Universal Semantic Communication

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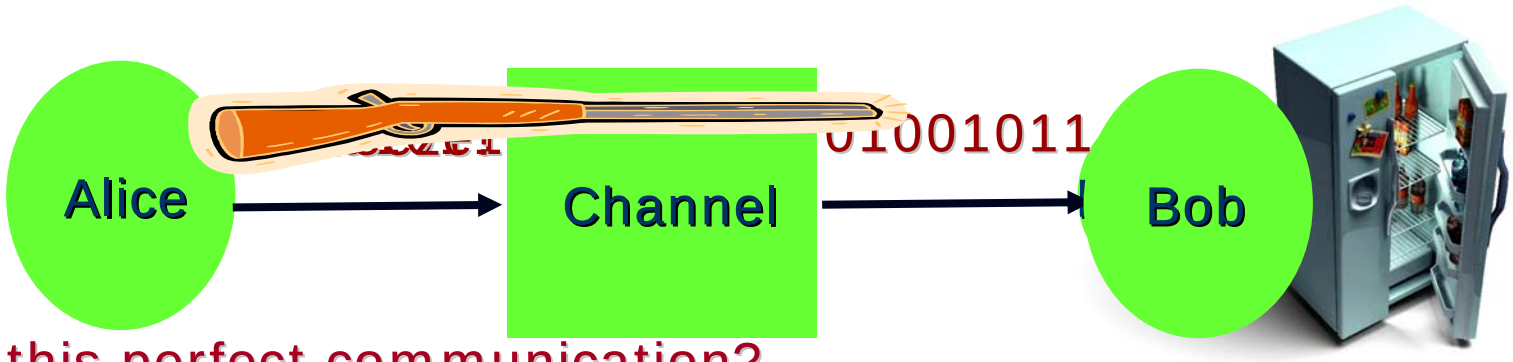
The Meaning of Bits



- Is this perfect communication?
- What if Alice is trying to send instructions?
  - Aka, an algorithm
  - Does Bob understand the correct algorithm?
  - What if Alice and Bob speak in different (programming) languages?
- Sales pitch: Crucial to the future of computing. More and more heterogenous computers, data, and software interact without human in the loop.

Some modelling

- Say, Alice and Bob know different programming languages. Alice wishes to communicate an algorithm to Bob.

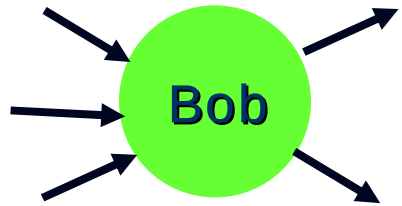
- Bad News: Can't be done
  - For every Bob, there exist algorithms A and A', and Alices, Alice and Alice', such that the two are indistinguishable to Bob.

- Good News: Need not be done.
  - From Bob's perspective, if A and A' are indistinguishable, then they are equally useful to him.

- What should be communicated? Why?

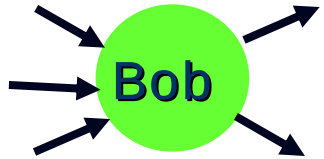
Modelling Bob and his perspective

- Bob: $\Omega \times \Sigma^k \to \Omega \times \Gamma^\ell$,
  where $\Omega =$ countable state space
  $\Sigma^k =$ input signals
  $\Gamma^\ell =$ output signals.



- Bob speaks to some environment (a multitude of entities).
- Why? Has some goal!
  - "Control": Wants to alter the state of the environment.
  - "Intellectual": Wants to glean knowledge (about universe/environment).
- Claim: By studying the goals, can enable Bob to overcome linguistic differences (and achieve goal).

Rest of the talk



- Part I: Bob wishes to solve hard problem, is computationally limited, and Alice can solve the problem.
- Part II: Bob is a teacher and wants to test student's ability.
- Part III: Generic goals.

Part I: A Computational Goal

Computational Goal for Bob

- Bob is prob. poly time bounded. Wants to decide membership in set S.
- Alice is computationally unbounded, does not speak same language as Bob, but is “helpful”.
- What kind of sets S?
  - E.g., undecidable?, decidable? PSPACE, NP, BPP?

Setup

Alice

Bob

$x \in S?$

$R \leftarrow \$\$\$$

Different from IP:

In IP Bob does not trust Alice, while here he does not understand her.

q_k →

$f(x, R, a_1, \ldots, a_k) = 1?$ ← a_k

Hopefully $x \in S \Leftrightarrow f(\cdots) = 1$

Intelligence & Cooperation?

- For Bob to have a non-trivial interaction, Alice must be:
  - Intelligent: Capable of deciding if x in S.
  - Cooperative: Must communicate this to Bob.
- Formally:

Alice is S-helpful
if $\exists$ probabilistic poly time (ppt) Bob B' s.t.
$\forall$ initial state of mind σ,
$A(\sigma) \leftrightarrow B'(x)$ accept w.h.p. iff $x \in S$.

Successful universal communication

- Bob should be able to talk to any S-helpful Alice and decide S.

