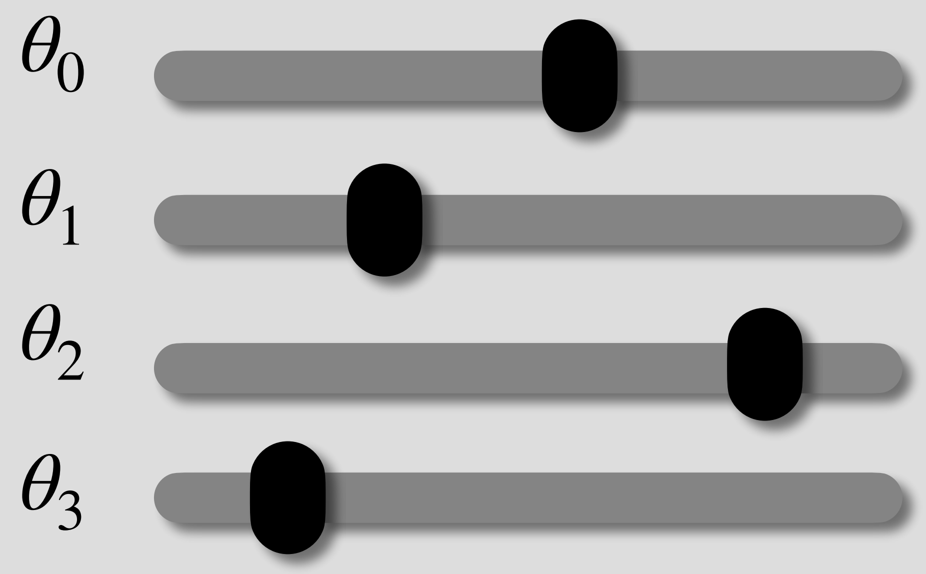
$$\min_v \; v^T K v - b^T v \qquad \text{s.t.} \; Cv = 0$$

KKT

$$\begin{pmatrix} K & C^T \\ C & 0 \end{pmatrix} \begin{pmatrix} v \\ q \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

Sensitivity Analysis

Optimization



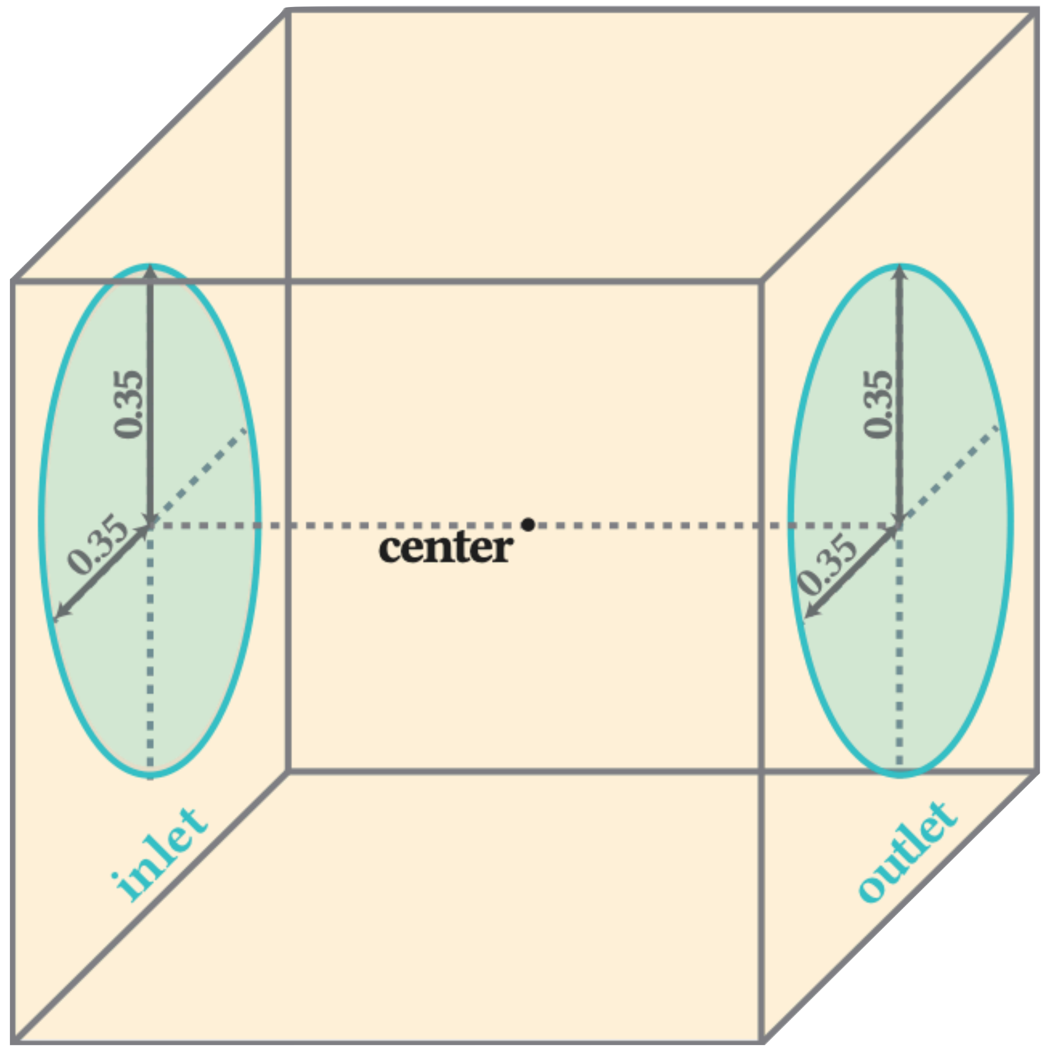
Anisotropic Mixture Model Parameterizaiton

