Decentralized Cooperative Trajectory Estimation for Autonomous Underwater Vehicles

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Abstract— **Autonomous agents that can communicate and
make relative measurements of each other can improve their
collective localization accuracies. This is referred to as cooper-
ative localization (CL). Autonomous underwater vehicle (AUV)
CL is constrained by the low throughput, high latency, and
unreliability of of the acoustic channel used to communicate
when submerged. Here we propose a CL algorithm specifically
designed for full trajectory, or maximum *a posteriori*, estimation
for AUVs. The method is exact and has the advantage that the
broadcast packet sizes increase only linearly with the number
of AUVs in the collective and do not grow at all in the case
of packet loss. The approach allows for AUV missions to be
achieved more efficiently since: 1) vehicles waste less time
surfacing for GPS fixes, and 2) payload data is more accurately
localized through the smoothing approach.**

I. INTRODUCTION

Accurate self-localization in underwater environments is
notoriously challenging. Typical underwater vehicles mis-
sions such as ship hull inspections [1], under-ice exploration
[2], and mine countermeasures [3] all benefit from improved
localization. In many cases surfacing for a GPS fix is
either dangerous or impractical. Without access to GPS and
in the absence of pre-installed infrastructure such as long
baseline (LBL) beacons, underwater localization is generally
achieved through a combination of Doppler velocity log
(DVL), inertial sensors, and compasses [4]. Integration of
velocity, acceleration, or angular rate sensor data to estimate
position will always result in unbounded growth in error
(referred to as dead reckoning).

Multi-AUV deployments are becoming common as ve-
hicles become cheaper and more autonomously capable. If
vehicles in a team have the ability to communicate and make
relative measurements of each other, then they can slow their
rate of position uncertainty growth [5]. In the literature this
is often referred to as cooperative localization [6].

CL will reduce the rate of position uncertainty growth for
the vehicles in the team (as compared with dead reckoning).
The rate of uncertainty growth decreases as the size of the
robot team increases, but is subject to the law of diminishing
returns [7]. In addition, the rate of uncertainty growth is
independent of the accuracy or frequency of the inter-vehicle
measurements [8].

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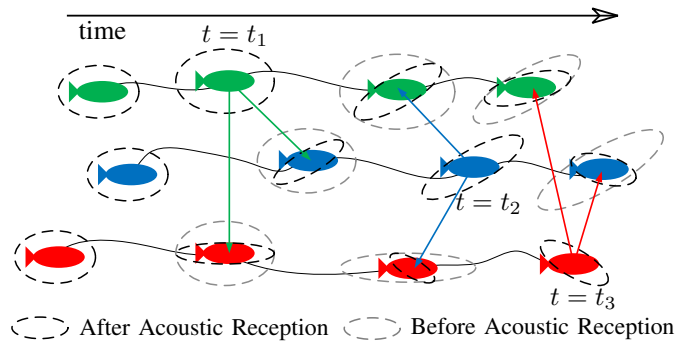`mae.seto@drdc-rddc.gc.ca`

Fig. 1. Conceptual figure showing acoustic communications amongst three
AUVs in a time division multiple access scheme. Green AUV transmits
at time t_1, followed by the blue one at time t_2, and finally the red one
at time t_3. Each reception enables the receiver to obtain a relative range
measurement of the sender based on the travel time of the packet and reduce
its location uncertainty in the direction of the sender (gray ellipse to black
ellipse).

Exact methods require that vehicles must either share their
filtered estimates and full covariances [9] or all raw propri-
oceptive and exteroceptive data [10] to the other members
of the team. For AUV CL this can be problematic since
communicating over any appreciable distance underwater
requires the use of the acoustic channel, which has severe
inherent challenges:

1) High latency: The speed of sound (SoS) in water is
   roughly 2×10^5 times slower than the speed of light
   in air.
2) Reduced bandwidth: On the order of 10-100 bytes/s
   and there is an inherent tradeoff between packet size
   and reliability.
3) Unacknowledged: Only one node (vehicle) can trans-
   mit at a time. Channel is shared using time-division
   multiple access (TDMA). Packet reception is not
   known to the sender unless an acknowledgement is
   sent in the next transmission.
4) Low Reliability: Packet drop rates from 20-50% or
   more depending on the environmental conditions are
   common.

In CL, vehicles must also make relative observations of
one another. Since transmission on the acoustic channel
propagates at the SoS in water (≈ 1500m/s), then relative
ranges between sender and receiver can be calculated by
measuring the time-of-flight (ToF) of the transmission and
the known SoS:

$$\text{Relative Range} = \text{ToF} \times \text{SoS}. \quad (1)$$

If vehicles can precisely synchronize their onboard clocks, for example by aligning with the GPS time signal at the surface and maintaining the time with a precise oscillator while submerged, then they can calculate these relative ranges through one-way travel ToF [11]. For a team of N AUVs, a broadcast acoustic packet can possibly result in $N - 1$ range measurements relative to the sender. Consequently, and unique to the underwater CL case, the relative measurements and the inter-vehicle communications are necessarily concurrent. A conceptual representation of AUV CL is shown in Fig. 1 where the colored arrows represent acoustic communications that result in relative measurements.

