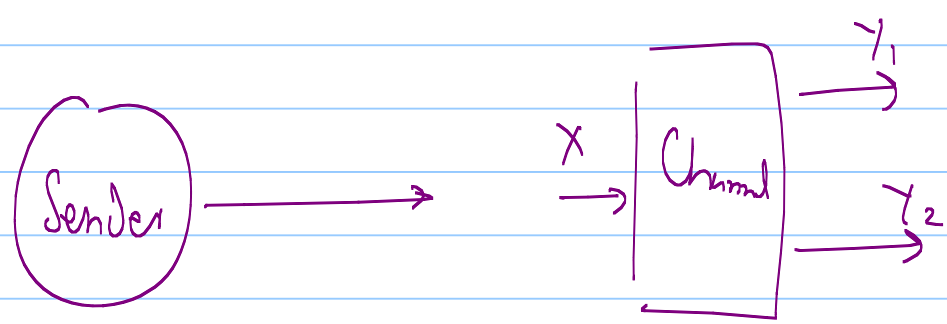
ST06 LECTURE 23

Note Title 5/9/2006

Today

Conclude : Broadcast Channel

Recall



Channel $\approx$ $P_{(Y_1, Y_2) | X}$

Fact: Channels with $P_{(Y_1 Y_2)|X}$ & $P'_{(Y_1 Y_2)|X}$

"equivalent" if $P_{Y_1|X} = P'_{Y_1|X}$

$P_{Y_2|X} = P'_{Y_2|X}$

"Equivalence": (R_1, R_2) achievable on P

$\Leftrightarrow$ (R_1, R_2) achievable on P'

Defn: (R_1, R_2) achievable on channel with P

if $\exists$ E, D_1, D_2 s.t. if

$(W_1, W_2) \in_U S_1 \times S_2$ $\qquad S_i := \{1 \ldots 2^{nR_i}\}$

for

$$(W_1, W_2) \xrightarrow{E} X^n \xrightarrow{\text{Chan}} Y_1^n, Y_2^n \xrightarrow{D_1, D_2} \hat{W}_1, \hat{W}_2$$

$$P_{err} \triangleq P_r\left[(W_1, W_2) \neq (\hat{W}_1, \hat{W}_2)\right] \longrightarrow 0$$

Degraded Channel

$$X \to Y_1 \to Y_2$$

Stochastically degraded channel : P

P s.d. if $\exists$ equivalent P' s.t. P' is Degraded

i.e $P'(Y_2 | Y_1, X) = P'(y_2 | y_1)$

& $P(y_2 | x) = P'(y_2 | x)$

$P(y_1 | x) = P'(y_1 | x)$

$\Leftarrow$ $\exists$ P' s.t.

$$P(y_2|x) = P'(y_2|x)$$

$$= \sum_{y_1} P'(y_1 y_2 | x)$$

$$= \sum_{y_1} P'(y_1|x) \, P'(y_2|y_1, x)$$

$$= \sum_{y_1} p(y_1|x)\, p'(y_2|y_1)$$

Thm: For (stochastically) degraded channel P

(Coding) : (R_1, R_2) acheivable $\Leftarrow \exists U$

$$U \to X \to Y_1 \to Y_2 \quad \text{s.t.}$$

$$R_1 \le I(X; Y_1 | U)$$

$$R_2 \le I(Y_2; U)$$

(Converse) : (R_1, R_2) acheivable $\Rightarrow \exists U$

with $|\Omega_U| \le \min\{ |\Omega_{X}|, |\Omega_{Y_1}|, |\Omega_{Y_2}| \}$

$$U \to X \to Y_1 \to Y_2$$

$$R_1 \le I(X; Y_1 | U)$$

$$R_2 \le I(Y_2; U)$$

(won't prove converse)

Coding Theorem Proof

Encoding E :

Step 1 :

Given W_2 : Generate $U^n = (U_1 \dots U_n)$

$U_i \sim P_U$ i.i.d.