$$\int \to \sum$$

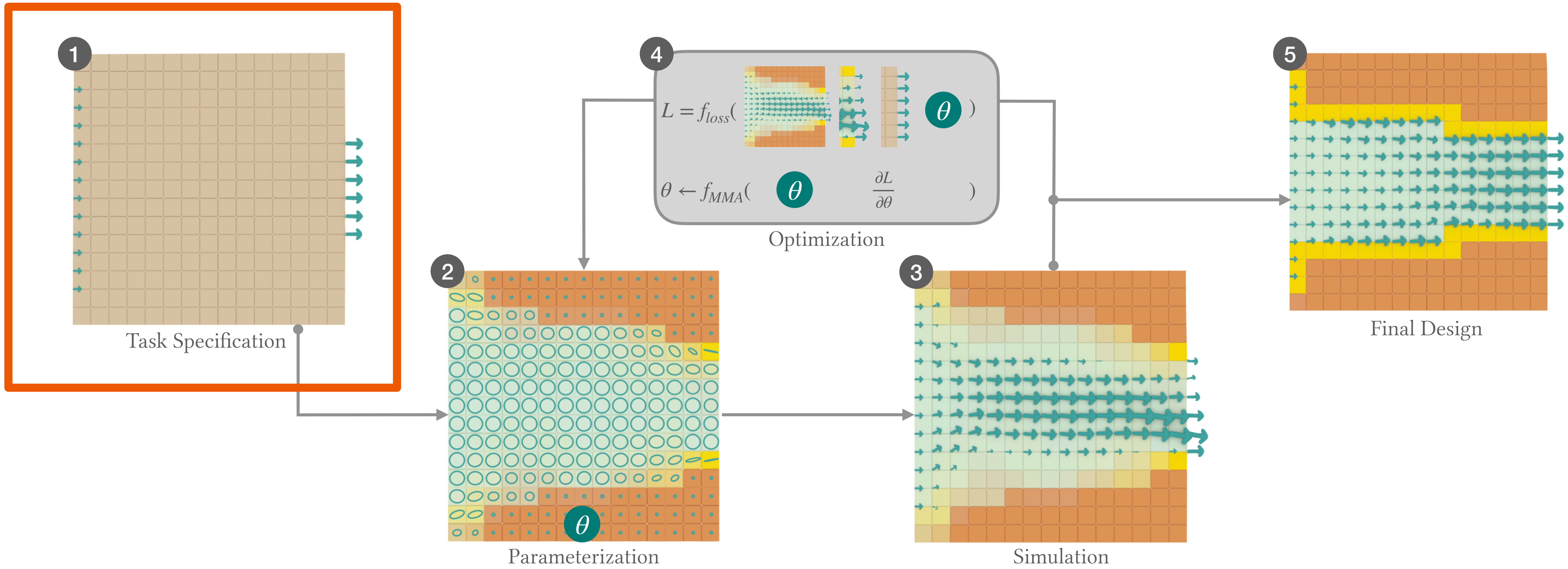
Discretization

Optimization

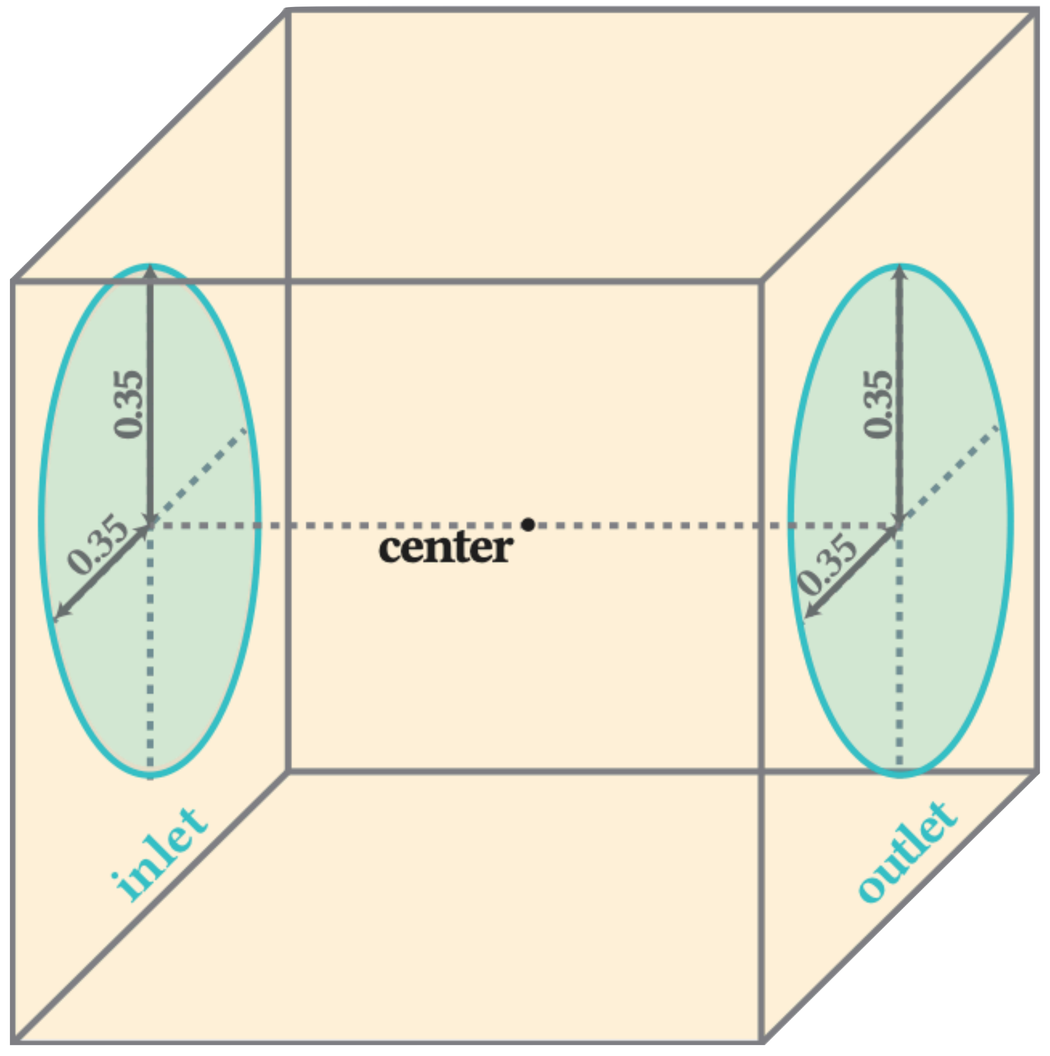
Optimization Pipeline



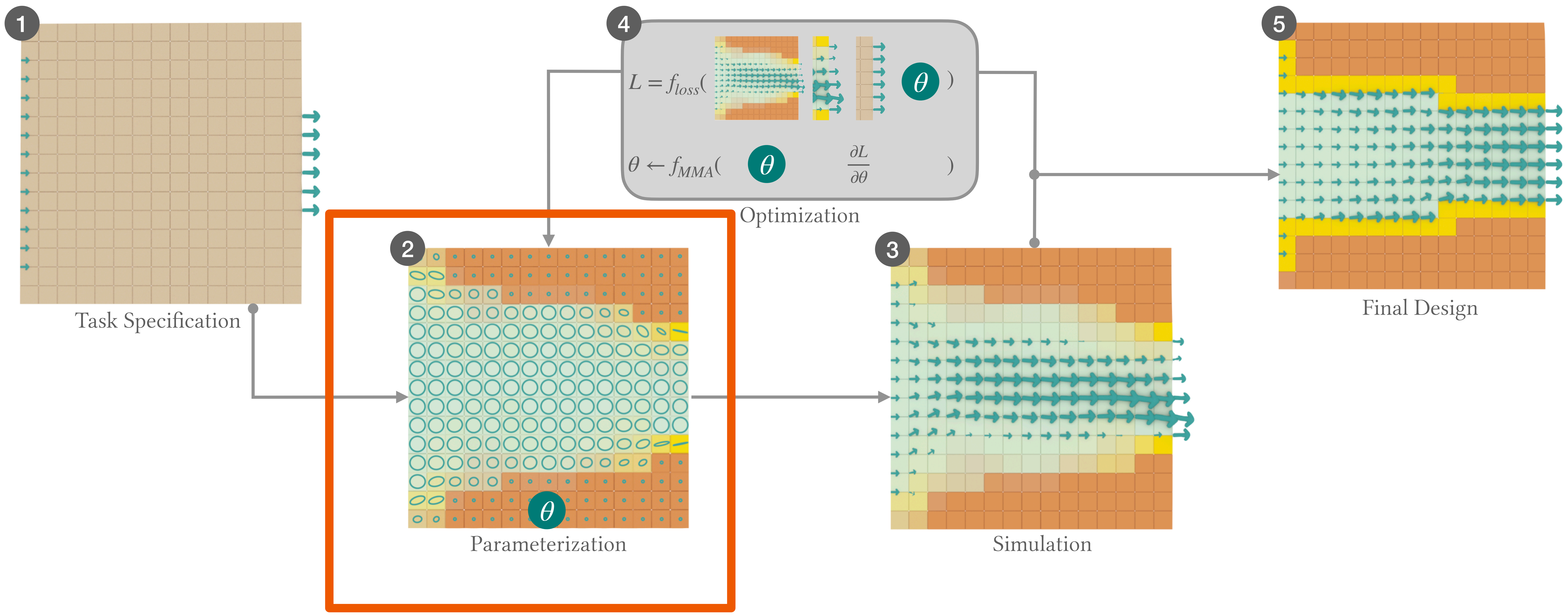
Task Specification



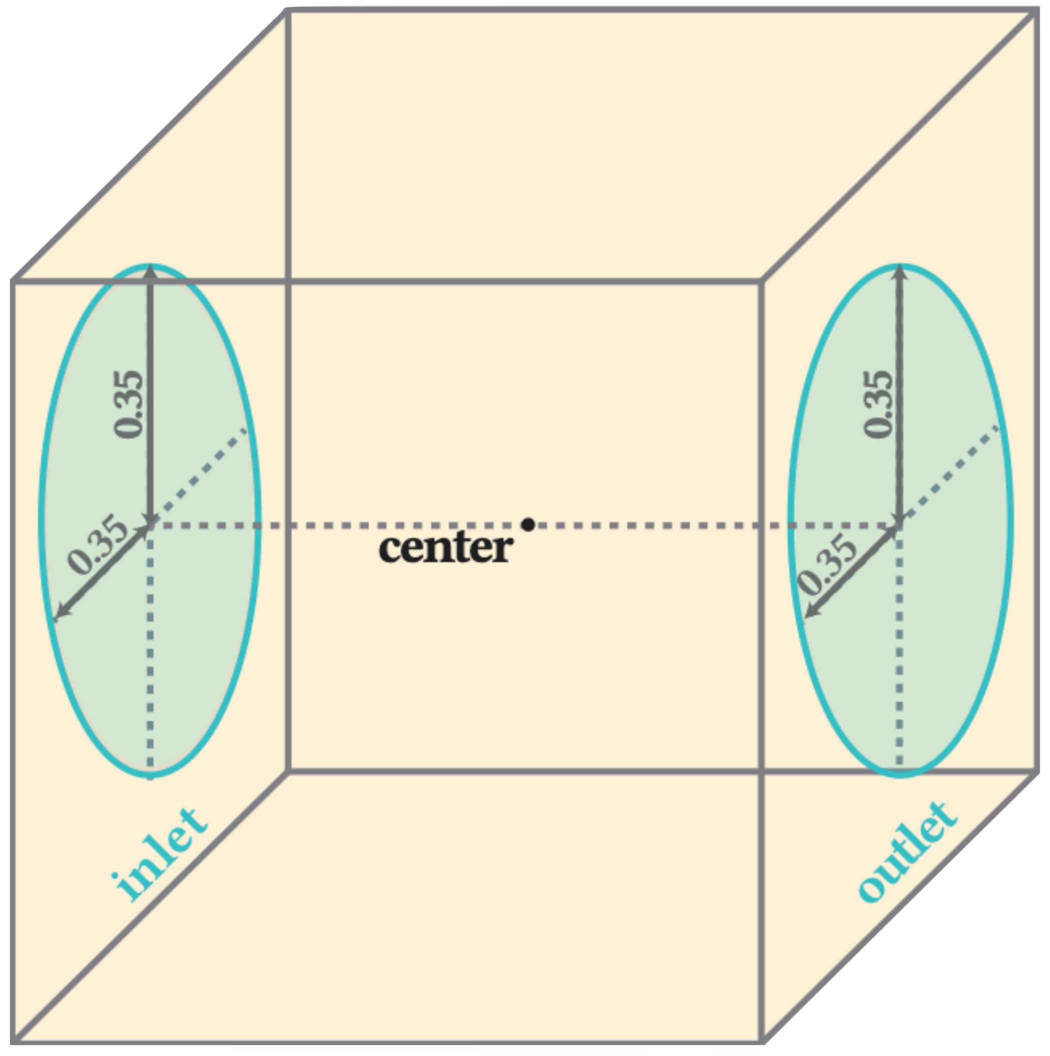
Optimization Pipeline



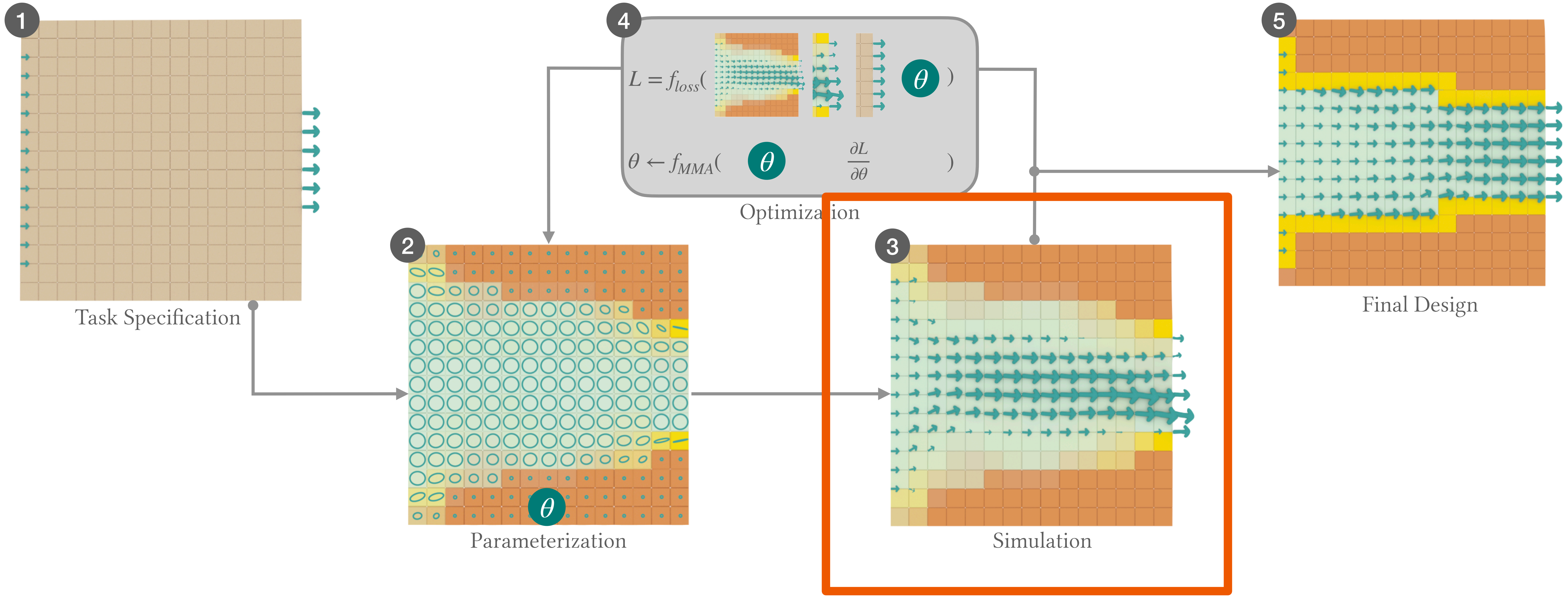
Task Specification



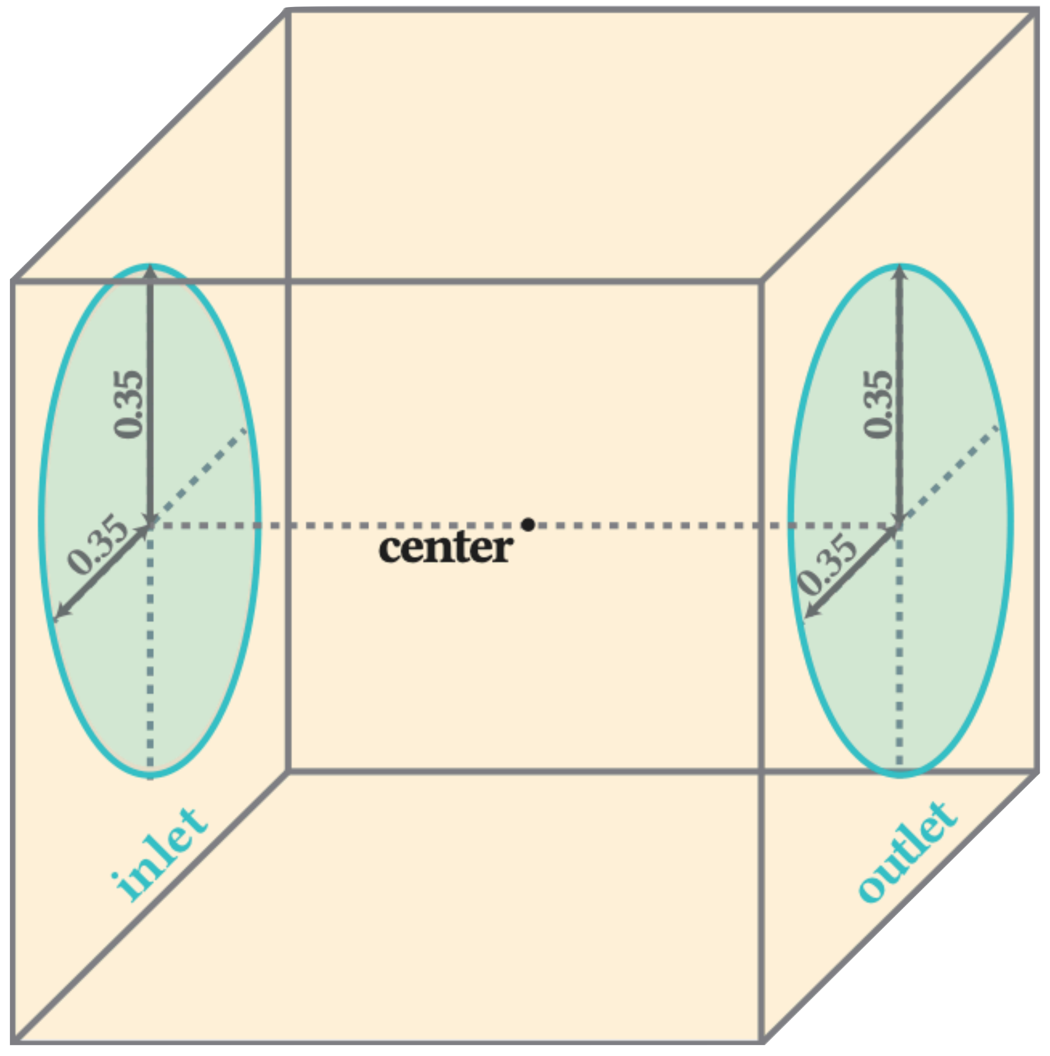
Optimization Pipeline