The key consideration in AUV CL is how to utilize the acoustic channel. However, design decisions made upstream, such as the choice of state estimator necessarily have a significant impact.

We proposed a method that draws inspiration from previous "multi-centralized" approaches where the full centralized state of the team is estimated onboard each robot [12], [13], [14]. However, in our case, the joint states estimated onboard each vehicle vary across the team. This is a result of the insight that agents need not estimate the poses of the others in between measurement/communication times nor their headings at any time since relative range are independent of heading. Additionally, using an approach similar to the "anti-factor" idea proposed in [15], our system is robust to communications failures without having to resend data by defaulting to send proprioceptive constraints that connect the current vehicle pose to the point of last known confirmed successful communication. In the case that the receiving vehicle already has some of the information contained within the factor that is transmitted, then a new correct and consistent factor can be generated through local subtraction. This is related to the "origin-state" method proposed in [16], but extends it by removing the need for relative measurements/communications to be unidirectional.

These design choices result in a CL scheme that has the following contributions, some of which are achieved by previous works, but none to our knowledge are able to claim in combination:

- Provides full multi-robot trajectory estimation
- Data packet size scales linearly with size of robot team
- Data packet size is constant in the case of communications failures
- Adaptive to the performance of the communications channel
- Provides consistent estimates (avoids overconfidence)
- Does not discard any measurement data and is therefore exact

Multi-AUV deployments can be beneficial in terms of being able to parallelize missions. Our proposed approach provides further benefits:

1) The need to surface to bound localization error is reduced since:

    a) Any vehicle surfacing will transfer the benefit to the entire team,

    b) Localization error grows more slowly when agents can cooperatively localize,

2) Payload data collection is more efficient by combining a trajectory estimation approach with adaptive planning [17].

In Sec. II, we provide a non-exhaustive review of CL literature with a particular focus on the underwater case. In Sec. III we formulate the centralized cooperative trajectory estimation problem as a non-linear least squares optimization. We show that the data transmission requirements to recover this fully centralized estimate vastly exceed the capabilities of the acoustic channel. In Sec. IV, we propose a decentralized version of the trajectory estimation problem and detail exactly what data should be transmitted and how the appropriate factors in the factor graph should be computed from the incoming packets. Experimental results are presented in Sec. V using real AUV navigation data from multiple AUVs and simulated acoustic communications under various conditions. We conclude in Sec. VI.

II. Cooperative Localization Literature

Perhaps the first work to exploit relative measurements between robots for localization was [5] where members of the team are divided into two groups which take turns remaining stationary as landmarks for the other. The term cooperative localization was coined in [6], where the necessity for some robots to be stationary was also removed. Subsequently, many have suggested different estimation algorithms such as distributed EKF [9], maximum likelihood [18], maximum *a posteriori* (MAP) [19], and particle filter [20]. Although many of these works cite the underwater case as a possible application domain, they all require communication capabilities that are infeasible underwater.

Recently, some works have specifically addressed the communications bandwidth issue through quantization of measurement data [21], [14], [13], or estimation of unknown correlations through covariance intersection [22]. The quantization-based approach is based on the sign-of-innovation Kalman filter and still requires transmission of at least 1 bit for every real-valued measurement. In addition, these approaches are not robust to unknown communications failures. The covariance intersection method in [22] can claim the same linear scalability of data throughput with the size of the robot team, however this method is approximate.

Several methods are capable of handling asynchronous communications such as [23], [12], [22]. For example, [23] provides a framework for deciding under what conditions raw data can be replaced by filtered estimates. Similarly in [12] a delayed-state filter is proposed. These works have two notably shortcomings for implementation underwater: first, filtering approaches will always require the transmission of the joint state covariance matrix which scales $\mathcal{O}(N^2)$ where N is the size of the robot team, and secondly, data backlog over extended periods of disconnectivity between nodes is problematic.
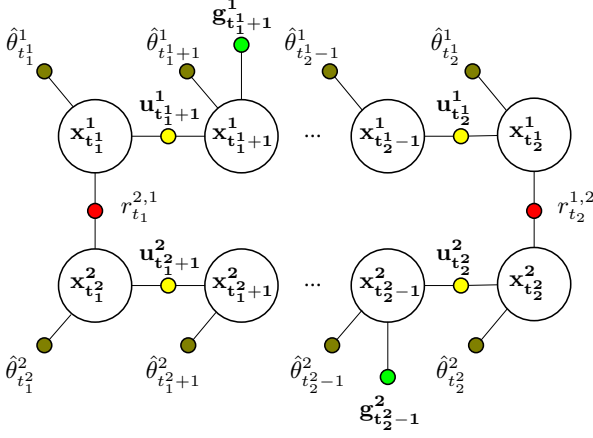
Fig. 2. Factor graph representation of multi-AUV cooperative trajectory estimation between two AUVs. Each vehicle estimates its own position through compass measurements $\hat{\theta}$, DVL-derived odometry, $\mathbf{u}$, and occasional GPS measurements $\mathbf{g}$. Vehicles can additionally improve their pose estimates through relative range observations r.

