ST86 LECTURE 22

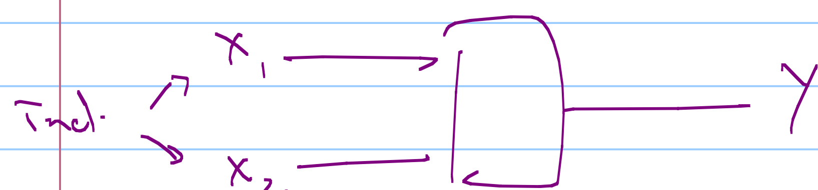
Note Title 5/4/2006

Today:

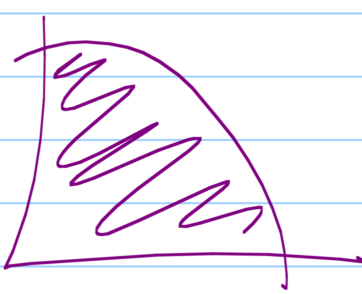
- (Degraded) Broadcast Channel

Review of "Network" I.T.

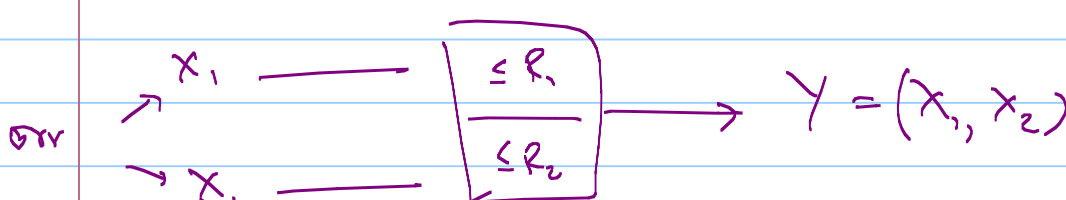
- Multiple Access Channel



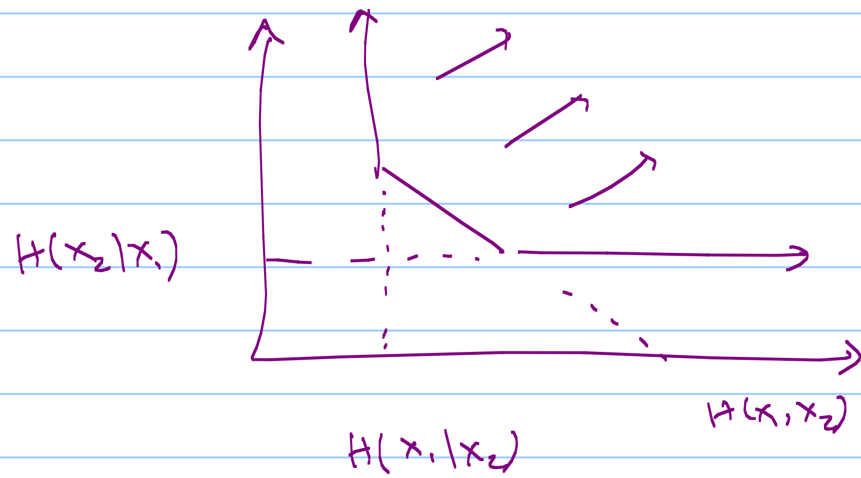
Rate region



- Correlated sources



Capacity needed



Together: Often enable us to achieve capacity

But not always best: Chung's Example

Correlated source produces $W_1 = W_2$ Power $\le P$

Pow $\le \sigma^2$

Channel $= (X_1, X_2) \longrightarrow X_1 + X_2 + Z$

Making X_1, X_2 ind. $\Rightarrow$ Signal power $= 2P$

$\Rightarrow$ Capacity $\div$ $R_1 + R_2 \leq \frac{1}{2}\log\left(1+\frac{2P}{\sigma^2}\right)$

Setting $X_1 = X_2$ $\Rightarrow$ Signal power $= 4P$

Capacity $= \frac{1}{2}\log\left(1+\frac{4P}{\sigma^2}\right)$

Can realize this if $W_1 = W_2$

Also talked about side channels.