- Formally,

  Ppt B is S-universal if for every $x \in \{0,1\}^*$

  – A is S-helpful $\Rightarrow$ $[A \leftrightarrow B(x)] = 1$ iff $x \in S$ (whp).

  A is not S-helpful $\Rightarrow$ Nothing!!

Or should it be ...

A is not S-helpful $\Rightarrow [A \leftrightarrow B(x)] = 1$ implies $x \in S$.

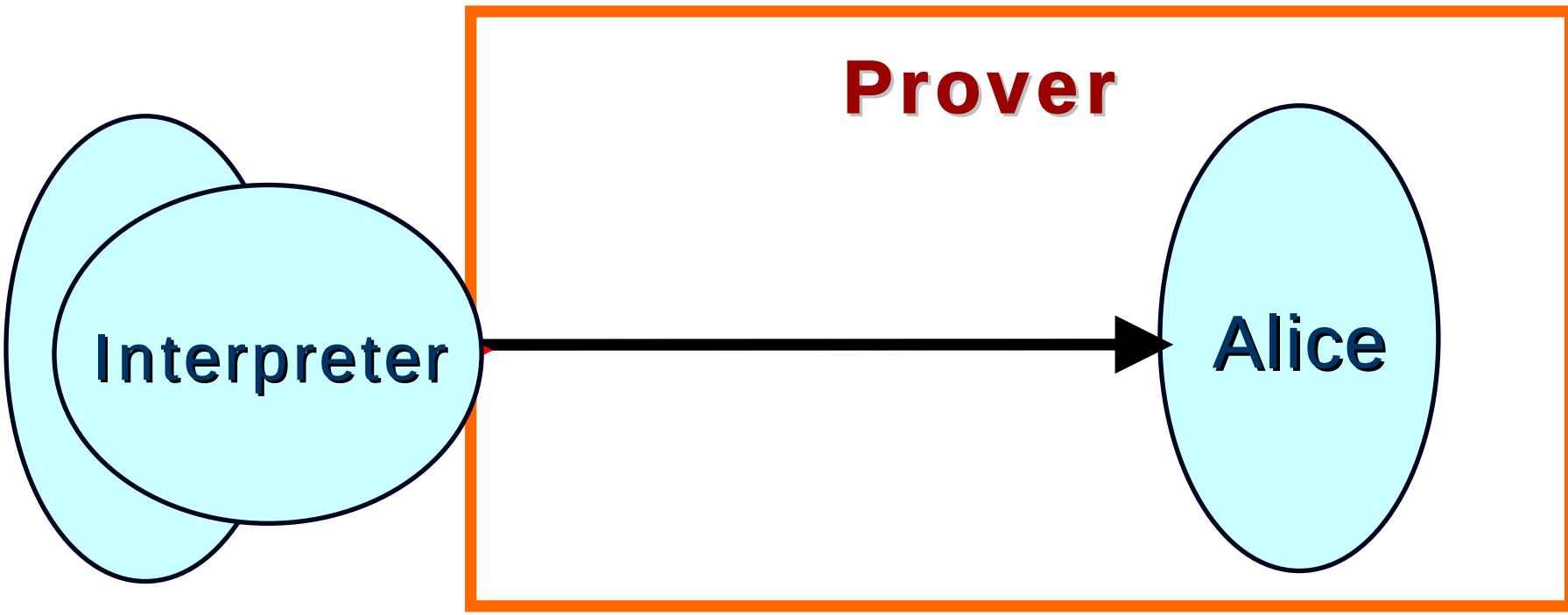
Main Theorem

- If S is PSPACE-complete, then there exists a S-universal Bob (generalizes to other checkable sets S).
- Conversely, if there exists a S-universal Bob, then S is in PSPACE.
- In other words:
  - If S is moderately stronger than what Bob can do on his own, then attempting to solve S leads to non-trivial (useful) conversation.
  - If S too strong, then leads to ambiguity.
  - Uses IP=PSPACE

Few words about the proof

- Positive result: Enumeration + Interactive Proofs

Guess: Interpreter; $x \in S$?



Proof works $\Rightarrow x \in S$; Doesnt work $\Rightarrow$ Guess wrong.

Alice S-helpful $\Rightarrow$ Interpreter exists!

