

A novel domestic electric water heater model for a multi-objective demand side management program[☆]

Liam Paull^{*}, Howard Li, Liuchen Chang

University of New Brunswick, Department of Electrical Engineering, Fredericton, NB, Canada E3B 5A4

ARTICLE INFO

Article history:

Received 11 September 2009
Received in revised form 3 May 2010
Accepted 10 June 2010
Available online 14 July 2010

Keywords:

Demand side management
Domestic hot water heater
Power system efficiency
Load control

ABSTRACT

This paper presents a novel domestic hot water heater model to be used in a multi-objective demand side management program. The model incorporates both the thermal losses and the water usage to determine the temperature of the water in the tank. Water heater loads are extracted from household load data and then used to determine the household water usage patterns. The benefits of the model are: (1) the on/off state of the water heater and temperature of the water in the tank can be accurately predicted, and (2) it enables the development of water usage profiles so that users can be classified based on usage behaviour. As a result, the amount of ancillary services and peak shaving that can be achieved are accurately predictable and can be maximized without adversely affecting users.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

The efficiency of the power grid is of great importance as environmental concerns related to the combustion of fossil fuels increase. One important way to reduce losses and increase power system stability is through advanced control algorithms on the load side. This idea is broadly referred to as demand-side management (DSM). Different objectives have been considered by past DSM programs found in the literature, such as peak shaving and valley filling [1], and providing ancillary services like synchronous reserve [2], frequency regulation [3], and voltage stability [4]. The pilot project that is underway in Saint-John, New Brunswick, tries to balance all of these objectives for maximum overall benefit. Other multi-objective DSM projects have been attempted but in these cases, the impact on the user is treated as an objective [5,6]. In the present project, a control objective function has been developed similar to the one presented in Ref. [7] where minimal impact on the users is treated as a constraint. In order to satisfy this constraint, a more advanced mathematical model of the domestic electric water heater (DEWH) is required.

DEWHs are ideal candidates for DSM projects because the hot water in the tanks acts as energy storage. In winter-dominated climates, the DEWH loads can contribute as much as 30% of the total

household load [8]. In addition, the DEWH load profile and average daily load profile follow a similar pattern, meaning that DEWH loads significantly contribute to peak load values [9]. DEWH loads have been used in the past to achieve DSM, most recently [10] among many others.

The heat transfer characteristics of the DEWH tank are well known, and are presented in Section 2. In most cases, when the DEWH is used for DSM, the individual DEWH heat transfer model is used to develop an aggregated DEWH model. For example, Ref. [11] uses a Monte Carlo rejection method to aggregate individual models, Ref. [12] uses a state-queuing model to account for uncertainties in user behaviour, and Ref. [9] replaces the deterministic parameters of the model with normal random variables. These aggregated models allow many DEWHs to be controlled together, thereby simplifying the control algorithms needed. However, aggregation of the model allows the temperature of the water in individual tanks to be at an unacceptable level at times when the user requires hot water. This adversely affects user comfort and can permit the growth of unwanted and potentially dangerous bacteria [13]. For the widespread acceptance and integration of the program, it is critical to avoid these situations.

This paper develops a predictive model that is not aggregated. Each household that is part of the DSM project has its own model with most parameters determined through inspection of the site. DEWHs will be controlled remotely using high frequency communications, and smart meters record household load data on 15 min intervals.

Although the aggregated model produces undesirable results, it is useful to classify similar users. The benefits are that control algorithms can be simplified, and users with habitual water usage

[☆] Thanks to the National Science and Engineering Research Council, the New Brunswick System Operator, and Saint John Energy for their support.

^{*} Corresponding author at: University of New Brunswick, Department of Electrical Engineering, P.O. Box 4400, Fredericton, NB, Canada E3B 5A4.