Step 2 :

Given W_1, U^n generate $X^n = (X_1 \dots X_n)$

$X_i \sim P_{X|U}(U_i)$ independently

Decoding

Receiver 1: if $\exists!\, \hat{w}_1, \hat{w}_2$ s.t.

$$\left(U(\hat{w}_2),\ X(\hat{w}_1, \hat{w}_2),\ Y_1^n\right) \text{ jointly typical,}$$

output $\hat{w}_1$ else error

Receiver 2: if $\exists\, \hat{\hat{w}}_2$ s.t.

$\left(U(\hat{\hat{w}}_2),\ Y_2\right)$ jointly typical output $\hat{\hat{w}}_2$ else error.

Analysis:

Receiver 2: Same as $U \to Y_2$ channel.

O.K. if $R_2 \leq$

Receiver 1:

1. (a) $(U(w_2), X(\hat{w}_1, w_2), Y_1)$ typical

$\Pr[\quad] \le 2^{-I(X;Y_1|U)\cdot n}$

(b) $(U(\hat{w}_2), X(\hat{w}_1, \hat{w}_2), Y_1)$ typical

$\Pr[\quad] \le 2^{-I((X,U);Y_1)\cdot n}$

$\rightarrow R_1 \le I(X;Y_1|U)$

$\rightarrow R_1 + R_2 \le I((X,U);Y_1)$

$$= \underbrace{I(X;Y_1|U) + I(U;Y_1)}$$

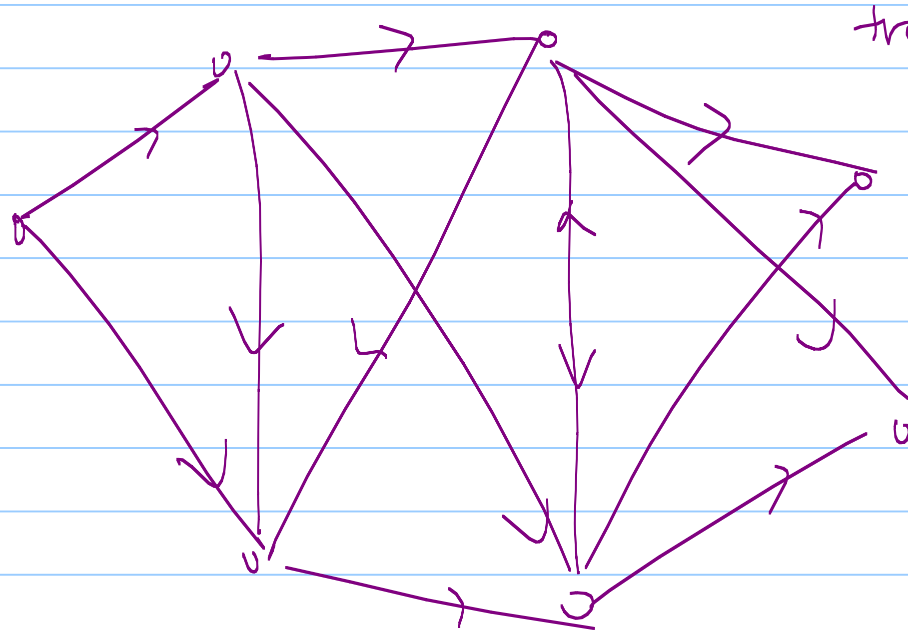
redundant since $R_1 \le I(X;Y_1|U)$

$R_2 \le I(U;Y_2) \le I(U;Y_1)$

