

# A WATER HEATER MODEL FOR INCREASED POWER SYSTEM EFFICIENCY

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## ABSTRACT

This paper presents a domestic hot water heater model to be used in a demand side management program. Water heater loads are extracted from household load data, and then used to develop household water usage data. The model incorporates both the thermal losses and the water used to determine the temperature of the water in the tank. The model will be used in the future to develop intelligent control algorithms to increase power system efficiency and reliability.

**Index Terms**— Demand Side Management, Domestic Hot Water Heater, Power System Efficiency, Load Control

## 1. INTRODUCTION

With surging fuel prices, the efficiency of the power grid is of utmost importance. One important way to reduce losses and increase stability is through the introduction of advanced control algorithms on the load side. This idea is broadly referred to as Demand-Side Management (DSM). The possible benefits include: peak shaving and valley filling to reduce losses in transmission, and providing ancillary services such as synchronous reserve or frequency regulation to decrease losses in generation.

Domestic electric water heaters (DEWH) are ideal candidates for DSM projects because the hot water in the tanks acts as energy storage. In winter-dominated climates, the DEWH loads can contribute as much as 30% of household load. In addition, the DEWH load profile and average daily load profile follow the same pattern, meaning that DEWH loads significantly contribute to peak load values [1]. The idea of using DEWH for DSM is not new but this is the first known project to investigate the tradeoffs between the different benefits described above.

The pilot project has been initiated in Saint-John, New Brunswick. DEWHs will be controlled remotely using high frequency communications. To be able to develop the multi-objective controller, an accurate predictive model of the DEWH is required. This is the focus of this paper.

There is extensive literature on the modeling of DEWHs. These models are developed with different objectives in mind and, as a result, some apply better than others to the present project. For example, in [2] the model assumes that every hot

water draw from the tank is of the same volume of water. This is not suitable here because the amount of water drawn from the tank is one of the things that will be determined.

A differential equation model of the water heater is presented in the literature [1, 3]. This model is based mainly on energy flow analysis and yields a method to determine the temperature of the water in the tank as a function of time.

$$\begin{aligned} T_H(t) = & T_H(\tau) e^{-(\frac{1}{R'C})(t-\tau)} \\ & + \{GR'T_{out} + BR'T_{in} + QR'\} \\ & \times [1 - e^{-(\frac{1}{R'C})(t-\tau)}] \end{aligned} \quad (1)$$

where

- $\tau$  : initial time (hour);
- $T_H(\tau)$  : initial temperature;
- $T_{in}$  : incoming water temperature ( $^{\circ}\text{F}$ );
- $T_{out}$  : ambient air temperature outside tank ( $^{\circ}\text{F}$ );
- $T_H(t)$  : temperature of water in tank at time  $t$  ( $^{\circ}\text{F}$ );
- $Q$  : energy input rate as a function of time (BTU/hour);
- $R$  : tank thermal resistance (hour\*ft<sup>2</sup>\*( $^{\circ}\text{F}$ )/BTU);
- $SA$  : surface area of tank;
- $G = SA/R$ ;
- $B$  : (density of water)\*(water usage)\*(specific heat of water);
- $C$  : (volume of tank)\*(density of water)\*(specific heat of water);
- $R' = 1/(B + G)$ ;

It is important to note that the values of  $Q$  and  $B$  are time dependent.  $Q$ , as the energy input, is dependent on whether the element is on or off.  $B$  is a function of the usage of water. Therefore, the value of  $\tau$  and  $T_H(\tau)$  must be updated every time there is a change in  $B$  or  $Q$ . All of the other parameters can be measured, and must be known for accurate prediction of the temperature.