E-mail address: Liam.Paull@unb.ca (L. Paull).

patterns can be identified. In previous literature, classification is performed on the household load data rather than the data for the remotely controlled load [14], which may introduce a significant source of error. The algorithm presented here allows the water usage data to be determined from the household load data, and a water usage profile can be developed as will be shown. It is much more effective to classify users based on their water usage patterns rather than the household load profile shape. It is important to note that although users are being classified for reduced control algorithm complexity, the individual models are still available so that the user comfort constraint is never violated.

The novel aspects of this model that make it advantageous over those presented in previous literature include:

- It can be implemented in near real time so that the on/off state of the individual heaters and the temperature of the water in the tanks can be accurately predicted. The result is that the exact amount of frequency regulation, synchronous reserve, or peak shaving that can be achieved is known before a control action is taken.
- Water usage profiles can be developed so that users may be classified in terms of water use rather than household load profile.

The result of these two benefits is that load control becomes predictable and the multiple objectives can be maximized with an absolute minimum impact on the users' comfort. It is the first known model to be developed that uses an analysis of past load data to determine exact water usage patterns, a facet which is integral in the modeling of the DEWH and is overlooked in all of the previous literature.

The previously established thermal model of the tank is presented in Section 2. The proposed methods of extracting water usage are presented in Section 3. The model is validated and results are shown in Section 4. Finally, the conclusions are presented in Section 5.

2. Background on DEWH thermal model

There is extensive literature on the modeling of DEWHs [9,11,15,16]. A differential equation model of the thermal characteristics of the water heater is presented in Refs. [9,11]. This model is based mainly on energy flow analysis and yields a method to determine the temperature of the water in the tank as a function of time. The differential equation describing the temperature of the water in the tank is

$$C\dot{T}_H(t) = -SA\left(\frac{1}{R}\right)[T_H(t) - T_{out}] - D \times W_D(t)C_p[T_H(t) - T_{in}] + Q(t) \quad (1)$$

A solution is given by

$$T_H(t) = T_H(\tau)e^{-(1/R'C)(t-\tau)} + \{GR'T_{out} + BR'T_{in} + QR'\} \times [1 - e^{-(1/R'C)(t-\tau)}] \quad (2)$$

where τ is the initial time (h); $T_H(\tau)$ is the initial temperature ($^{\circ}\text{F}$); T_{in} is the incoming water temperature ($^{\circ}\text{F}$); T_{out} is the ambient air temperature outside tank ($^{\circ}\text{F}$); $T_H(t)$ is the temperature of water in tank at time t ($^{\circ}\text{F}$); $Q(t)$ is the energy input rate as a function of time (W); R is the tank thermal resistance ($\text{m}^2 \text{ } ^{\circ}\text{F}/\text{W}$); SA is the surface area of tank (m^2); $G = SA/R$ ($\text{W}/^{\circ}\text{F}$); $W_D(t)$ is the water demand as a function of time (L/h); C_p is the specific heat of water ($\text{J}/(^{\circ}\text{F}\text{kg})$); D is the density of water = 1 kg/L ; $B(t) = D \times W_D(t) \times C_p \times 1 \text{ h}/3600 \text{ s}$; Q : (volume of tank) $\times D \times C_p$ ($\text{J}/^{\circ}\text{F}$); $R' = 1/(B/G)$ ($\text{W}/^{\circ}\text{F}$).

It is important to note that the values of Q and B are time dependent. Q , as the energy input, is dependent on whether the element is on or off, and B is a function of the usage of water. Therefore, the

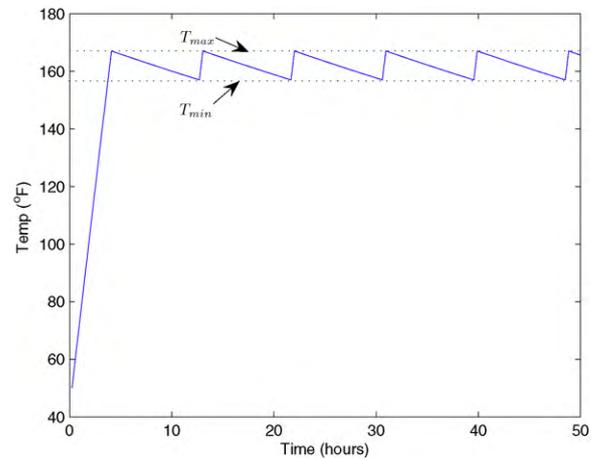


Fig. 1. Temperature of water in the tank with no water usage.

value of τ and $T_H(\tau)$ must be updated every time there is a change in B or Q . All of the other parameters are measured from the site and must be known for accurate prediction of the temperature.