Try coding à la Slepian-Wolf as an exercise ....

Today

Broadcast Channels: Single source

Many receivers.

Source sends X

R_1 receives Y_1

R_2 " Y_2

Channel characterized by

$P^1_{Y_1|X}$ & $P^2_{Y_2|X}$

Goal of Receivers = ?

Some common info;

Some disjoint info

Rate triple $\underline{(R_0, R_1, R_2)}$ acheivable

if $\exists$ encoder $\underset{\sim}{X}$, decoders D_1, D_2 s.t.

$$\underline{X} : S_0 \times S_1 \times S_2 \to \Omega_X^n$$

$$D_1 : \Omega_{Y_1}^n \to S_0 \times S_1$$

$$D_2 : \Omega_{Y_2}^n \to S_0 \times S_2$$

$$\left(S_i = \{1 \ldots 2^{nR_i}\} \right)$$

s.t.

$$(W_0, W_1, W_2) \to X(W_0, W_1, W_2) \xrightarrow{\text{Channel}} (Y_1, Y_2)$$

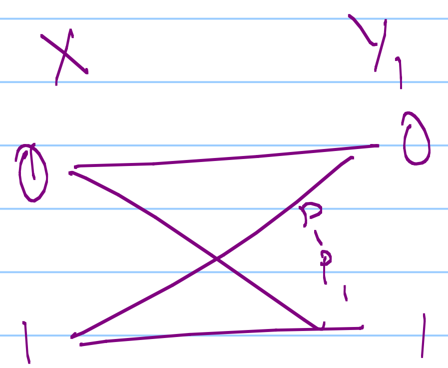
$$\downarrow D_1 \qquad D_2$$

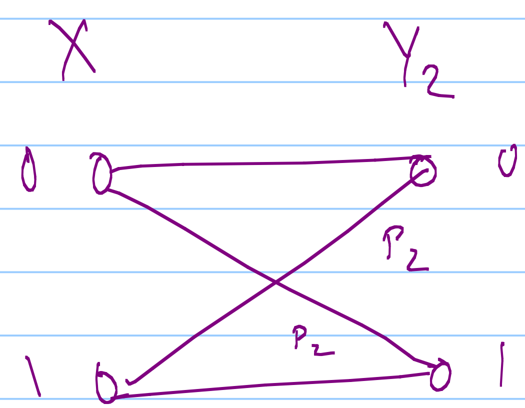
$$(\hat{W}_0, \hat{W}_1) \qquad (\hat{\hat{W}}_0, \hat{W}_2)$$

$$\Pr\left[(W_0, W_0, W_1, W_2) \neq (\hat{W}_0, \hat{\hat{W}}_0, \hat{W}_1, \hat{\hat{W}}_2) \right] \to 0$$

Example

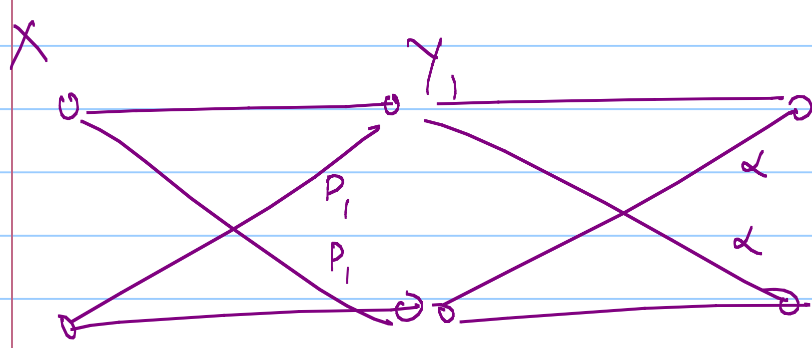
$$\Omega_X = \Omega_{Y_1} = \Omega_{Y_2} = \{0, 1\}$$





$p_1 \leq p_2$

Consider special case where



s.t. $\quad p_1(1-\alpha) + (1-p_1)\alpha = p_2$

Claim: $\forall\ p_1 \le p_2 \le \frac{1}{2}$ such an

$\alpha \le \frac{1}{2}$ exists

Latter channel $\equiv$ any other channel $P_{(Y_1 Y_2)|X}$

with same $P_{Y_1|X}$

$P_{Y_2|X}$

So what is the rate region of BSC Broadcast channel?