A. The Underwater Case

For AUV CL the communications channel is the fundamental limitation. There are two basic approaches: either vehicles transmit pose estimates (distributions) or raw measurements. In the former, a key consideration is accounting for the correlations that are induced between vehicles as neglecting these will inevitably result in inconsistency and divergence. A hierarchical approach sidesteps this problem by restricting communication and relative ranging to be one-way. For example, [24], where one or more support vehicles are referred to as communications and navigation aids (CNA), and [25], [16] where vehicles are separated into "servers" and "clients". The necessity to transmit a full joint covariance matrix can also be avoided through the interleaved update approach in [26], however the estimates from this approach are overly conservative. In the case of transmission of raw data, the issue becomes how to selectively transmit data since sensor frequencies are generally orders of magnitude higher than the communication frequencies. In [10], a keyframe-style approach is used, where only a subset of the relative measurements are used and the remaining communication slots are used to marshal data. The keyframe rate is chosen *a priori* based on the expected performance of the communication channel. Unexpectedly poor communication performance or long periods of disconnectivity will always result in data backlogging and algorithm failure. In our approach we transmit raw data but we combine measurements together to avoid this backlogging problem.

III. CENTRALIZED COOPERATIVE TRAJECTORY ESTIMATION

We begin by formulating the centralized trajectory estimation problem. Specifically we consider a 2D kinematic motion model for a torpedo-style AUV since depth can be accurately observed with a pressure sensor. When submerged, AUVs dead reckon using a DVL sensor that measures the velocity relative to the seabed and a compass.

Let the pose of vehicle i at time t be represented by: $\mathbf{x}_t^i = [x_t^i, y_t^i, \theta_t^i]^T$. The centralized trajectory estimator state is $\mathbf{x}_c \triangleq \mathbf{x}_{1:T}^{1:N}$ where N is the number of vehicles in the collective and T is the present time. Each vehicle propagates an estimate of its own pose using velocity data, $\mathbf{u}_t^i = [v_t^i, w_t^i]^T$, where v and w are the forward and starboard returns from the DVL:

$$\mathbf{x}_t^i = f(\mathbf{x}_{t-1}^i, \mathbf{u}_t^i) + \zeta_t^i \ , \ \zeta_t^i \sim \mathcal{N}(0, \Sigma_t^i)$$
$$= \mathbf{x}_{t-1}^i + \begin{bmatrix} \Delta t \mathcal{R}(\theta_{t-1}^i) \mathbf{u}_t^i \\ 0 \end{bmatrix} + \zeta_t^i \quad (2)$$

where Δt is the reciprocal of the frequency of the DVL sensor, $\mathcal{R}(\theta_{t-1}^i)$ is the standard 2x2 rotation matrix, and the additive noise covariance, Σ_t^i, is calculated as:

$$\Sigma_t^i = \begin{bmatrix} \Delta t^2 \mathcal{R}(\theta_{t-1}^i) \begin{bmatrix} \sigma_{vv}^2 & 0 \\ 0 & \sigma_{ww}^2 \end{bmatrix} \mathcal{R}(\theta_{t-1}^i)^T & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 0 \end{bmatrix}$$

where σ_{vv} and σ_{ww} are the RMS error values of the DVL sensor in the forward and starboard directions respectively.

The heading is assumed directly observable through compass measurements $\hat{\theta}_t^i$:

$$\hat{\theta}_t^i = \theta_t^i + \gamma \ , \ \gamma \sim \mathcal{N}(0, \sigma_{\hat{\theta}\hat{\theta}}^2) \quad (3)$$

When an AUV is at the surface, position is directly observable through GPS measurements $\mathbf{g}_t^i$

$$\mathbf{g}_t^i = [x_t^i, y_t^i]^T + \xi \ , \ \xi \sim \mathcal{N}(0, \Xi) \quad (4)$$

where Ξ is the diagonal matrix of RMS squared values for the error of the GPS sensor.

Vehicles communicate with each other using the acoustic modem and share the channel through time division multiple access (TDMA). In our implementation the TDMA sequence is decided beforehand. However, it is possible to devise flexible schemes whereby slots can be chosen dynamically. In the fixed case that we are using there is no need to send any vehicle identifier since the packet origin can be inferred from the TDMA sequence. Vehicles synchronize their onboard clocks to the GPS time signal before submerging and then maintain the time onboard with precise clocks [27]. AUV j sends acoustic transmission $k = 1..K$ at time $t_k \triangleq t_k^j$ and it is received on vehicle i at time $t_k + \Delta_k^{i,j} \triangleq t_k^i$ where $\Delta_k^{i,j}$ is the TOF of the acoustic packet. The resulting range measurement is represented by the RV $r_{t_k}^{i,j}$. It should be noted that in reality the acoustic transmission is sent from point to point in 3D space. We project the range onto the 2D plane which requires knowledge of both vehicles' depths, d_i and d_j:

$$r_{t_k}^{i,j} \triangleq r_{2D} = (r_{3D}^2 - (d_i - d_j)^2)^{\frac{1}{2}}$$

The range measurement model is given by:

$$r_{t_k}^{i,j} = h(\mathbf{x}_{t_k^i}^i, \mathbf{x}_{t_k^j}^j) + \delta_{t_k}^{i,j} \ , \ \delta_{t_k}^{i,j} \sim \mathcal{N}(0, \sigma_{rr}^2)$$
$$= ||[x_{t_k^i}^i, y_{t_k^i}^i]^T - [x_{t_k^j}^j, y_{t_k^j}^j]^T||_2 + \delta_{t_k}^{i,j} \quad (5)$$

where σ_{rr}^2 is the covariance of the range measurement and is assumed to be constant with time and independent of range, a claim experimentally validated in [11].