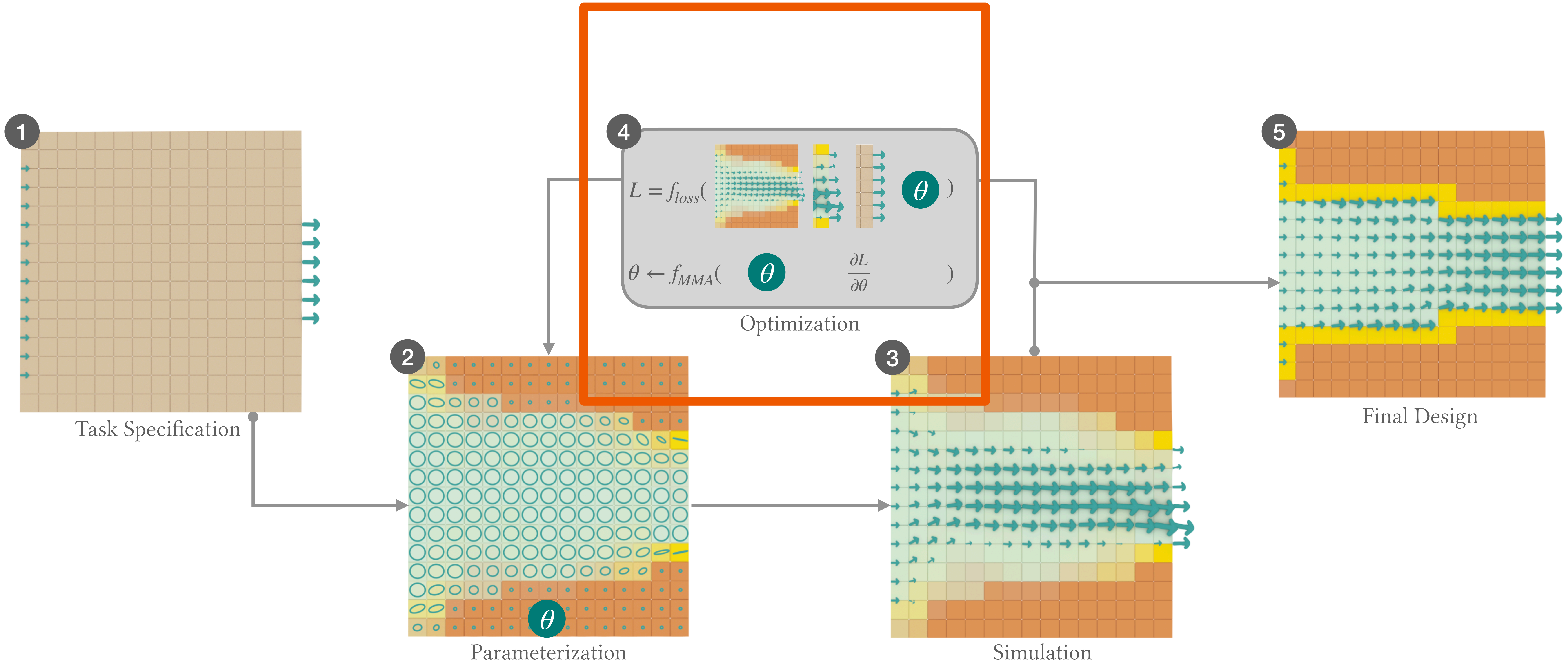
Task Specification



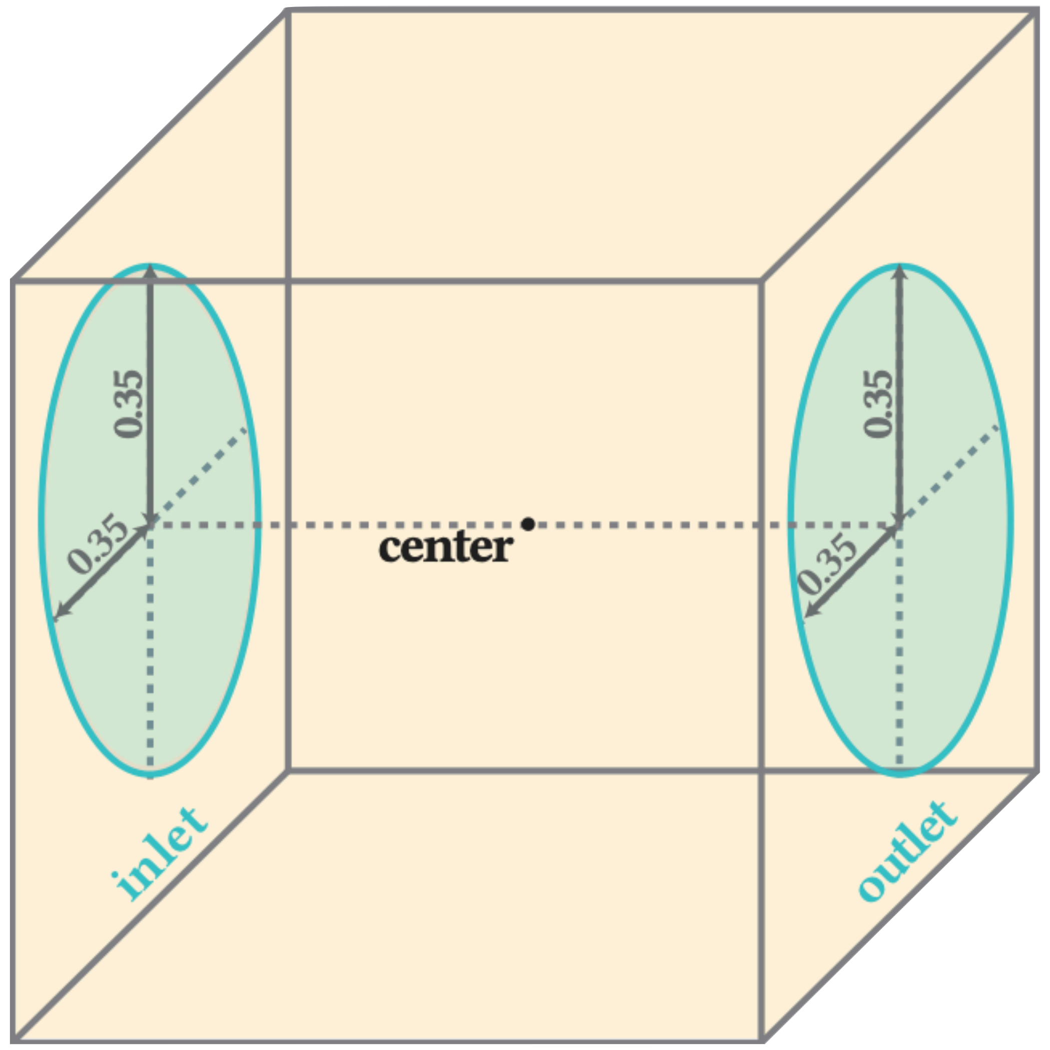
Optimization Pipeline



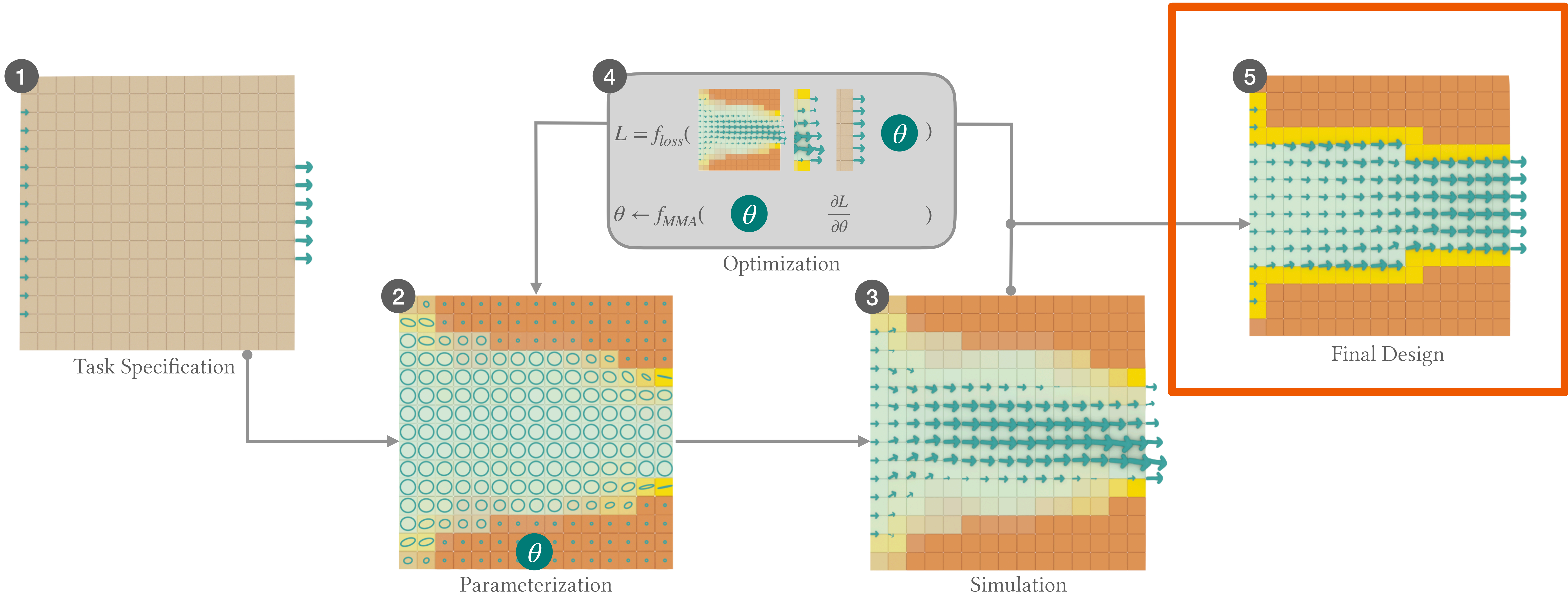
Task Specification



Optimization Pipeline



Task Specification



Optimization

$$\min_{\theta} \; L_f + kL_r$$

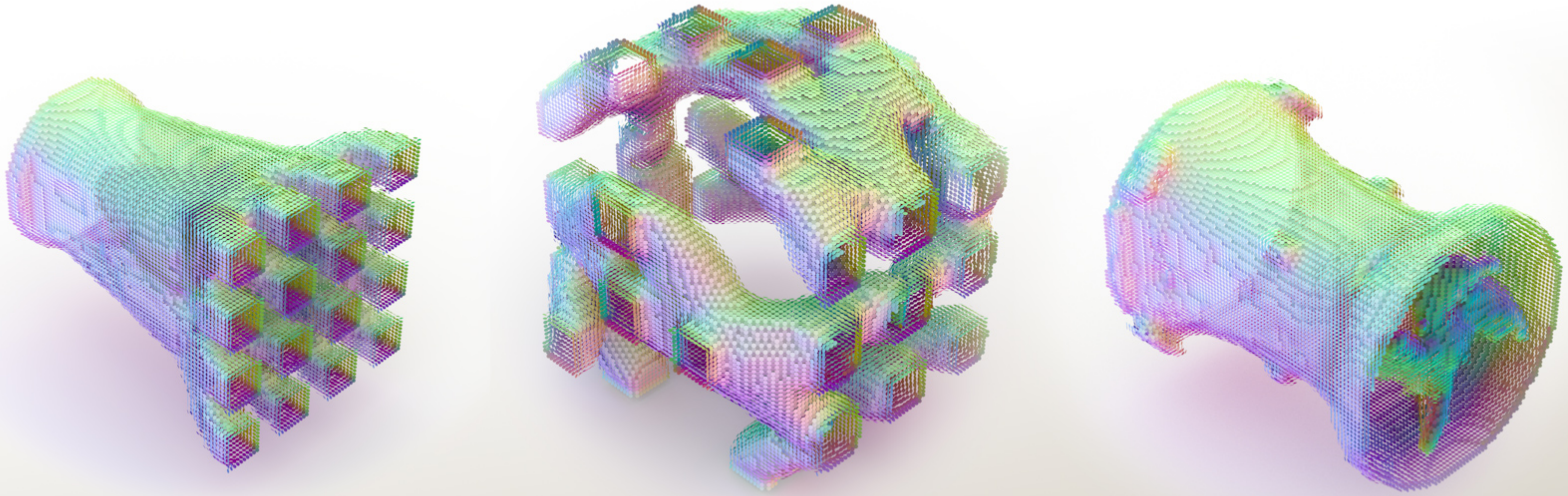
subject to

$$V \leq V_{frac}$$

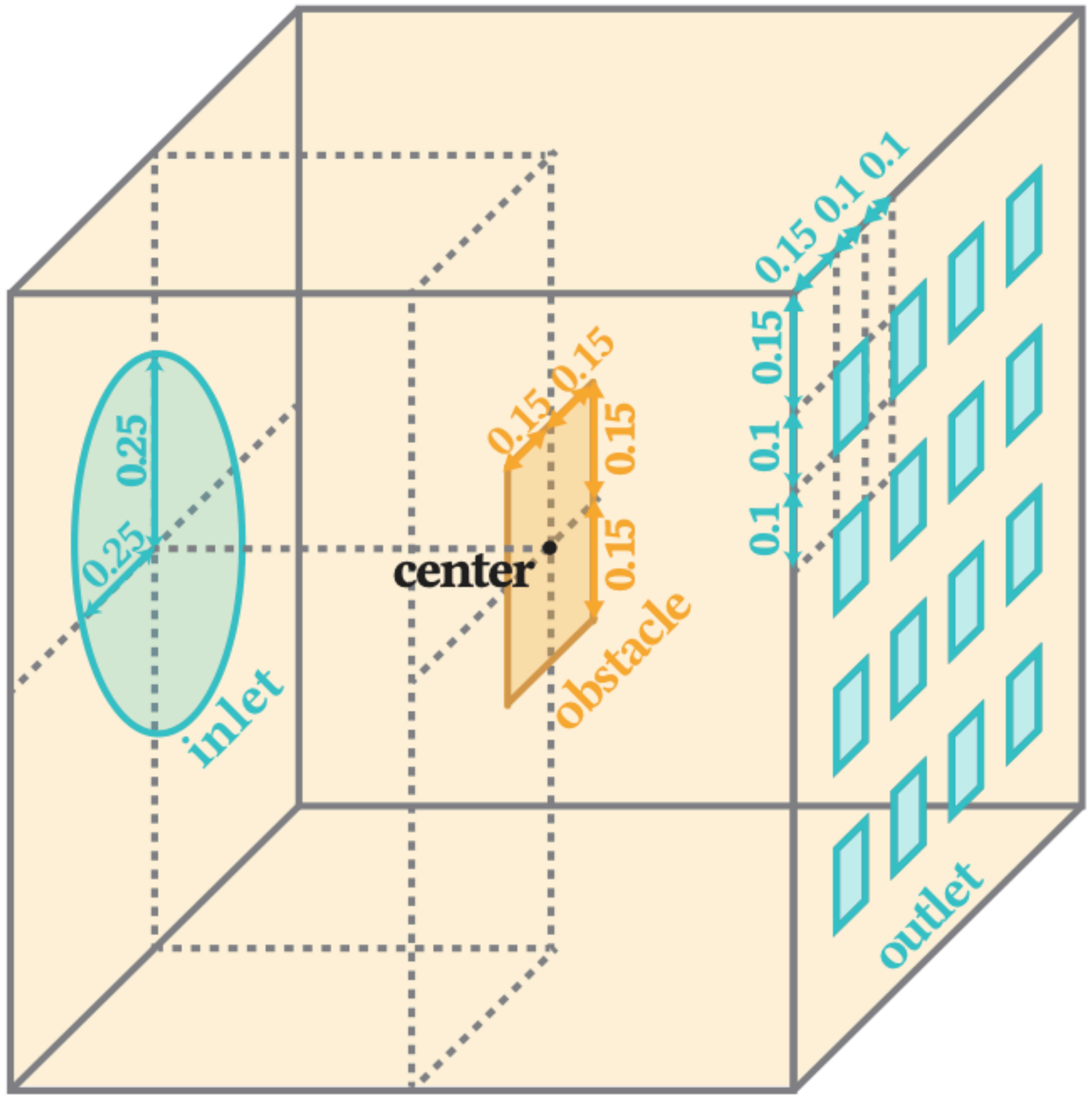
$$\theta \in [\theta_{\min}, \theta_{\max}]$$

L_f	**Functional Objective**
L_r	**Compliance, Direction and Anisotropic Regularizer**
θ	$\theta_c = (\rho_c, \epsilon_c, \alpha_c), \forall \Omega_c \in \Omega$
Initialization	$\rho = V_{frac}, \;\; \epsilon = 1$
Optimizer	**Method of Moving Asymptotes (MMA)**

Applications



Tree Diffuser



Goal

Generate a fluidic diffuser that transports fluids from one inlet into 16 outlets, bypassing a small obstacle