The element turns on and off in a hysteresis fashion as the temperature varies between the minimum and maximum setpoints, T_{min} and T_{max} . The value of Q is either 3000 W or 0 , and is determined from (3):

$$Q(t + \Delta t) = \begin{cases} 3000 \text{ W}, & T_H(t) < T_{min} \\ 0, & T_H(t) > T_{max} \\ Q(t), & \text{else} \end{cases} \quad (3)$$

During no water usage operation of the DEWH, the temperature of the water exponentially decays and rises between the two temperature setpoints as shown in Fig. 1.

The other time dependent variable, B , which is a function of the water usage, is more difficult to predict. Past projects treat the water usage as known [11] or as constant [9]. The assumption that the water usage is constant leads to significant errors in the prediction of the temperature of the water in the tank. The present study will use past household load data to develop a water usage model for accurate prediction of individual water heater temperature.

3. Proposed method

The development of the water usage profile requires three steps. First, the DEWH load data must be extracted from the household data. Next, the water usage amounts are determined from the DEWH load. Last, the water heater profile is determined from large amounts of DEWH load data.

3.1. Extracting water heater load data from household load data

It is known that the water heater elements in the study are 3 kW . The household load data is analyzed for drops or jumps that account for this 3 kW load. In general, there are two types of scenarios under which the water heater element turns on: (1) the temperature of the water has dropped below the minimum setpoint as a result primarily of conduction heat losses; and (2) a large amount of water has been drawn and replaced with colder incoming water. In the first case, the temperature of the water in the tank is at the minimum setpoint, and the element will be on for a consistent and predictable amount of time. In the second case, the water temperature can be far below the minimum temperature setpoint, and the length of time that the element is on will be variable, depending on the amount of water drawn.

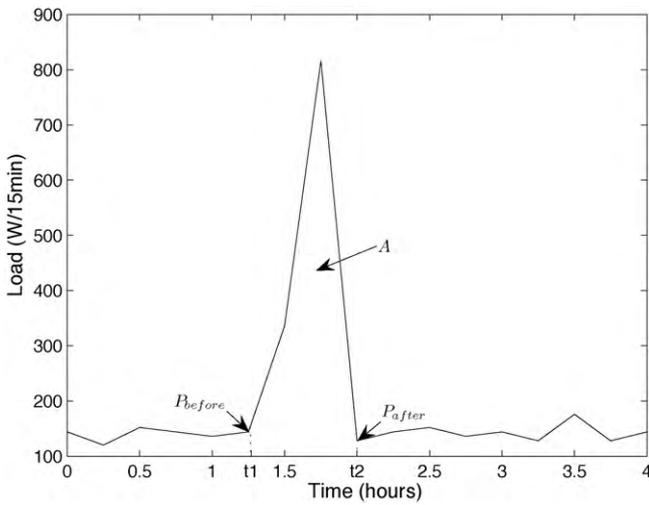


Fig. 2. Zoomed in view of household load spike when DEWH turns on for short period.