By moving all non-noise terms onto the left hand side of equations (2)-(5) and following the method in [28] we can factorize the joint probability over vehicle trajectories, inputs, and measurements, as a product of conditionals:

$$p(\mathbf{x}_c, \mathbf{u}_{1:t}^{1:N}, \mathbf{g}_{1:t}^{1:N}, \hat{\theta}_{1:t}^{1:N}, r_{t_1:t_k}^{1:N,1:N}) \propto \prod_{t=1}^{T} \prod_{i=1}^{N} p(\mathbf{x}_t^i | \mathbf{x}_{t-1}^i, \mathbf{u}_t^i)$$

$$\prod_{t=1}^{T} \prod_{i=1}^{N} p(\mathbf{g}_t^i | \mathbf{x}_t^i) \prod_{t=1}^{T} \prod_{i=1}^{N} p(\hat{\theta}_t^i | \theta_t^i) \prod_{k=1}^{K} \prod_{i=1}^{N} \prod_{\substack{j=1 \\ i \neq j}}^{N} p(r_{t_k}^{i,j} | \mathbf{x}_{t_k^i}^i, \mathbf{x}_{t_k^j}^j)$$

(6)

Note that for convenience we have omitted the priors since in the field the AUV prior location is initialized with GPS on the surface and is encapsulated by $\mathbf{g}$.

We represent the joint probability given in (6) as a Gaussian factor graph as shown in Fig. 2 and follow the procedure in [28] to represent the problem as a non-linear least squares optimization problem and solve for $\mathbf{x}_c^*$, the MAP estimate of all vehicle trajectories:

$$\mathbf{x}_c^* = \underset{\mathbf{x}_c}{\operatorname{argmin}} \{ \sum_{t=1}^{T} \sum_{i=1}^{N} \frac{1}{2} ||f(\mathbf{x}_{t-1}^i, \mathbf{u}_t^i) - \mathbf{x}_t^i||_{\Sigma_t^i}^2 +$$

$$\sum_{t=1}^{T} \sum_{i=1}^{N} \frac{1}{2} ||[x_t^i, y_t^i]^{\mathrm{T}} - \mathbf{g}_t^i||_{\Xi}^2 + \sum_{t=1}^{T} \sum_{i=1}^{N} \frac{1}{2} ||\theta_t^i - \hat{\theta}_t^i||_{\sigma_{\hat{\theta}\hat{\theta}}^2}^2 +$$

$$\sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{1}{2} ||h(\mathbf{x}_{t_k^i}^i, \mathbf{x}_{t_k^j}^j) - r_{t_k}^{i,j}||_{\sigma_{rr}^2}^2 \},$$

(7)

where the standard squared Mahalanobis distance notation $||e||_{\Sigma}^2 = e\Sigma^{-1}e^{\mathrm{T}}$ is used. In the implementation, (7) can solved incrementally [29].

A. Data Throughput Required for Centralized Trajectory Estimate

The centralized multi-vehicle MAP estimate is obtained by solving (7). This requires knowledge of all proprioceptive and exteroceptive measurement data from all vehicles for all time.

1) No Comms Dropouts: If the DVL and compass frequencies are 10Hz and each piece of data can be encoded with 1byte (8 bits) and the TDMA slot length is 10s and the number of vehicles in the team is N, then each vehicle would potentially need to transmit (8bits/piece of data*30 pieces of data /second * 10seconds/slot * N slots)*N vehicles=21.6Kbits of data per transmission for a modest team size of $N = 3$. Even in the case of further one-bit quantization as proposed in [21], the amount of data per transmission is still 2.7Kbits of data. Such throughput rates are unachievable in water.

2) With Comms Dropouts: In the inevitable case that there are communications dropouts, the data required to be transmitted is unbounded and grows linearly with time. In the worst case all vehicles would need to transmit all their sensor data from the start of the mission.
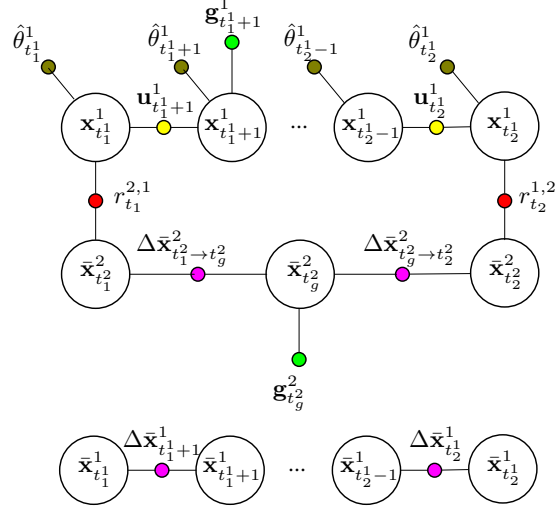

Fig. 3. Factor graph representation of decentralized multi-AUV trajectory estimation. Vehicle 1 now maintains 2 factor graphs. **Top:** The new local multi-AUV factor graph. **Bottom:** The dead-reckoning (DR) position graph. Marginalization is performed on the DR position graph to compute the factors that other members of the team require in order to generate their own local multi-AUV trajectory estimates.