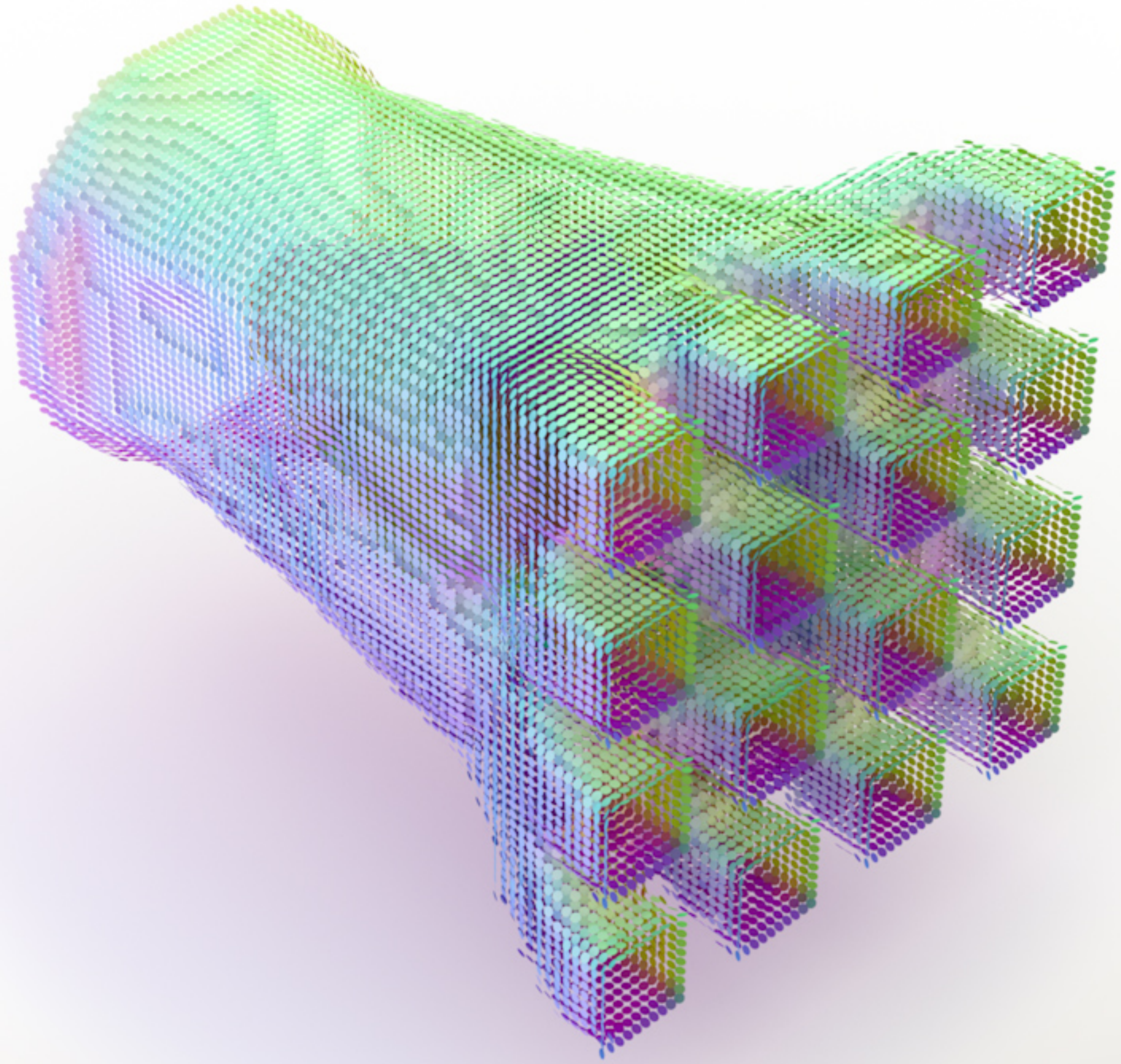
Resolution

80x80x80

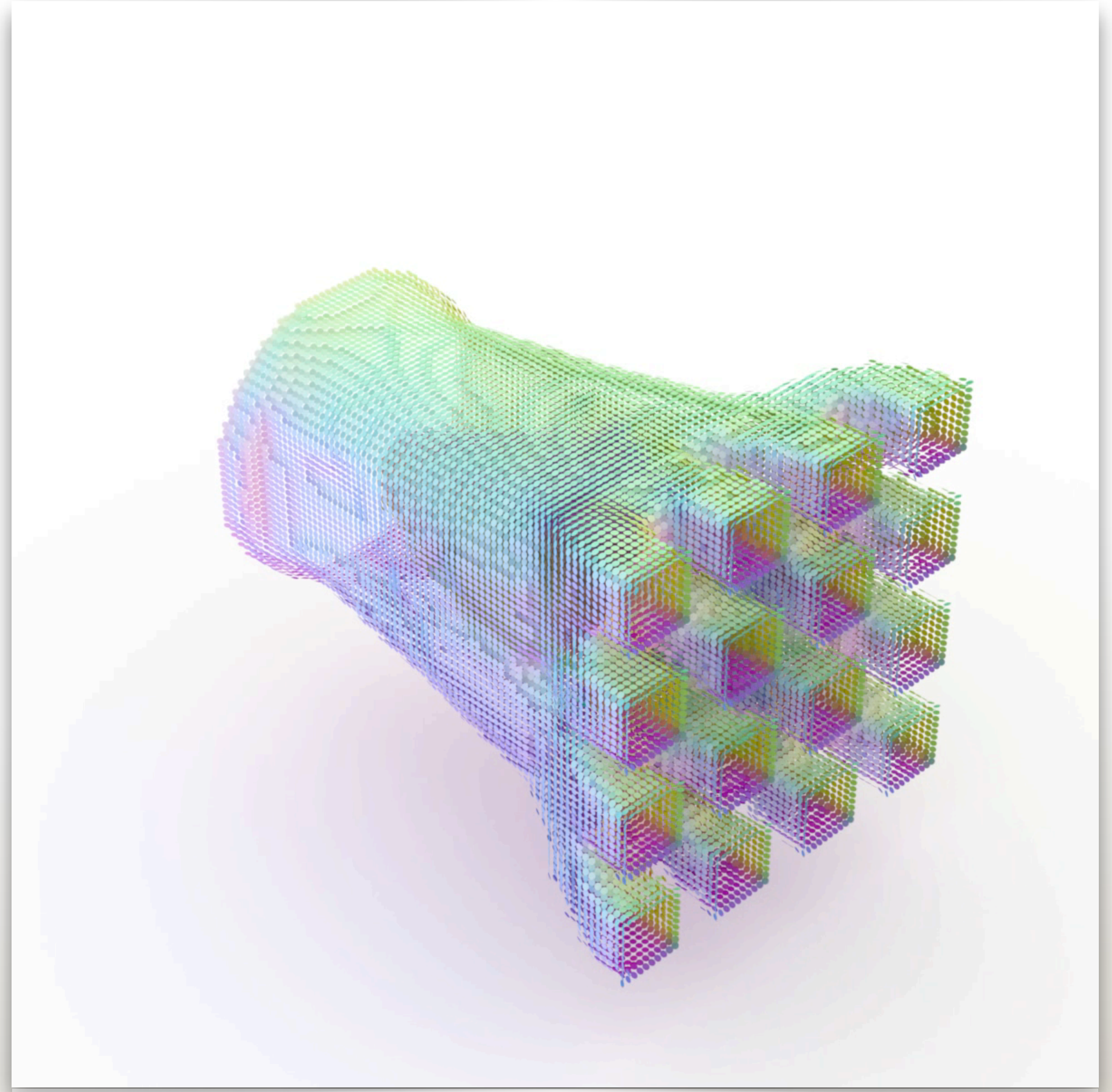
Volume Fraction

0.25

Tree Diffuser

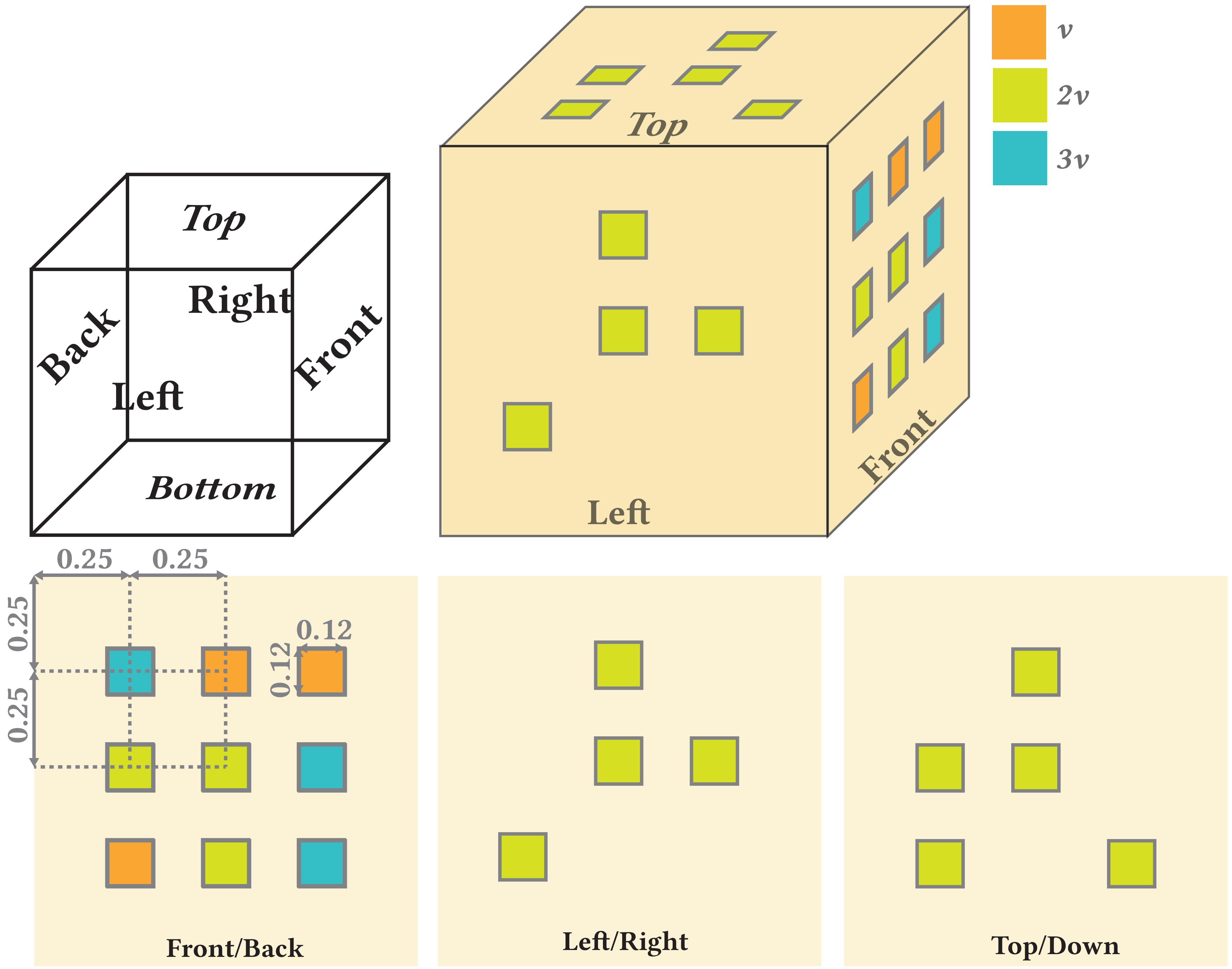


Final Design



Cross Section Visualization

Fluid Circuit



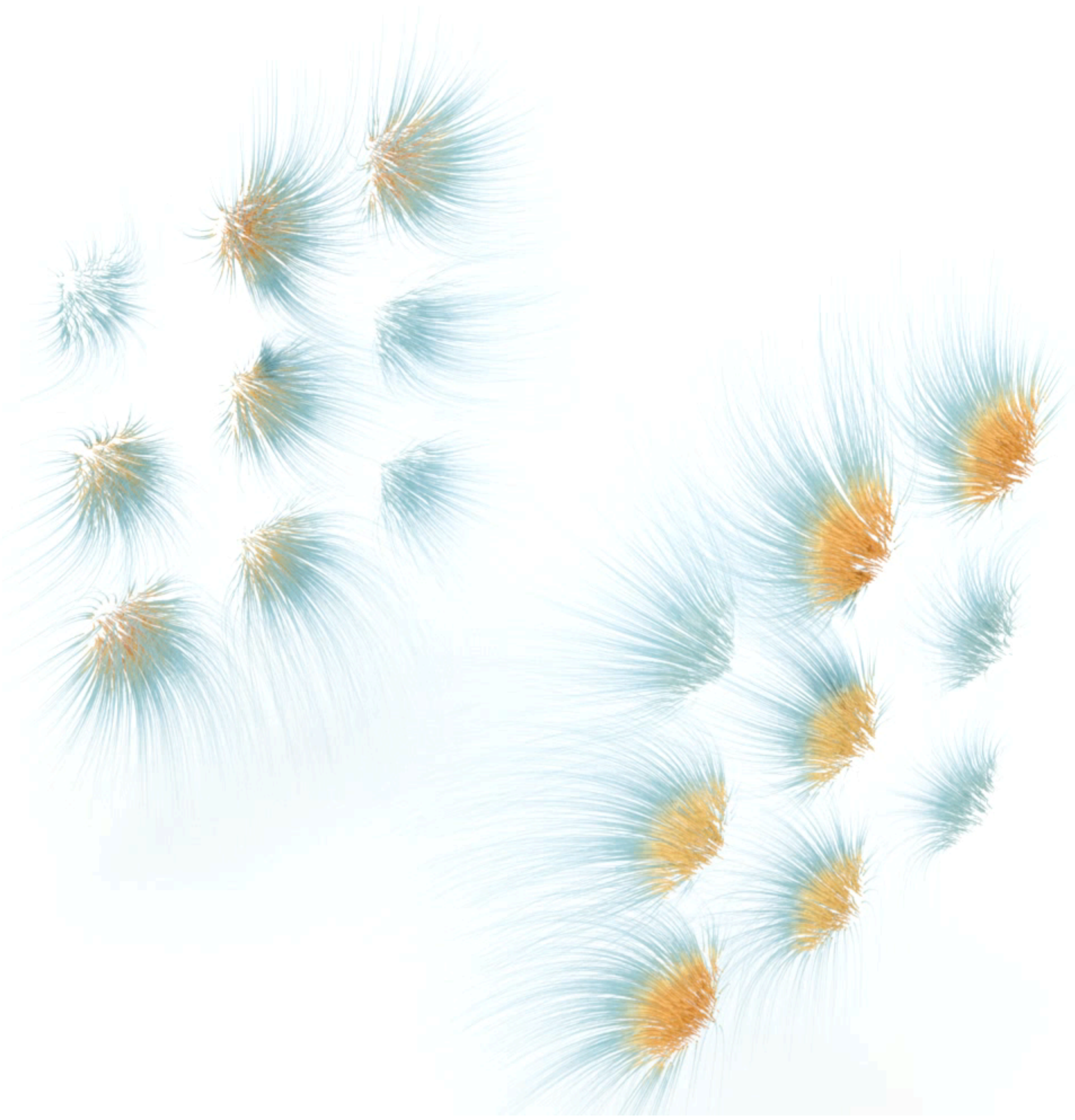
Goal: Connect inlets (two faces) with varying velocities to produce equal flows at the outlets (four faces).