In the first case, given that the on-time for the element is known, the amount of power used is consistent and can be calculated. The time required to heat the water from the minimum setpoint to the maximum setpoint can be determined from an analysis of the system assuming no water usage by measuring the rise time in Fig. 1. This time is given as t_{short} . From the values of t_{short} and Q , the amount of energy, E_{short} that is required to heat the water from the minimum to maximum setpoint can be calculated from (4). The load data is scanned for any spikes and their areas are calculated. If the area, A , calculated using (5) of any spike is approximately equal to E_{short} plus the baseline loading over that period of according to (6) then it is determined that the DEWH turned on. The units of A and E_{short} are $W * min$. The baseline loading is calculated by doing an interpolation between the baseline points before and after the spike, P_{before} , and P_{after} , respectively. $P(t)$ is the load data as a function of time, and t_1 and t_2 are as shown in Fig. 2.

$$E_{short} = t_{short} \times Q \tag{4}$$

$$A = \int_{t_1}^{t_2} P(t)dt \tag{5}$$

$$A \approx E_{short} + .5(P_{before} + P_{after}) \times (t_2 - t_1) \tag{6}$$

Larger water usage events result in the DEWH element being on for a longer and more unpredictable amount of time. Under normal operating conditions, the element can be on from .5 to almost 4 h. To detect these events, jumps or drops of 750 W/15 min (3 kW/h) are identified by doing a difference between P_{on} and P_{off} as shown in Fig. 3. Once a jump is identified, the data for the next .5–4 h segment

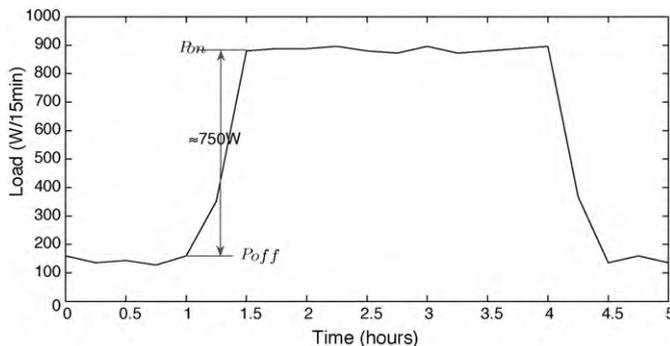


Fig. 3. Zoomed in view of household load during a long turn on period of the DEWH.

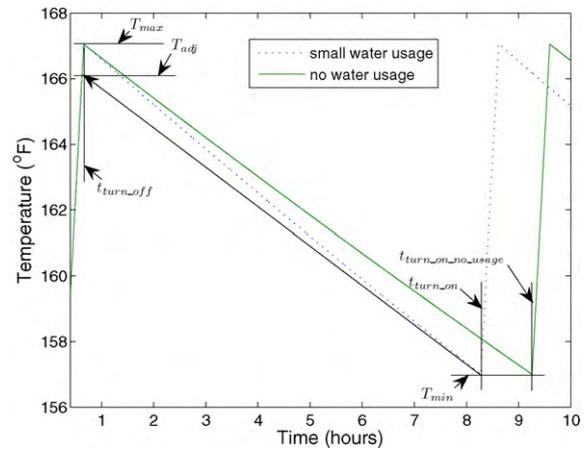


Fig. 4. Determining the water used during a normal cycle.

is carefully analyzed and the most likely choice for the DEWH turn off is chosen. If a drop of 750W is identified, then the previous .5–4 h is examined for the most likely turn on point.

The two previous methods are combined to form a water heater load profile.

3.2. Using the DEWH load to determine hot water usage

To determine the hot water usage, it is useful to separate water usage events into two categories: large water draws, such as showers are large enough to cause the heater element to turn on almost immediately, and small water draw events, such as washing hands, which shorten the normal hysteresis cycle.

3.2.1. Small water draw events

If the temperature of the water in the tank is at the minimum setpoint when the heater turns on, the assumption cannot be made that no water has been used. If a small amount of water is used then the element will have to turn on sooner than otherwise. The amount of water used during this cycle can be calculated. It is assumed that the water used during the time when the element is off is used uniformly throughout the cycle.