IV. PROPOSED DECENTRALIZED MULTI-AUV TRAJECTORY ESTIMATION

Here we propose a modified version of (7) where the amount of data required to be passed between vehicles is feasible within the restrictive acoustic channel and accounts for the challenges enumerated in Sec. I. The key to the approach is that each vehicle can treat the others as moving beacons and only needs to estimate their positions at communication/measurement times in order to obtain all of the benefits of cooperative trajectory estimation locally.

We begin with a few shorthand notation definitions. The position of vehicle i at time t is given by $\bar{\mathbf{x}}_t^i \triangleq [x_t^i, y_t^i]^{\mathrm{T}}$. With a slight abuse of notation let the position of vehicle i at transmission time t_k be given by $\bar{\mathbf{x}}_k^i \triangleq [x_{t_k^i}^i, y_{t_k^i}^i]^{\mathrm{T}}$

Each vehicle j locally maintains two factor graphs as shown in Fig. 3. The first consists of own-vehicle poses for all time and other vehicle positions for all communications/measurement times:

$$\mathbf{x}_d^j \triangleq [\bar{\mathbf{x}}_{1:K}^{1:j-1}, \mathbf{x}_{1:T}^j, \bar{\mathbf{x}}_{1:K}^{j+1:N}]^{\mathrm{T}} \quad (8)$$

The second is a dead-reckoning (DR) position graph that is used to estimate only own-vehicle position: $\bar{\mathbf{x}}_{1:T}^j$ using compass and DVL sensor data directly (as opposed to estimating heading):

$$\bar{\mathbf{x}}_t^j = \bar{\mathbf{x}}_{t-1}^j + \Delta\bar{\mathbf{x}}_t^j + \bar{\zeta}_t^j , \; \bar{\zeta}_t^j \sim \mathcal{N}(0, \bar{\Sigma}_t^j) \quad (9)$$

where $\Delta\bar{\mathbf{x}}_t^j \triangleq \Delta t \mathcal{R}(\hat{\theta}_t^j) \mathbf{u}_t^j$ and:

$$\bar{\Sigma}_t^j = \Delta t^2 \begin{bmatrix} \mathcal{R}(\hat{\theta}_t^j) & \mathcal{R}'(\hat{\theta}_t^j)\mathbf{u}_t^j \end{bmatrix} \mathbf{Q} \begin{bmatrix} \mathcal{R}(\hat{\theta}_t^j) & \mathcal{R}'(\hat{\theta}_t^j)\mathbf{u}_t^j \end{bmatrix}^{\mathrm{T}}$$

(10)

where $\mathbf{Q}$ is diagonal matrix with diagonal elements $\sigma_{uu}, \sigma_{vv}, \sigma_{\hat{\theta}\hat{\theta}}$.

This DR position graph is used to generate the factors that will be transmitted to other vehicles. From the DR position graph we can generate a change in position factor (estimate and associated covariance) from any start time to any end time by marginalizing out intermediate position nodes. In this case marginalization is equivalent to performing a compounding operation, and since we are only considering positions and not orientations, this operation is equivalent to simple addition. For example to combine position factors from time t_1 to t_2:

$$\bar{\mathbf{x}}_{t_1}^j = \bar{\mathbf{x}}_{t_2}^j + \Delta\bar{\mathbf{x}}_{t_1 \to t_2}^j + \bar{\zeta}_{t_1 \to t_2}^j \quad (11)$$

where:

$$\Delta\bar{\mathbf{x}}_{t_1 \to t_2}^j \triangleq \sum_{t=t_1}^{t_2} \Delta\bar{\mathbf{x}}_t^j \quad (12)$$

and

$$\bar{\zeta}_{t_1 \to t_2}^j \triangleq \sum_{t=t_1}^{t_2} \bar{\zeta}_t^j \sim \mathcal{N}(0, \bar{\Sigma}_{t_1 \to t_2}^j) \quad (13)$$

with:

$$\bar{\Sigma}_{t_1 \to t_2}^j = \sum_{t=t_1}^{t_2} \bar{\Sigma}_t^j \quad (14)$$

Each vehicle uses its own local DR position graph combined with the bookkeeping algorithm described below to determine which set of factors should be transmitted such that other vehicles in the team will be able to generate a local estimate of the multi-vehicle trajectory.

A. Bookkeeping

Bookkeeping is required for vehicles to know which local factors should be generated to guarantee consistency of the multi-vehicle estimates maintained by others. Each vehicle i maintains a set of $N-1$ incoming ($\mathcal{C}_{in}^i$) and outgoing ($\mathcal{C}_{out}^i$) confirmed contact points. These contact points are the times of most recent confirmed successful communications to and from each other vehicle in the team.

Incoming contact points are easily detectable based on the times at which communications are received. Outgoing contact points necessitate the use of communicated acknowledgment bits that are sent in subsequent data packet transmissions. In the case that an acknowledgement communication also fails, the contact point time will not be updated, in essence assuming that the previous outgoing communication had failed. However, in the case that this implied assumption is incorrect, the receiving vehicle will still be able to recover the appropriate factor from the data sent using the subtraction property for change in position factors (see Sec. IV-C).