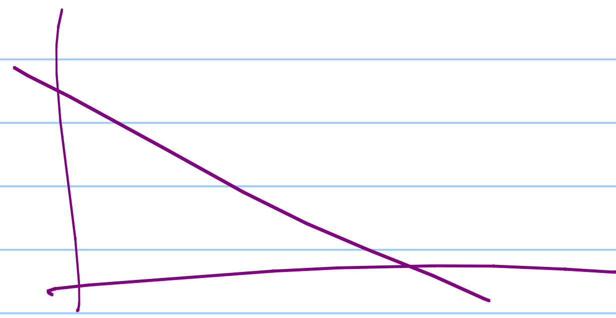
How to encode?

Fix $R_0 = 0$

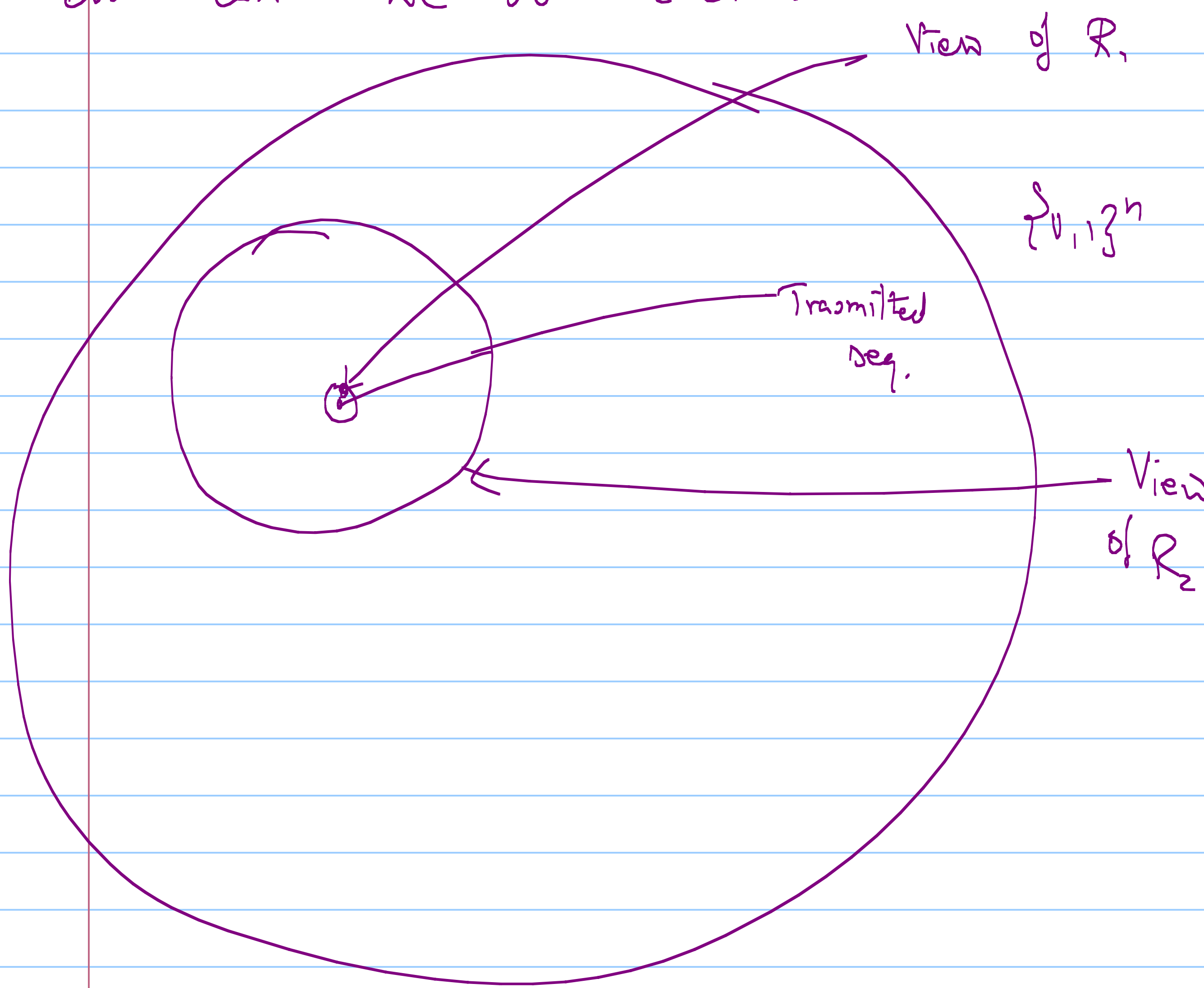
(Aside: $(0, R_1, R_2)$ acheivable

$\Rightarrow (R_0, R_1, R_2 - R_0)$ acheivable)