From analysis of the DEWH load data, the exact times that the heater element turns on are known. In addition, from a simulation of the model similar to that of Fig. 1, the amount of time that would normally elapse for the water to decay from T_{max} to T_{min} is known. As shown in Fig. 4, if the time when the element turns off is t_{turn_off} , then the temperature of the water in the tank at this time is known to be T_{max} . Under a no water usage situation, the element would turn on at time $t_{turn_on_no_usage}$. If instead it turns on at a sooner time, t_{turn_on} , then it can be concluded that some water was used. To determine the amount of water used over the cycle, then a curve is constructed that is parallel to the no water usage curve determined by (2), and runs through the point (t_{turn_on}, T_{min}) . The temperature where this curve intersects the line $t = t_{turn_off}$ is denoted as T_{adj} . This would be the temperature that the water in the tank would have needed to be at for the tank to have turned back on at t_{turn_on} without any water usage. Given this temperature, the size of the tank, and the temperature of the incoming water, T_{in} , the amount of water drawn can be determined from (7):

$$\text{water used} = \text{volume of tank} \times \frac{T_{max} - T_{adj}}{T_{max} - T_{in}} \text{ (L)} \tag{7}$$

3.2.2. Large water draw events

The other type of water use is the major water draw event that causes the temperature of the water to drop far below T_{min} . From

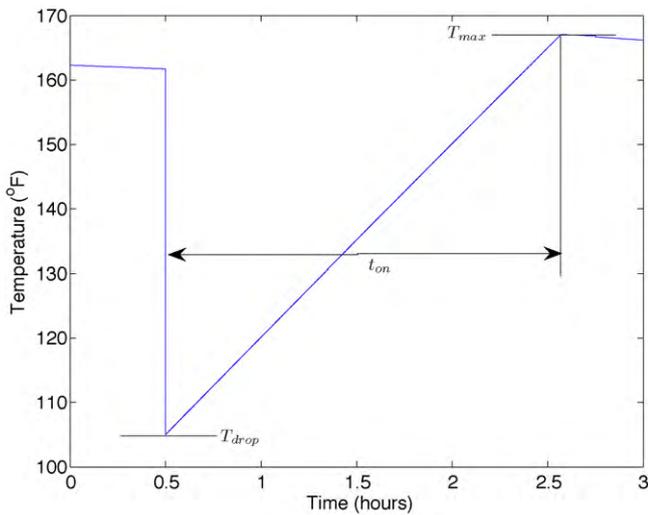


Fig. 5. Determining the water used during large water draw event.

examining the DEWH load data, the time that the element is on for, t_{on} , can be determined. It is also known that when the element is shut off, the temperature of the water in the tank is at T_{max} . By finding a curve that satisfies (2) and runs from the point that the element turns off backwards for time t_{on} , then the temperature that the water in the tank dropped to, T_{drop} , can be determined, as shown in Fig. 5. Similar to the small water use case, a ratio can be used to determine the amount of water that was drawn, as shown in (8):

$$\text{water used} = \text{volume of tank} \times \frac{T_{max} - T_{drop}}{T_{max} - T_{in}} \text{ (L)} \quad (8)$$

Once all of the water usages have been calculated, they are combined and used in the value of B in (2). Fig. 6 shows the total water used and the corresponding temperature of the water in the tank during a two day period of one household.

4. Model simulation, validation, and results

The DEWH load extraction system is validated by comparing the actual measured DEWH load with the calculated load. The predictions of the temperature of the water in the tank are validated by ensuring that the rise and fall of the temperature corresponds correctly with the known state of the element. Several water usage

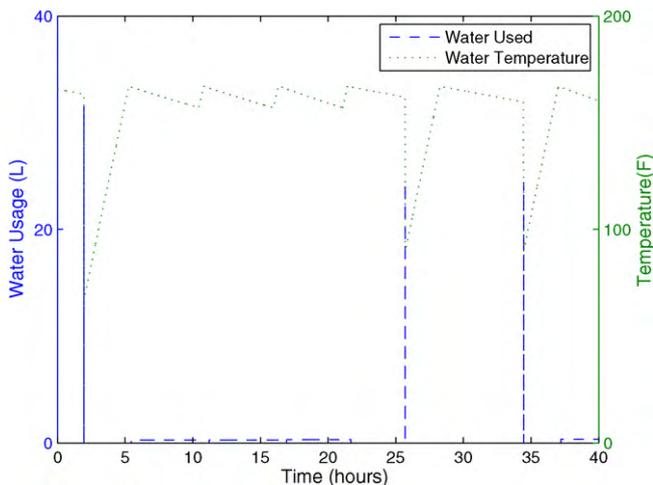


Fig. 6. Water use and temperature of water in tank.