In most cases in the literature, the authors use the individual DEWH model to develop an aggregated DEWH model. In [3] this is accomplished through a monte carlo rejection method. In [4], a state-queuing model is presented to account for uncertainties in user behavior. In [1] the deterministic parameters of the model (1) are replaced by normal random variables. The main constraint of the DSM program is to ensure that users are not affected by ensuring that the water in the

DEWH is always at an acceptable temperature for use. By aggregating the model, it becomes impossible to satisfy this constraint in general. There will always be users, although they may be in the minority, that will have their DEWHs turning off at undesirable times. In addition, to adequately provide ancillary services, which is part of the multi-objective nature of this project, it is important to know exactly how much frequency regulation or synchronous reserve can be provided. This is only possible if the temperature and state of individual water heaters is known.

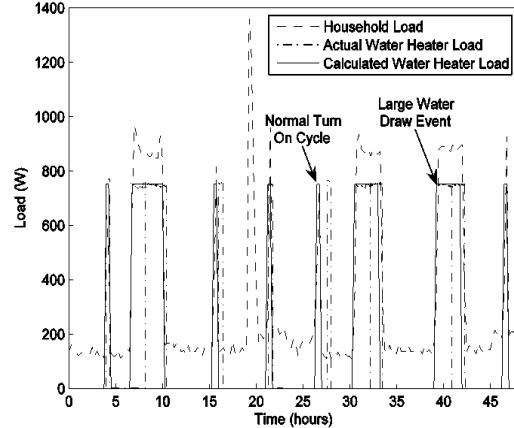
This paper develops a predictive model that is not aggregated. Instead, each household that is part of the DSM project has its own model and the parameters are determined exactly through inspection of the site. Smart meters have been recording household data for some time now at these households, and this data is used to develop a user profile that determines when the most likely times are that water will be used in a particular household. The system is described in the following sections: section 2 will discuss how the DEWH load data can be extracted from the household load data, section 3 will discuss how the water usage data can be determined from water heater load data, Section 4 will discuss how the temperature of the water in the tank can be determined from the water usage data, and Section 5 will present how the water usage profiles are developed.

## 2. EXTRACTING WATER HEATER LOAD DATA FROM HOUSEHOLD LOAD DATA

Smart meters have been installed and record the number of watts used by the household on 15-minute intervals. This data is available on the web in real-time. Extracting the water heater load from the household load data is the first step to developing the water usage profile.

It is known that the water heater elements in the study are 3kW. The household load data is analyzed for drops or jumps that account for this 3kW load. In general, there are two types of scenarios under which the water heater element turns on: 1) the temperature of the water has dropped below the minimum setpoint as a result primarily of conduction heat losses; and 2) a large amount of water has been drawn and replaced with colder incoming water. In the first case, the temperature of the water in the tank is at the minimum setpoint, and the element will be on for a consistent and predictable amount of time. In the second case, the water temperature can be far below the minimum temperature setpoint, and the length of time that the element is on will be variable, depending on the amount of water drawn.

In the first case, given that the on-time for the element is known, the amount of power used is consistent and can be calculated. If a spike occurs in the household load data with an area that approximately equals this value, then it is likely that this is the situation. In the case of larger water draws, the household load data is inspected for jumps or drops of 750W



**Fig. 1.** Extraction of Water Heater Load from Household Load

(3kW/4). An example of the data received from the smart meters and how the water heater load is extracted is shown in Fig. 1.

The actual water heater load was measured with an ammeter for comparison. It is clear from Fig. 1 that the system's performance is quite good. It is important to note that this data was taken during the summer, when the other household loads are quite low. During the winter, particularly in households with electric heating, it can be more difficult to extract the DEWH load. It is notable that the large water draw events require the water element to stay on for a much longer time. During the normal cycle turn ons, the element is on for about 18 minutes, but after a large water draw it can be on for 2-3 hours.

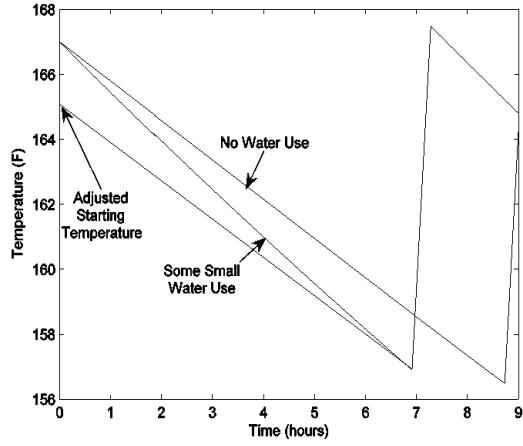
## 3. USING THE DEWH LOAD TO DETERMINE HOT WATER USAGE

To determine the how water usage, it is useful to separate water usage events into two categories: large water draws, such as showers, and small water draw events, such as washing hands.