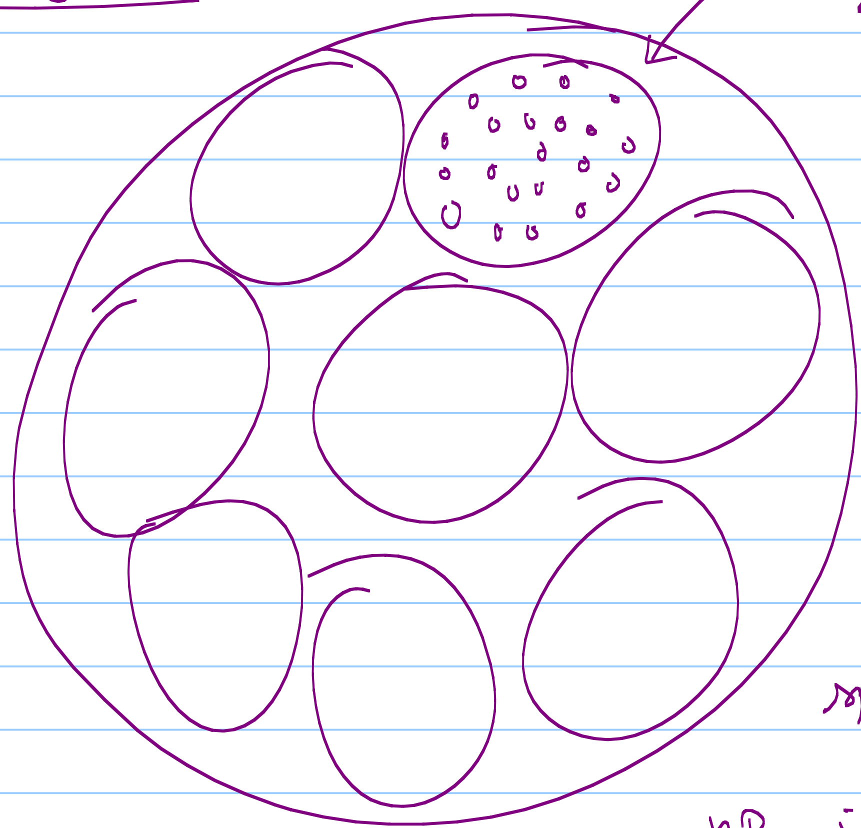
Clearly $R_1 + R_2 \le 1 - H(p_2)$ acheivable



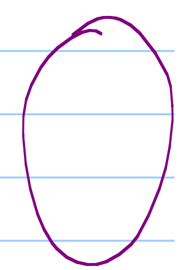
But can we do better?