Resolution: 80x80x80

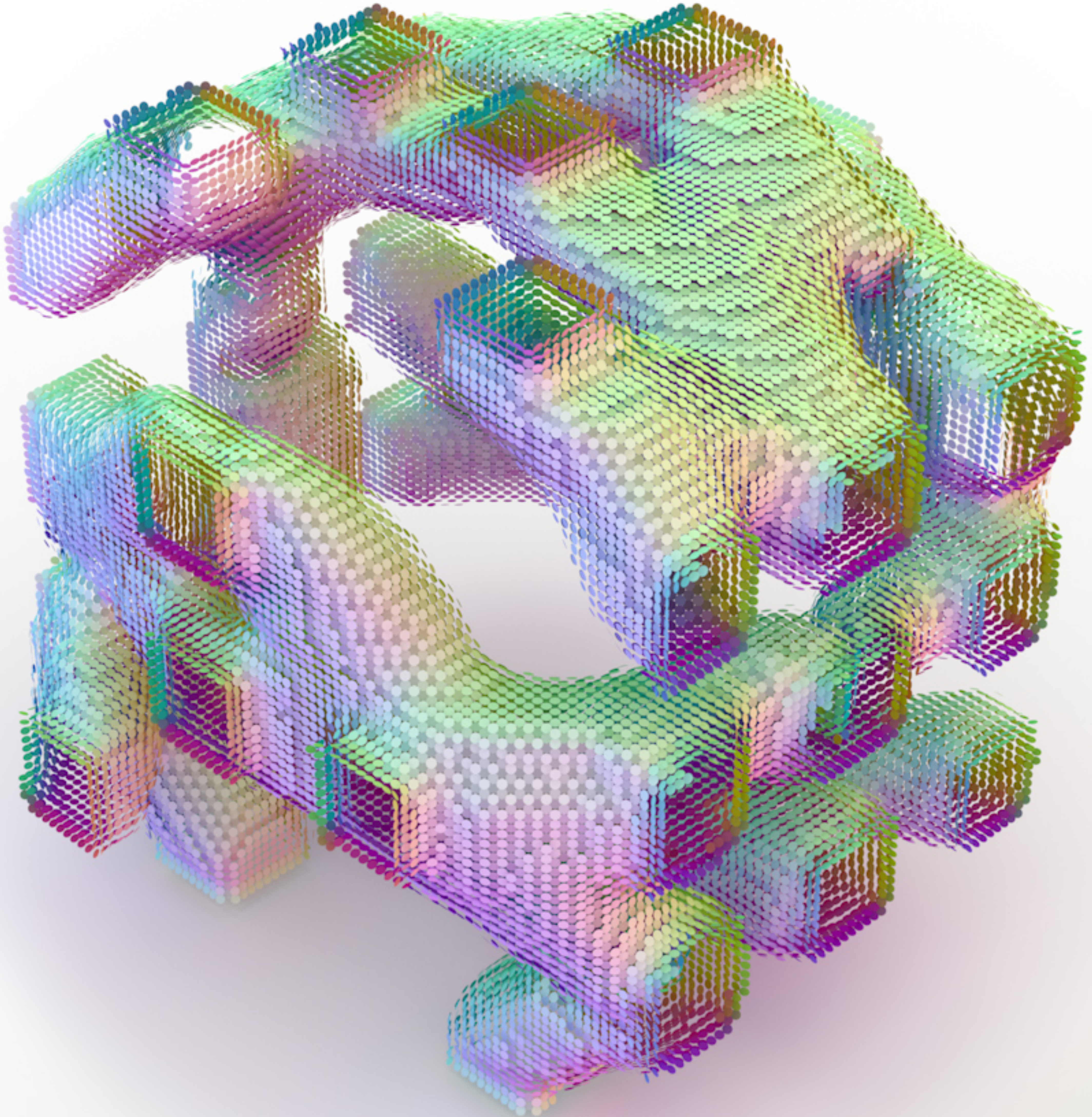
Volume Fraction: 0.25, 0.3

Fluid Circuit

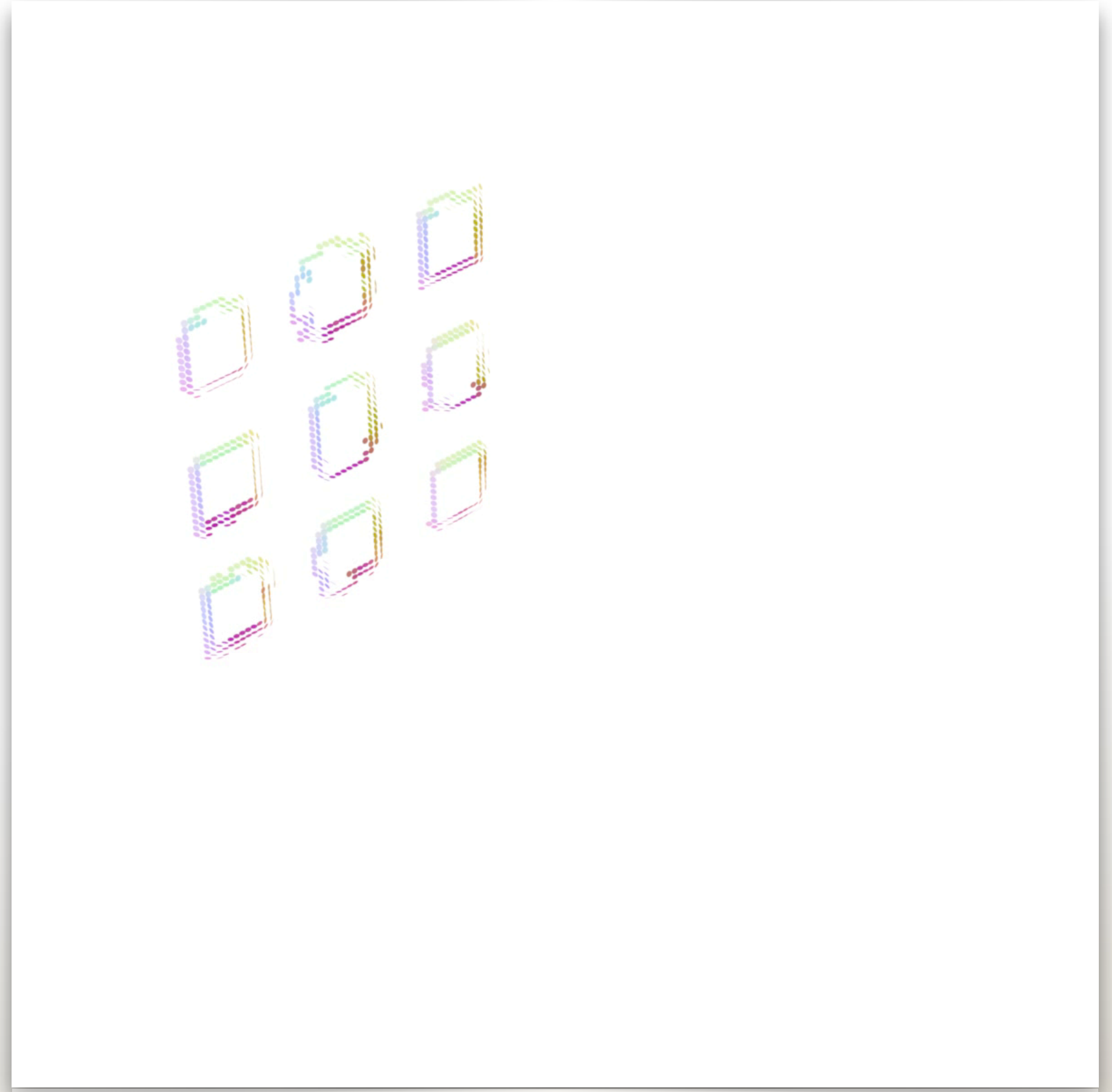
Optimization Iteration Visualization



Fluid Circuit

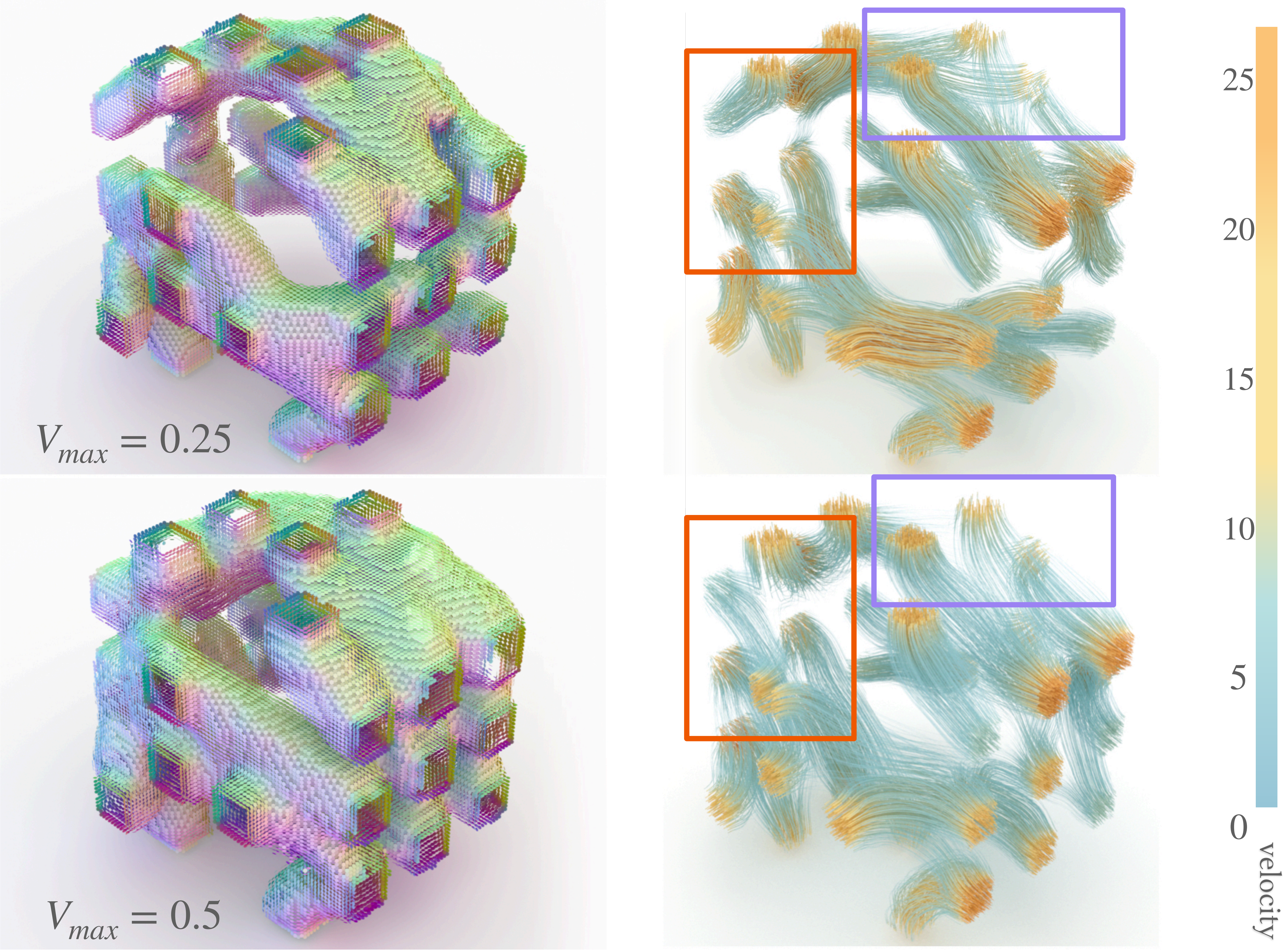


Final Design

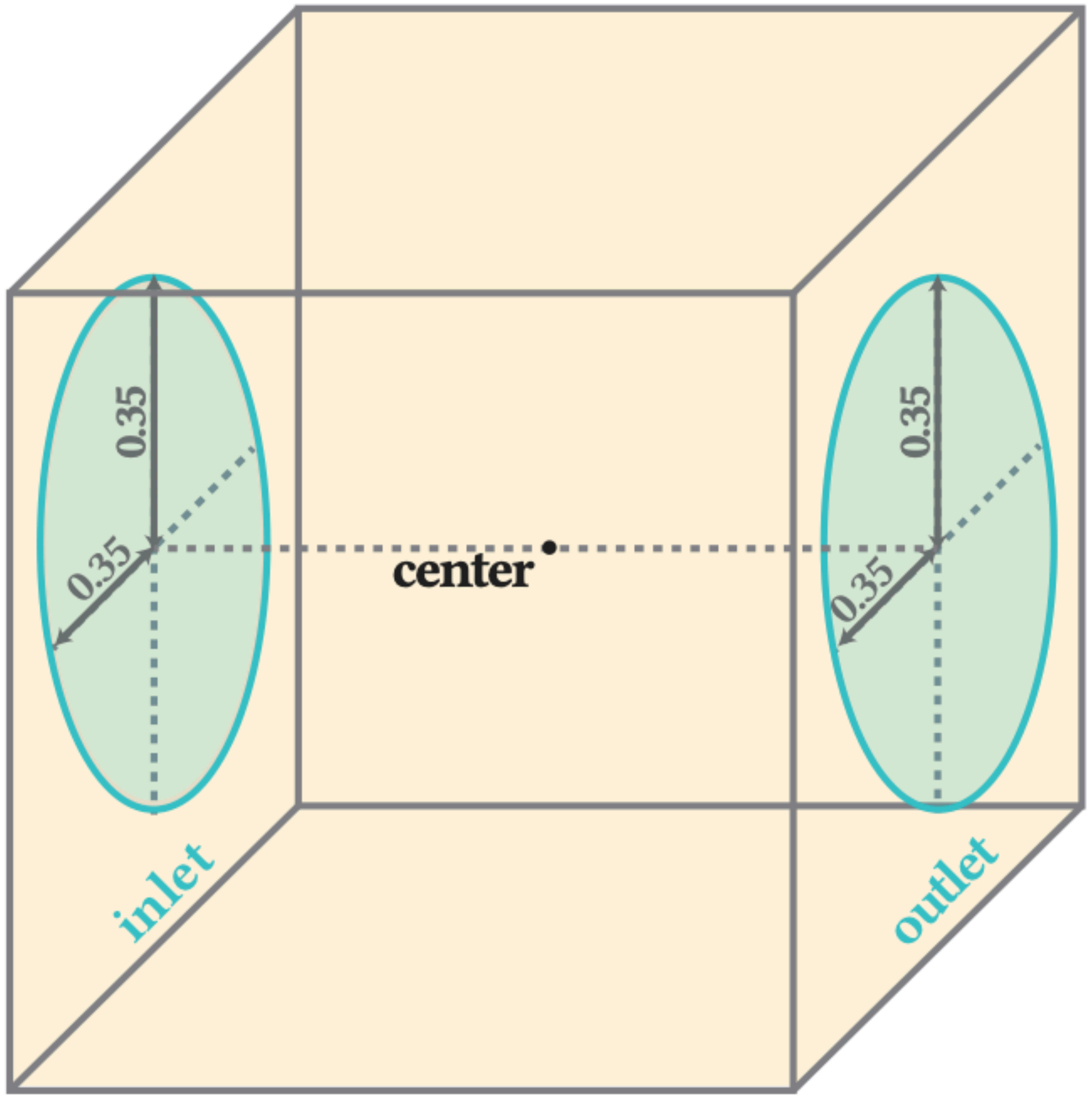