### 3.1. Small Water Draw Events

When the element is only on for the short period of time, it is because the element only had to heat the water from the minimum temperature setpoint to the maximum temperature setpoint. However, this does not mean that no water has been used since the last time the element shut off. As is shown in Fig. 2, if there is a small amount of water used over the cycle it causes the water heater element to turn on slightly sooner than it otherwise would have.

It is assumed that the water that is used during the time when the element is off is used uniformly throughout the cy-



**Fig. 2.** Decaying Water Temperature With and Without Small Water Use

cle. This assumption is not correct, but it is impossible to pinpoint the exact time of water usage from the information given. From the known time when the element actually turned on, the adjusted starting temperature is calculated by finding the starting temperature that causes the element to turn on at that time, assuming no water drawn. Once this adjusted starting temperature is known, the amount of water used in the cycle can be calculated using:

$$\text{Water Used} = \text{Volume of Tank} * \frac{(T_{MAX} - T_{adj})}{(T_{MAX} - T_{IN})} \quad (2)$$

where

$T_{MAX}$  : maximum temperature setpoint

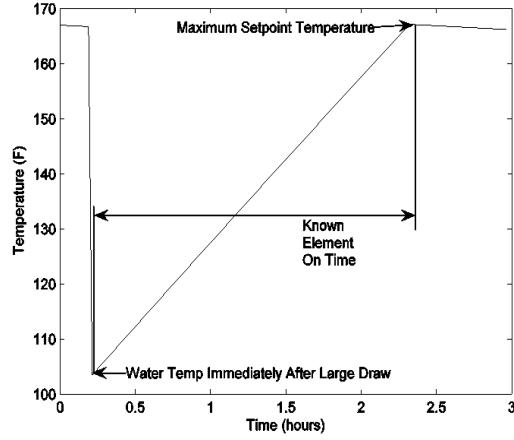
$T_{adj}$  : adjusted starting temperature as shown in Fig. 2

$T_{IN}$  : incoming water temperature

### 3.2. Large Water Draw

The other type of water use is the major water draw event, such as: taking a shower, running the dishwasher, or some other event that draws a large amount of hot water in short amount of time. It is assumed in this case that the water draw is essentially instantaneous, and that when the water is drawn, the temperature of the water in the tank is halfway between the two setpoints. It is known from the data how long the element is on for and that when the element shuts off that the water is at the maximum setpoint. From this information a curve can be fit to determine what the temperature of the water must have been at the moment of the large draw, when the element turns on. Refer to Fig. 3.

Once the temperature at the end of the large draw is known, the amount of water used can be calculated using (2).



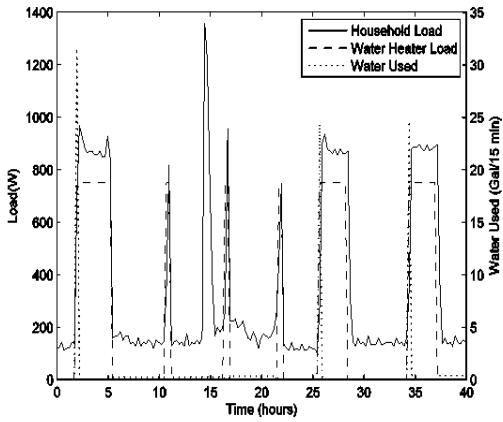
**Fig. 3.** Temperature of Water in Tank During a Large Water Draw Event