NETWORK INFORMATION THEORY

General Setup

Many nodes acting as receivers + transmitters



Intermediate nodes can

1. Store & forward
2. Recompute & forward.

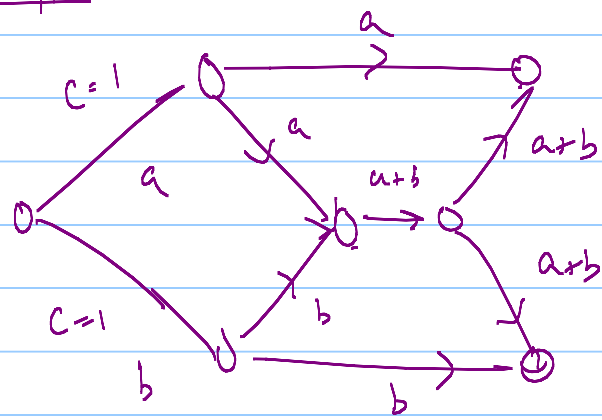
1 $\neq$ 2

Example

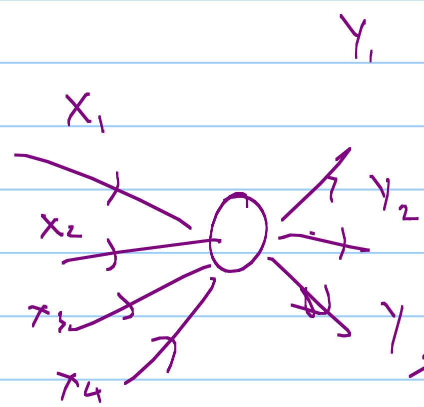
S

$R = 2$ bits

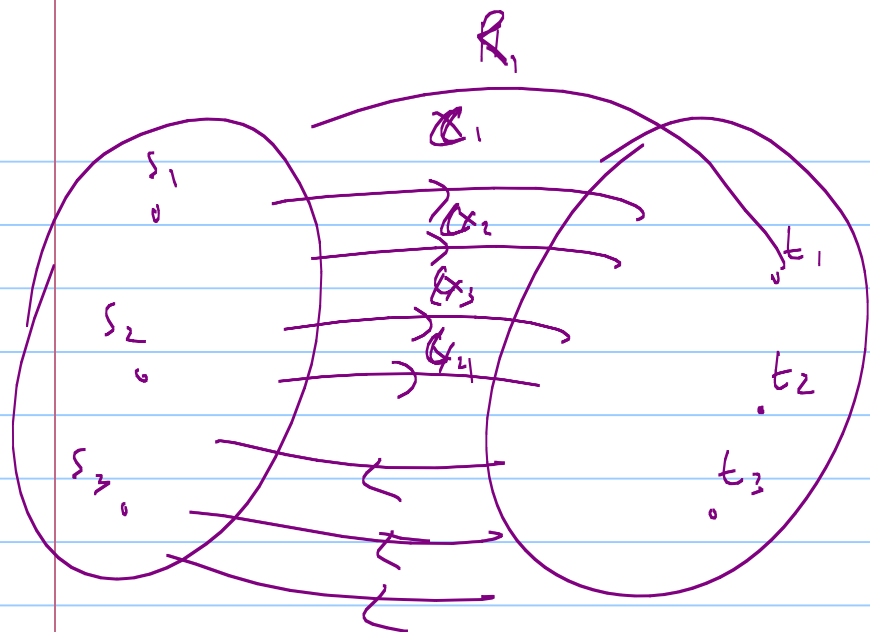
a, b



What can be acheived ?



$$H(Y_1, Y_2, Y_3 \mid X_1 X_2 X_3 X_4) = 0.$$



$$R_1 + R_2 + R_3 \leq C_1 + C_2 + C_3 + C_4$$

Not always acheivable.

Don't even know what kind of entropies acheivable.

$\exists$ given $H_{\{1\}}, H_{\{2\}}, \ldots H_S \ldots H_{\{1 \ldots m\}}$

$S \subseteq \{1 .. m\}$

Does there exist a distribution

$P(x_1 \ldots x_m)$ s.t. $\forall S$

$$H_S = H(\{X_i\}_{i \in S})$$

H_S's satisfy some constraints (Chain Rule)

$$H_{S \cup T} \leq H_S + H_T$$

But other constraints apply! [Li, Yeung ...]

even better

$$H_{S \cup T} + H_{S \cap T} \leq H_S + H_T$$

(See [Zeger's talk].)