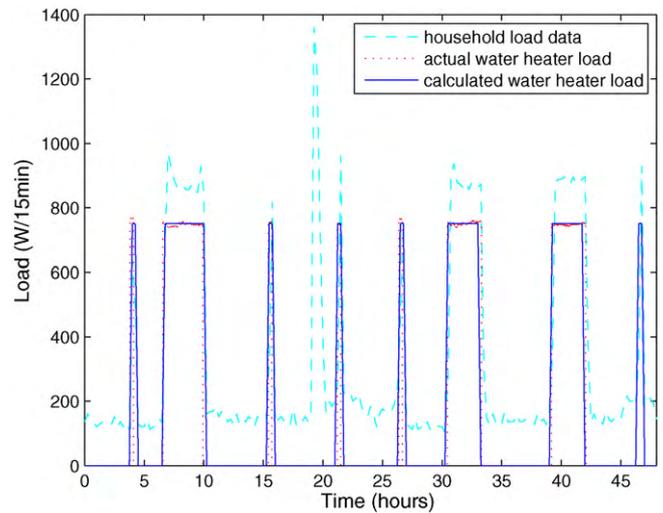


Fig. 7. Extraction of water heater load from household load data.

profiles are also developed using averaged data to illustrate how these profiles may be used for accurate classification of users based on water usage habits. These profiles are validated by interviews with the users.

4.1. Water heater load extraction

In Fig. 7 the smart meter data is presented, as well as the calculated and actual water heater load values. The actual water heater load was determined by logging the power usage of the DEWH with a power meter. The calculation of the water heater load for this time period is very accurate as can be seen in the figure.

4.2. Simulation of model with water usage

Fig. 1 demonstrates the pattern of the water temperature under no water usage conditions. In Section 3, a method for determining water usage from household loads is developed. To validate this method, it must be verified that the element is indeed on when the temperature is predicted to be increasing, and that the element is off during the times that the temperature is predicted to be decreasing. The small water use and large water use events are combined to generate the value of B in (2) and then the model is simulated and compared with the water heater load in Fig. 8. It is clear that

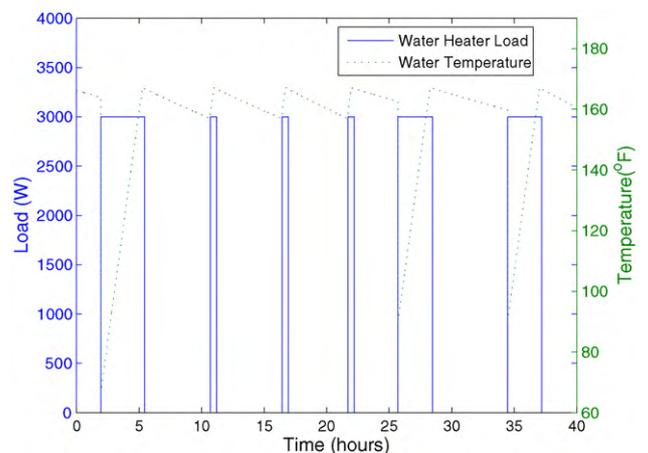


Fig. 8. Temperature of water in tank and water heater load.