As an example, for the case of fully successful transmissions for an entire cycle depicted in Fig. 1, the incoming contact point time sets after the communication at time t_3 are given by:

$$\begin{aligned}
\mathcal{C}_{in}^1 &= \{-, t_2^1, t_3^1\} \\
\mathcal{C}_{in}^2 &= \{t_1^2, -, t_3^2\} \\
\mathcal{C}_{in}^3 &= \{t_1^3, t_2^3, -\}
\end{aligned} \quad (15)$$

Algorithm 1 Generating data packet for acoustic transmission on-board vehicle i at time t_K^i

1: Transmission queue is empty
2: **for all** $j = 1..N, j \neq i$ **do**
3:     Calculate $\Delta\bar{\mathbf{x}}_{\mathcal{C}_{in}^i[j] \to t_K^i}$ and $\Delta\bar{\mathbf{x}}_{\mathcal{C}_{out}^i[j] \to t_K^i}$ and associated covariances $\bar{\Sigma}_{\mathcal{C}_{in}^i[j] \to t_K^i}^i$ and $\bar{\Sigma}_{\mathcal{C}_{out}^i[j] \to t_K^i}^i$ and add them to transmission queue.
4:     Add range measurement $r_{\mathcal{C}_{in}^i[j]}^{i,j}$ to transmission queue.

5: **end for**
6: **if** have GPS update, $\mathbf{g}_{t_g}^i$ with ($t_g > \min\{\mathcal{C}_{in}^i, \mathcal{C}_{out}^i\}$) **then**
7:     Add $\mathbf{g}_{t_g}^i$, $\Delta\bar{\mathbf{x}}_{t_g \to t_K^i}^i$, and $\bar{\Sigma}_{t_g \to t_K^i}^i$ to the transmission queue.
8: **end if**
9: Add acknowledgment bits to the transmission queue.
10: Push data to the modem hardware for transmission.

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and the outgoing contact point times sets are given by:

$$\begin{aligned}
\mathcal{C}_{out}^1 &= \{-, t_1^1, t_1^1\} \\
\mathcal{C}_{out}^2 &= \{t_0^2, -, t_2^2\} \\
\mathcal{C}_{out}^3 &= \{t_0^3, t_0^3, -\}
\end{aligned} \quad (16)$$

where a '$-$' represents the entry in the set that corresponds to the vehicle on which it resides ($\mathcal{C}_{in}^i[i] = \mathcal{C}_{out}^i[i] = $ '$-$'). All contact points in this case were initialized to t_0. Note that the outgoing contact point times for $\mathcal{C}_{out}^3$ are still t_0 because AUV 3 (red) has no knowledge about whether the communications that it sent at time t_3 were successful until it gets a confirmation through the acknowledgement bits.

B. Acoustic Packet Transmission

Consider the case where vehicle i makes an acoustic transmission at time t_K. The following data should be included in the data packet:

- The change in position factors from incoming and outgoing contact point times to the present time (line 3),
- Range data associated with each of the incoming contact point times (line 4) as well as corresponding vehicle depth,
- A local GPS measurement if one has been made since the oldest contact point and a change in position to the time of the GPS measurement, t_g (line 7),
- A set of $N-1$ acknowledgment bits (line 9),

The packet generation is described in Algorithm 1.

C. Acoustic Packet Reception

Upon reception of an acoustic packet on vehicle j sent from vehicle i, the receiver must generate the correct factors to compute the MAP estimate of $\mathbf{x}_d^j$. Generating the correct change in position factors that relate the positions of other vehicles to own-vehicle poses can possibly require performing a subtraction operation on the the change in position factor. For example, consider the case where AUV j receives two

Algorithm 2 Reception of acoustic data packet K on AUV j sent from AUV i at time t_K

1: Calculate new range factor $r_{t_K}^{i,j}$ from TOF and synchronized clocks
2: **for all** new range factors $r_{t_k}^{i,p}$ received or calculated **do**
3:    **if** $\bar{\mathbf{x}}_{t_k^i}^i$ or $\bar{\mathbf{x}}_{t_k^p}^p$ not in state space **then**
4:       Add to state space
5:    **end if**
6:    Add range factor
7: **end for**
8: **if** GPS factor $\mathbf{g}_{t_g}^i$ received **then**
9:    **if** $\mathbf{x}_{t_g}^i$ not in state space **then**
10:       Add to state space
11:    **end if**
12:    Add GPS factor
13: **end if**
14: **for all** Change in position factors **do**
15:    **if** Factor can be decomposed using subtraction property **then**
16:       Decompose
17:    **end if**
18:    Add change in position factor
19: **end for**
20: $\mathcal{C}_{in}^i[j] \leftarrow t_K$
21: **if** acknowledgment bit '1' received for last transmission time t_k **then**
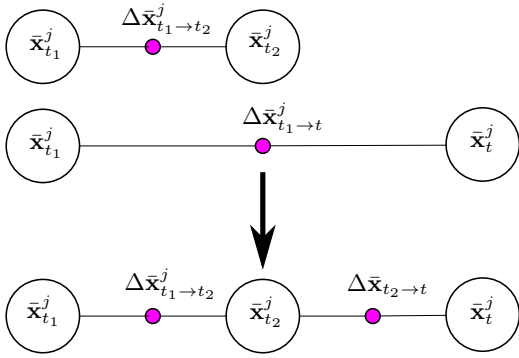22: $\mathcal{C}_{out}^i[j] \leftarrow t_k$
23: **end if**