Cross Section Visualization

Fluid Circuit

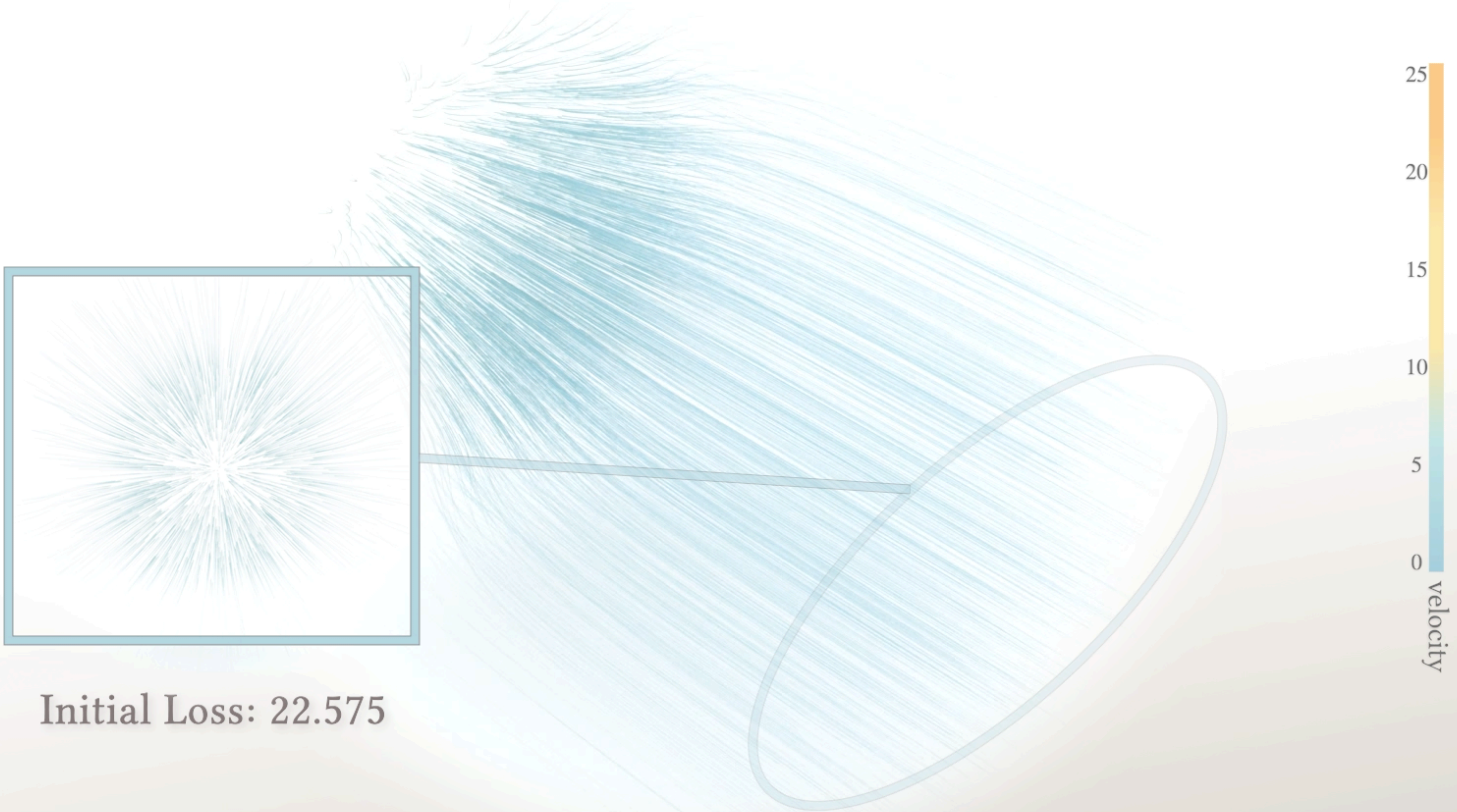


Fluid Twister



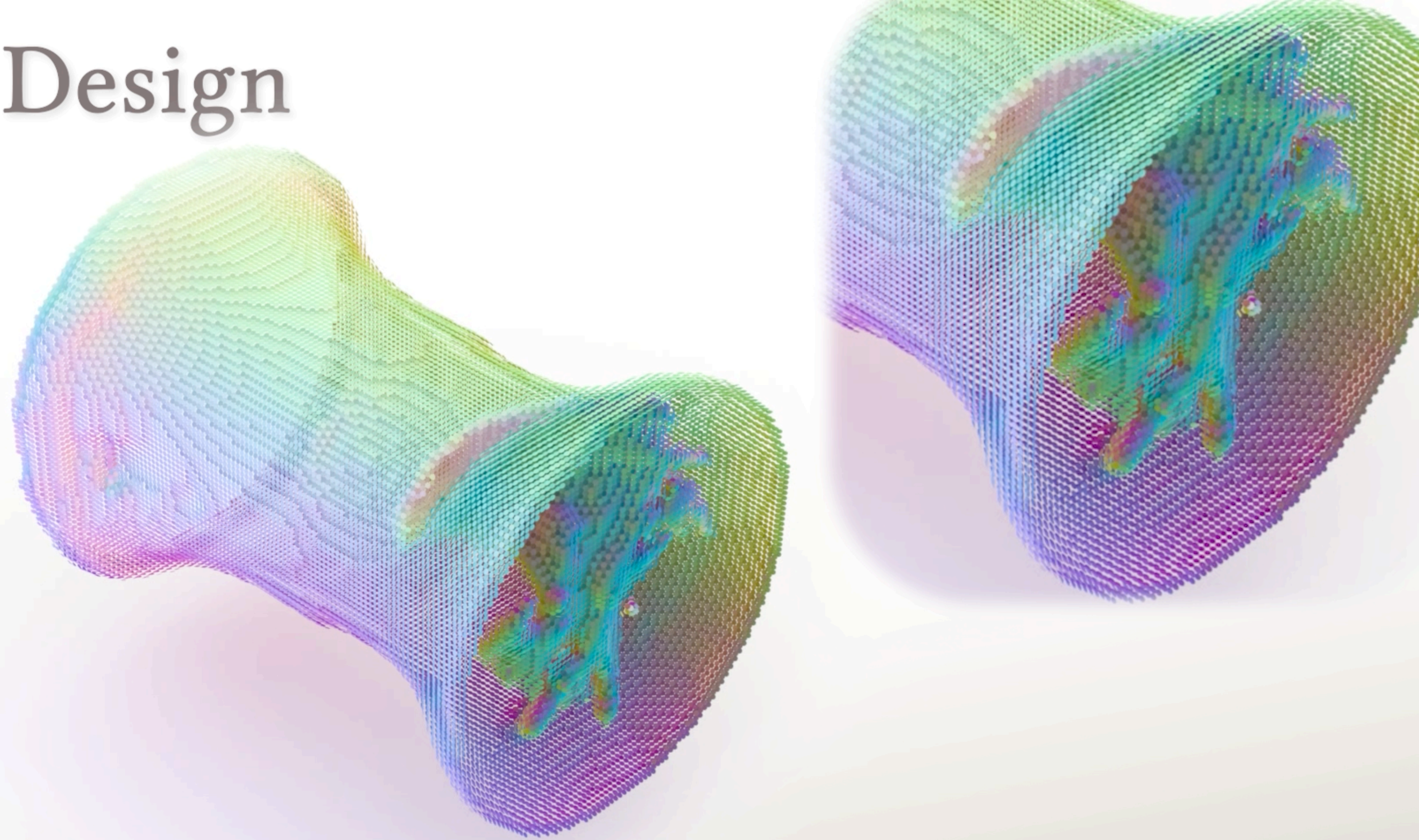
Goal	Generate a twisting flow in the yz-plane at the outlet of the domain from a circular-shaped constant inlet with inflow velocity
Resolution	100x100x100
Volume Fraction	0.3