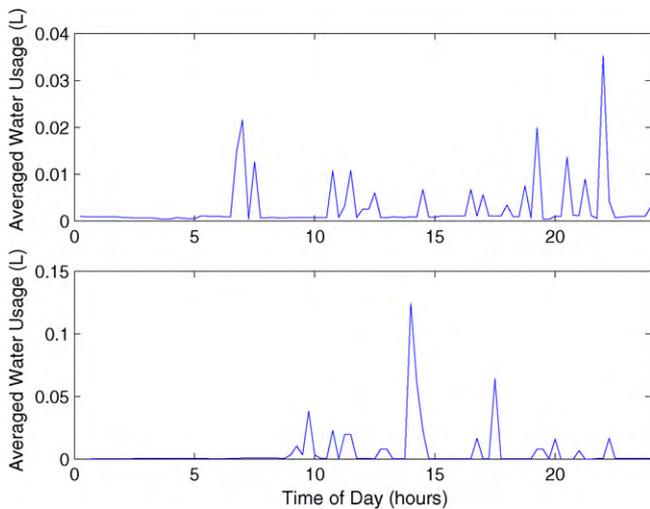


Fig. 9. (Top) Weekday water usage profile and (bottom) weekend water usage profile.

the predictions as to when the water heater element will turn on and off are very accurate.

4.3. Development of water usage profile

As previously stated, water usage profiles are used to classify users more accurately than household load profiles. A water usage profile is developed by analyzing large amounts of hot water usage data. The water usage profile shown in Fig. 9 is generated by averaging the water usage data for one month, in this case July 2008, for each hour of the day. As a result, the water usage profile shows when the user has used the most water during that month. Peaks in the water usage profile represent times of day when the user is more likely to use hot water. It is determined in this case that the weekday and weekend patterns for this user are distinctly different, and, consequently, the two time periods are treated separately. It is known from conducting an interview with the members of this household that the two adults work a regular workday from Monday to Friday, and they tend to use quite a lot of water later in the evenings for laundry because they have two small children. They rarely work weekends, and most of the adult laundry is done

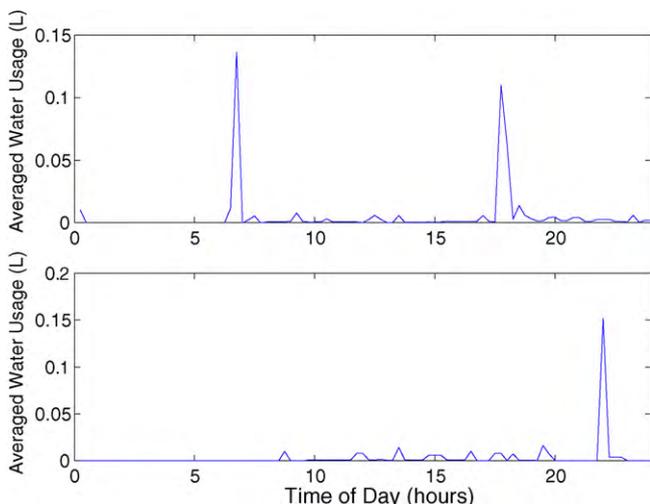


Fig. 10. (Top) Weekday water usage profile and (bottom) weekend water usage profile.

during the day on weekends. The water usage profile reflects this pattern.

Another sample water usage profile is shown in Fig. 10.

The second household contains only a single male. This type of user is very valuable to the load control project because he is so predictable. The figure shows the user's water profile has sharp spikes in the morning around 7 a.m. and around 6 p.m. on weekdays. On weekends the majority of hot water usage is late at night. The sharp spikes in certain areas, as compared to the rest of the profile, which is flat, indicate that the user is very regular in his habits. These users' tanks can be controlled with very low risk that they will be affected, as long as the water is hot in their tank at the points in the profile where the sharp spikes occur. Again, these results were verified by conducting an interview with this user who confirmed that this profile matches his usage patterns very closely.

5. Conclusion

A multi-objective demand side management (DSM) program has been initiated that uses remote control of domestic electric hot water heaters to achieve added power system efficiency and reliability. It has been noted in the past that the widespread acceptance of DSM programs relies on minimal impact to the users. As a result, the DEWH loads cannot be controlled in an aggregate manner as they have in previous literature because users will be adversely affected. To achieve minimal impact, the model of the DEWH must be able to predict the temperature of the water in individual users' tanks in real-time. In addition, the exact knowledge of which tanks are on or off at a given time allows the controller to make better calculations about how much of each of the multiple objectives (peak shaving, frequency regulation, synchronous reserve, etc.) can be achieved at a given time.