### 4. USING WATER USAGE DATA TO DETERMINE WATER TANK TEMPERATURE

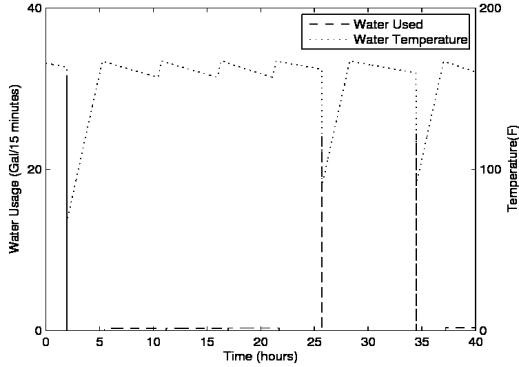
Once the water usage has been determined, it is used in (1) to calculate the value for  $B$ . The value of  $Q$ , or the incoming energy, is determined by the temperature in the tank in a hysteris fashion. If the element is *on* then once the temperature drops below the minimum temperature setpoint, the element switches to *off* and the value of  $Q$  is set to zero. The reverse happens when the temperature reaches the maximum temperature setpoint. The result is that the water usage data is determined entirely from the household load data, and the temperature of the water in the tank is determined entirely from the water usage data. Fig. 4 shows the water usage being calculated from the water heater load values. It can be seen that there are large spikes that represent large water usage events, and the rest of the time, the water usage values are fairly low. Fig. 5 shows how the temperature of the water is calculated from the water usage and the model developed in (1). Note the huge drops in water temperature when large water draw events occur. The accuracy of the model is verified by the fact that the calculated times that the water heater element is *on* correspond exactly with the times that the element was *on*.

### 5. DEVELOPMENT OF WATER USAGE PROFILE

It is desired to be able to predict the temperature of the water in water tanks such that intelligent decisions can be made about when they can be turned on or shut off without compromising the user's hot water service. This is achieved by doing the above analysis to determine the hot water usage on a large amount of data, and then developing a water usage profile for the user. The water usage profile shown in Fig. 6 was done by taking an average of the water used for one month,



**Fig. 4.** Daily Household Load, Water Heater Load, and Calculated Water Use



**Fig. 5.** Using Water Usage to Calculate Water Temperature

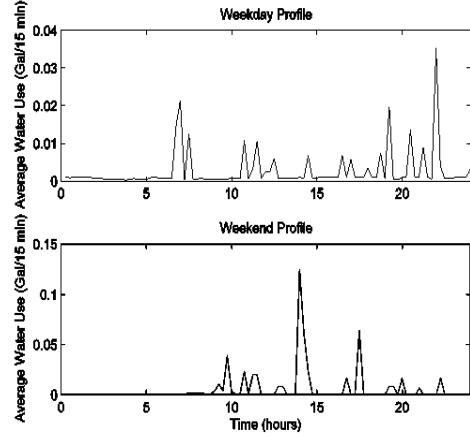
in this case July 2008, and then averaging it. The weekdays and weekends have been separated because it is clear that the behavior of the user is different.

It is known that this user works a regular workday, and tends to use quite a lot of water later in the evenings for laundry because they have small children. The water usage profile reflects this pattern.

Similar water usage profiles will be developed for all users in the DSM program. Intelligent decisions can be made about which loads to shed or pick up to have maximum performance of the system with minimum impact on the user.

## 6. CONCLUSION

A novel multi-objective demand side management program is being developed that uses domestic hot water heaters to achieve added power system efficiency and reliability. To achieve the goals of the project, a detailed model of the individual hot water heater is required. Water heater loads are extracted from household load data that is provided at 15 minute



**Fig. 6.** a. Weekday Water Usage Profile b. Weekend Water Usage Profile

intervals by smart meters installed at the dwelling. The water heater load is used to determine the household water usage data. Significant amounts of this data are combined to develop a water usage profile for the household that will help us estimate better when the water heater elements are most likely to turn on or off. In further work, advanced control algorithms will be developed to make intelligent decisions on how to control the water heaters for maximum gains based on the model developed here.

## 7. REFERENCES

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