Optimization Visualization

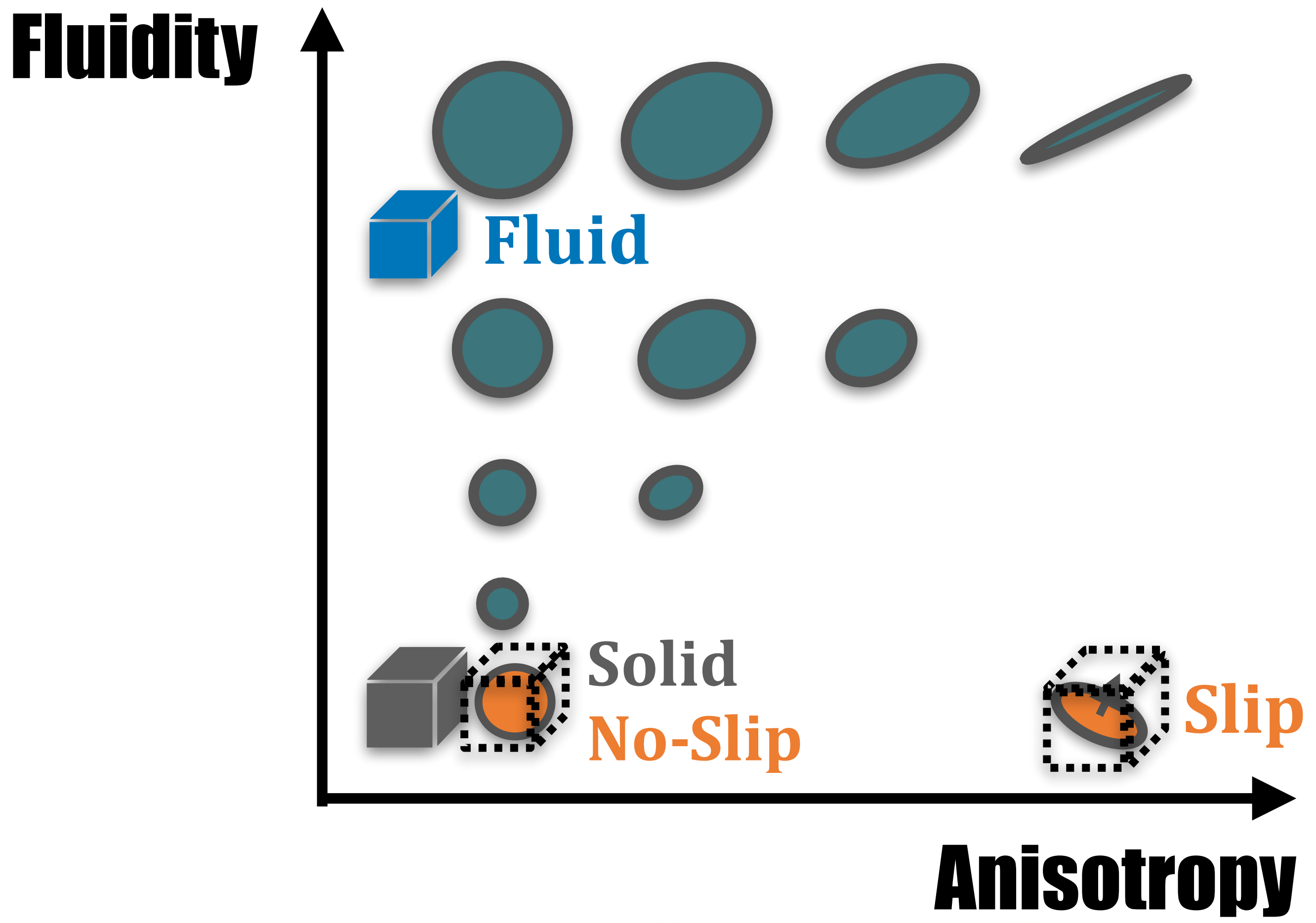


Initial Loss: 22.575

Final Design



Anisotropic Mixture Model



Arbitrary Topology ✓

Sharp Interface ✓

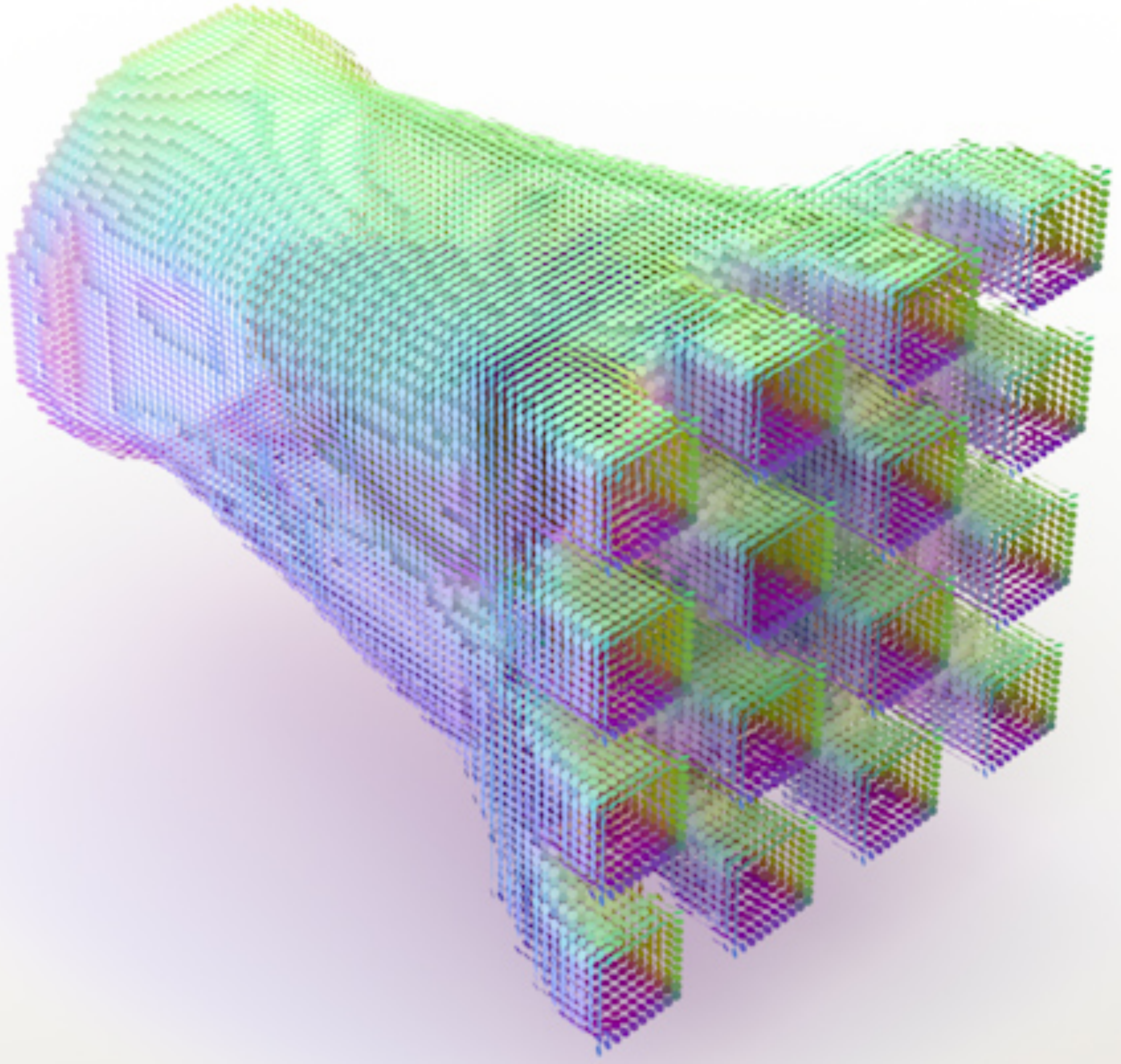
Flexible Boundary Modeling ✓

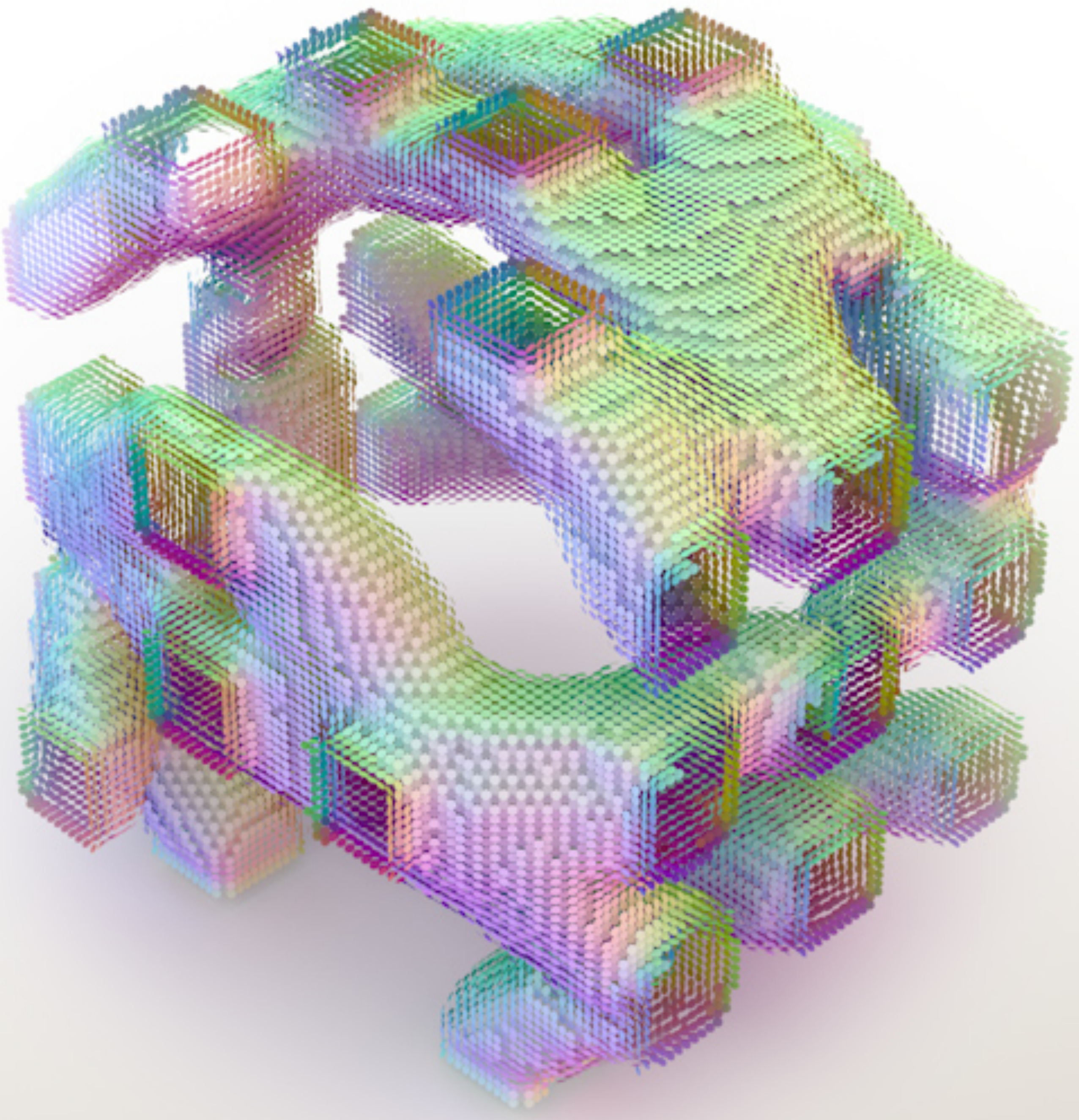
Future Works

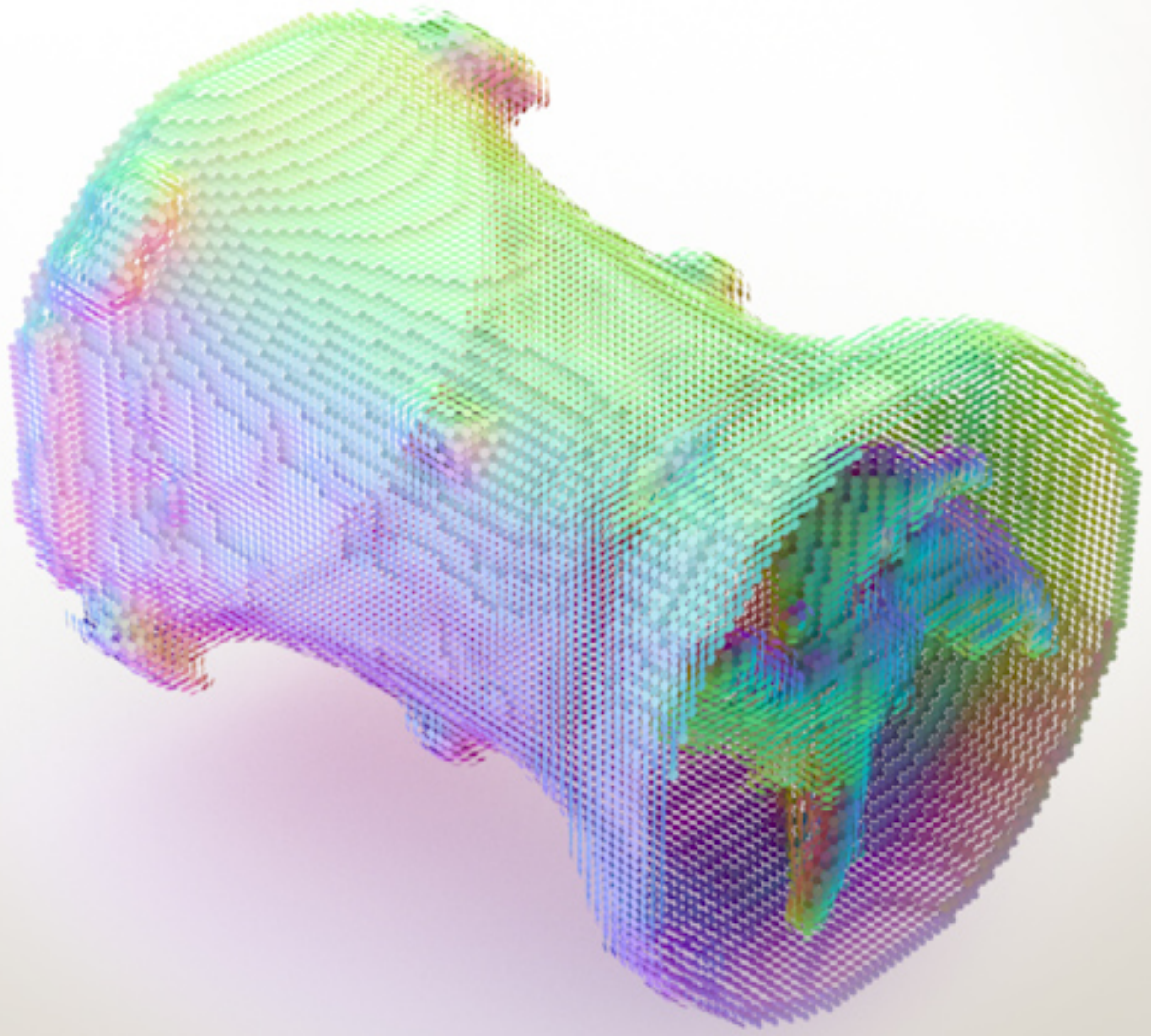
Extension to Navier-Stokes flow

Scalability via Iterative, Multi-Resolution solvers

Built-In Manufacturability







people.csail.mit.edu/liyifei/topostokes/

liyifei@csail.mit.edu