In order to make these accurate predictions of the temperature of the water in the tanks, a predictive model is built that generates a water usage profile for each user. The household load data that has been logged for some time by smart meters is processed to extract only the electric water heater loads. From a detailed analysis of the electric water heater load data, the water usage is determined. Once large amounts of water usage data are obtained, a water usage profile is calculated for each user by averaging the data for each hour of the day. This water usage profile is integral in allowing the controller to coordinate which water heaters should be shut off at which times to have maximum benefit with the minimum impact on users.

References

- [1] J. van Tonder, I.E. Lane, Load model to support demand management decisions on domestic storage water heater control strategy, *IEEE Trans. Power Syst.* 11 (4) (1996) 1844–1849.
- [2] K.-Y. Huang, Y.-C. Huang, Integrating direct load control with interruptible load management to provide instantaneous reserves for ancillary services, *IEEE Trans. Power Syst.* 19 (3) (2004) 1626–1634.
- [3] X. Xiong, W. Li, A new under-frequency load shedding scheme considering load frequency characteristics, 2006, *Int. Conf. Power Syst. Technol.* (2006) 1–4.
- [4] I. Hiskens, B. Gong, Mpc-based load shedding for voltage stability enhancement, in: *Proceedings of the 44th IEEE Conf. Decision and Control*, 2005, pp. 4463–4468.
- [5] C.H.A. Alvaro Gomes, A.G. Martins, A multiple objective evolutionary approach for the design and selection of load control strategies, *IEEE Trans. Power Syst.* 19 (2) (2004) 1173–1180.
- [6] C.A.H. Jorge, A. Martins, A multiple objective decision support model for the selection of remote load control strategies, *IEEE Trans. Power Syst.* 15 (2) (2000) 865–872.
- [7] A.M.B. Jianming Chen, N. Lee S Fred, R. Adapa, Scheduling direct load control to minimize system operational cost, *IEEE Trans. Power Syst.* 10 (4) (1995) 1994–2001.
- [8] B.L.M.H. Nehrir, V. Gerez, A customer-interactive electric water heater demand-side management strategy using fuzzy logic, in: *IEEE Power Eng. Soc. 1999 Winter Meeting*, vol. 1, 1999, pp. 433–436.

- [9] M. Nehrir, R. Jia, D. Pierre, D. Hammerstrom, Power management of aggregate electric water heater loads by voltage control, in: 2007 IEEE Power Eng. Soc. Gen. Meeting, 2007, p. 4275790.
- [10] C.A.G. Grayson, C. Heffner, M. Moezzi, Innovative approaches to verifying demand response of water heater load control, *IEEE Trans. Power Deliv.* 21 (1) (2006) 388–397.
- [11] P.D.M. Nehrir, V. Gerez, Development of a monte carlo based aggregate model for residential electric water heater loads, *Elect. Power Syst. Res.* 36 (1) (1996) 29–35.
- [12] N. Lu, D. Chassin, S. Widergen, Modeling uncertainties in aggregated thermostatically controlled loads using a state queueing model, *IEEE Trans. Power Syst.* 20 (2) (2005) 725–733.
- [13] M. Lacroix, Electric water heater designs for load shifting and control of bacterial contamination, *Energy Conver. Manage.* 40 (1999) 1313–1339.
- [14] S.C.H.T. Yang, P. Peng, Genetic k-means-algorithm-based classification of direct load-control curves, *IEE Proc. Gener. Transm. Distrib.* 152 (4) (2005) 489–495.
- [15] I. Lane, N. Beute, A model of the domestic hot water load, *IEEE Trans. Power Syst.* 11 (4) (1996) 1850–1855.
- [16] J. Laurent, R. Malhame, A physically-based computer model of aggregate electric water heating loads, *IEEE Trans. Power Syst.* 9 (3) (1994) 1209–1217.