Fig. 4. The subtraction property illustrated on the change of position factors. The top two change in position factors are received and are decomposed into the single bottom position chain.

change in position factors from vehicle i at time t, $\Delta\bar{\mathbf{x}}_{t_1 \to t}^i$ and $\Delta\bar{\mathbf{x}}_{t_2 \to t}^i$ with $t_1 < t_2$. Then $\Delta\bar{\mathbf{x}}_{t_1 \to t_2}^i$ can be recovered using:

$$\Delta\bar{\mathbf{x}}_{t_1 \to t_2}^j = \Delta\bar{\mathbf{x}}_{t_1 \to t}^j - \Delta\bar{\mathbf{x}}_{t_2 \to t}^j \qquad (17)$$

This is a valid operation since the factors are built using simple addition in (12) and (14). For a visual depiction refer to Fig. 4. This is required when a previous transmission that was assumed to have failed was actually successful.

An overview of the method for processing the received data is given in Algorithm 2. The key is that vehicle j can recover the appropriate range, GPS, and changes in position

needed for its own local multi-vehicle factor graph. The multi-vehicle factor graph is guaranteed to remain connected and consistent at all times because change in position factors originate from times of known communication and therefore relative measurement.

D. Centralized → Decentralized

To obtain a decentralized multi-vehicle trajectory estimate, each AUV, j locally solves the following non-linear least squares problem:

$$
\begin{aligned}
\mathbf{x}_d^{j\,*} = \operatorname*{argmin}_{\mathbf{x}_d^j}\Big\{ &\sum_{t=1}^{T} \frac{1}{2}||f(\mathbf{x}_{t-1}^j, \mathbf{u}_t^j) - \mathbf{x}_t^j||_{\Sigma_t^j}^2 + \\
&\sum_{k_1=1}^{K}\sum_{k_2=k_1}^{K}\sum_{\substack{i=1..N\\i\neq j}} \frac{1}{2}||\bar{\mathbf{x}}_{t_{k_1}}^i + \Delta\bar{\mathbf{x}}_{t_{k_1}\to t_{k_2}}^i - \bar{\mathbf{x}}_{t_{k_2}}^i||_{\bar{\Sigma}_{t_{k_1}\to t_{k_2}}^i}^2 + \\
&\sum_{t=1}^{T}\sum_{i=1}^{N} \frac{1}{2}||\bar{\mathbf{x}}_t^i - \mathbf{g}_t^i||_{\Xi}^2 + \sum_{t=1}^{T} \frac{1}{2}||\theta_t^i - \hat{\theta}_t^i||_{\sigma_{\theta\theta}^2}^2 + \\
&\sum_{k=1}^{k}\sum_{i_1=1}^{N}\sum_{\substack{i_2=1\\i_2\neq i_1}}^{N} \frac{1}{2}|||\bar{\mathbf{x}}_k^{i_1} - \bar{\mathbf{x}}_k^{i_2}||_2 - r_{t_k}^{i_1,i_2}||_{\sigma_{rr}^2}^2 \Big\}.
\end{aligned}
$$

$$(18)$$

which is identical to (7) except that the odometry factors have been re-organized into own-vehicle odometry (first term) and other-vehicle changes in position (second term) and that the compass factors have been removed for all other vehicles (fourth term).

E. Data Throughput Required for Decentralized Trajectory Estimate

Data to be transmitted is at most:

- $2(N-1)$ change in position factors (comprising value and associated covariance),
- $N-1$ range factors and depths,
- one GPS factor with associated change in position,
- $N-1$ acknowledgment bits.

Scaling is linear with respect to the size of the AUV team N and constant with respect to time t even in the worst case of communications dropouts.

V. EXPERIMENTS

A. Setup

Navigation data was collected over several days in October 2012 at our test site in Nova Scotia, Canada. The Iver2 AUV shown in Fig. 5 is equipped with a SonTek DVL, a 3-axis compass, and a WHOI micromodem for acoustic communications [30]. The vehicle operates in a frontseat/backseat configuration where our cooperative trajectory estimation algorithm runs on a backseat 1.6GHz Atom processor running the MOOS-IvP middleware [31] and communicating with the frontseat via the iOceanServerComms application [32]. The cooperative trajectory optimization algorithm uses the open source iSAM [29] and Goby software [33]. We used the data

Fig. 5. The Iver2 AUV in the water at the field site near Dartmouth, Nova Scotia, Canada.
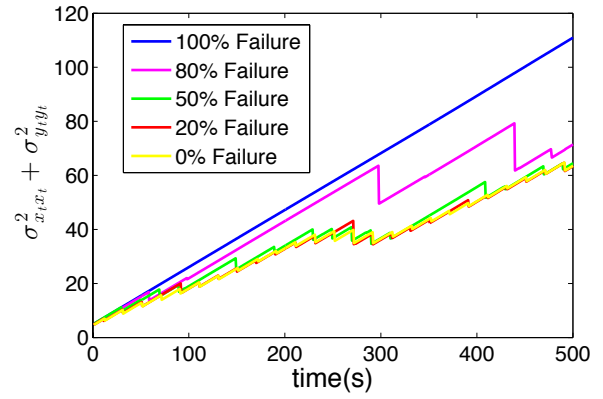