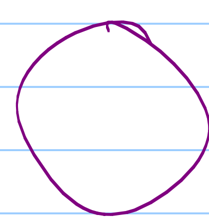


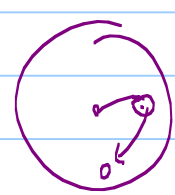
Encoding idea



"Pack" 2^{nR_2} spheres of radius $\sim \gamma \cdot n$ into $\{0,1\}^n$

Then "pack" 2^{nR_1} spheres of radius nP_1 into each 

R_1 : finds o & so knows its position relative to 

R_2 : Views  as corruption ; adds P_2 error ; decodes informal analysis

$$\frac{\mathrm{Vol}((\gamma * p_1), n)}{\mathrm{Vol}(p_1, n)} = 2^{n(H(\gamma * p_1) - H(p_1))}$$

$$\boxed{R_1 \le H(\gamma * p_1) - H(p_1)}$$

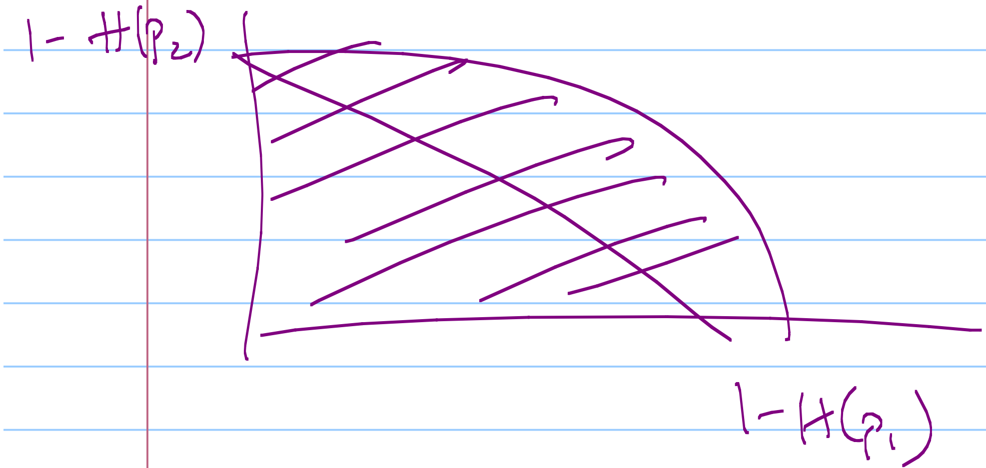
$1 - H(\gamma * p_2) \ge R_2 \leftarrow$ first shift perturbs γ fraction of coordinates; next one perturbs p_2 fraction.

(In lecture, might as well switch $\gamma \to \beta$)

$$R_1 + R_2 = \max_{\gamma} \left\{ 1 - H(\gamma * p_2) + H(\gamma * p_1) - H(p_1) \right\}$$

$$\ge 1 - H(0 * p_2) + H(0 * p_1) - H(p_1) = 1 - H(p_2)$$

Optimizing γ yields



More generally

$$X \to Y_1 \to Y_2 \quad [\text{Degraded B.C.}]$$

(R_1, R_2) achievable if $\exists\, U$

$$U \to X \to Y_1 \to Y_2 \quad \text{s.t.}$$

$$R_2 \le I(U; Y_2)$$

$$R_1 \le I(X; Y_1 \mid U)$$

Encoding P_U

Given (w_1, w_2)

let . $U(w_2)_i \sim P_U$ i.i.d. over i, w_2

Let $X(w_1, w_2)_i \sim P_{X|U}(U(w_2)_i)$ i.i.d over i, w_1, w_2

Transmit $X(w_1, w_2)$

Decoding : Typical Set

Analysis : Two seperate codes

R_1 err $\rightarrow 0$ if $\underset{\text{small}}{I(X; Y_1 | U)}$

R_2 err $\rightarrow 0$ if $\underset{\text{small}}{I(X; Y_2 | U)}$