Fig. 6. The instantaneous (filtered) location uncertainty of AUV 1 using the proposed method for different dropout rates.
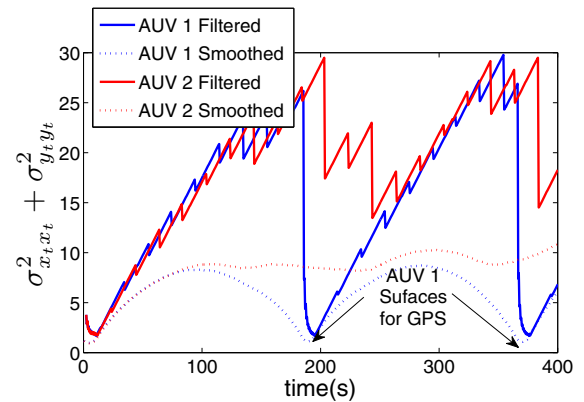

Fig. 7. The instantaneous (filtered) and smoothed uncertainties of two vehicles cooperatively localizing using the proposed method. AUV 1 surfaces for GPS twice thus bounding the uncertainty for both vehicles.

gathered from the multiple runs to simulate a large multi-vehicle experiment by synchronizing the data and playing it all back at once and simulating inter-vehicle ranging and communications. As such, we can directly control the performance of the acoustic communications channel and evaluate the performance of our algorithms under different conditions.

B. Results

Recall the originally stated benefits of the proposed algorithm which were:

1) AUVs minimize surfacing for GPS fixes,
2) Uncertainty is reduced over the entire AUV trajectory.

Here we demonstrate how each of these two objectives have been achieved notwithstanding the challenges of acoustic communications.

1) Objective 1 - Less Frequent Surfacing for GPS Fixes: This objective is achieved in the proposed method in two ways that compound. First, the fact that vehicles are communicating and making relative measurements causes their instantaneous location uncertainty to grow more slowly. Consider Fig. 6 which shows the local filtered location estimate uncertainties for one vehicle using the proposed algorithm. From the figure, even for a 20% success rate of data communications (red plot) there is a significant advantage over no communication (100% failure). The algorithm opportunistically exploits the few successfully received packets without any backlogging of data resulting from all the failed ones.

Second, the surfacing of one vehicle for a GPS fix can bound the localization error of all vehicles in the team. In Fig. 7, both the instantaneous, or filtered, covariances and the smoothed covariances obtained by re-optimizing the trajectory at time $t = 500s$ are shown. The uncertainty on the vertical axis of Fig. 7 is represented as the sum of the autocovariances: $\sigma^2_{x_t x_t} + \sigma^2_{y_t y_t}$. AUV 1 surfaces for GPS twice at which point the instantaneous location uncertainty is reduced to $\approx 3m$. In between the GPS fixes, the uncertainties

of the two vehicles are similar, meaning that AUV 2 obtained most of the benefit of surfacing without ever having to do so.

2) Objective 2 - Uncertainty Reduced over Entire Trajectory: The benefit of full trajectory estimation is shown in Fig. 7 by comparing the instantaneous location uncertainty (solid lines) with the smoothed estimate uncertainty (broken lines). In this case, since inter-AUV measurements are intermittent and provide information about the AUV location directly, smoothing has a large effect.

Fig. 8 shows the smoothed estimate uncertainties for the cases from Fig. 6 that are computed at time $t = 500s$. Even for a success rate of only 50%, the smoothed estimate uncertainties are very close to the optimal case (100% success).

VI. CONCLUSION

We have presented a cooperative trajectory estimation scheme for teams of autonomous underwater vehicles which is able to opportunistically exploit the underwater acoustic communications channel. Normally, mission planners require AUVs to surface when their location uncertainty reaches a threshold. Therefore, the longer an AUV's location uncer-
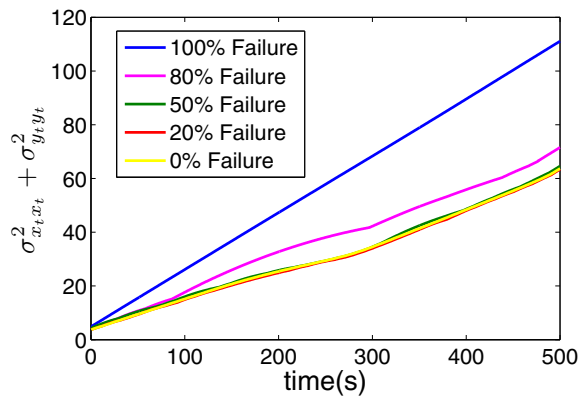
Fig. 8. The location uncertainty of the smoothed estimate at time $t = 500s$ of AUV 1 using the proposed method for different dropout rates.

tainty is maintained below the threshold, the less frequently it needs to surface. In addition, gathered sensor data will be more accurately localized through full trajectory estimation.

Future work in this direction includes a large multi-vehicle deployment and extension to the full cooperative simultaneous localization and mapping scenario.

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