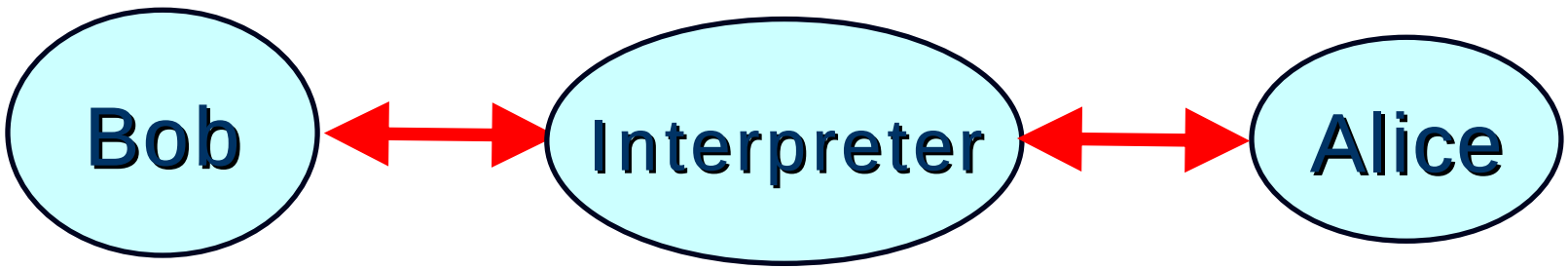
Proof of Negative Result

- L not in PSPACE implies Bob makes mistakes.
  - Suppose Alice answers every question so as to minimize the conversation length.
    - (Reasonable effect of misunderstanding).
  - Conversation comes to end quickly.
  - Bob has to decide.
  - Conversation + Decision simulatable in PSPACE (since Alice's strategy can be computed in PSPACE).
  - Bob must be wrong if S is not in PSPACE.
  - Warning: Only leads to finitely many mistakes.

Part II: Intellectual Curiosity

Setting: Bob more powerful than Alice

- What should Bob's Goal be?
  - Can't use Alice to solve problems that are hard for him.
  - Can pose problems and see if she can solve them. E.g., Teacher-student interactions.
  - But how does he verify "non-triviality"?
  - What is "non-trivial"? Must distinguish ...



Scene 2

Setting: Bob more powerful than Alice

- Concretely:
  - Bob capable of TIME(n^{10}).
  - Alice capable of TIME(n^3) or nothing.
  - Can Bob distinguish the two settings?
- Definition:

Alice is $n^{3-\epsilon}$-helpful

if $\exists$ Bob $B' \in \text{TIME}(n^{3-\epsilon})$ s.t.

$\forall\ S \in \text{TIME}(n^3)$, and $\forall$ initial state of mind σ,

$A(\sigma) \leftrightarrow B'(x_1, \ldots, x_n)$ computes $S(x_1), \ldots, S(x_n)$.

:

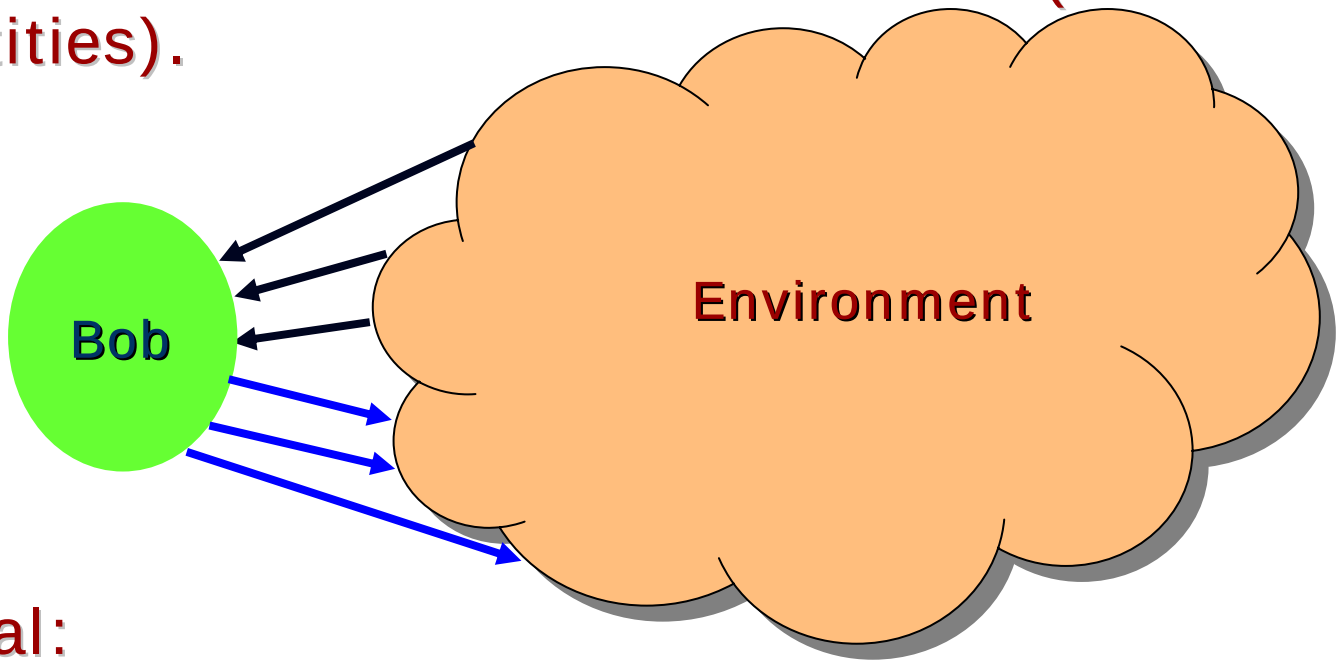
- Theorem: There exists a universal Bob that distinguishes helpful Alices from trivial ones.

- Moral: Language (translation) should be simpler than problems being discussed.

Part III: Generic Goals

Generically

- Bob interacts with an environment (collection of entities).

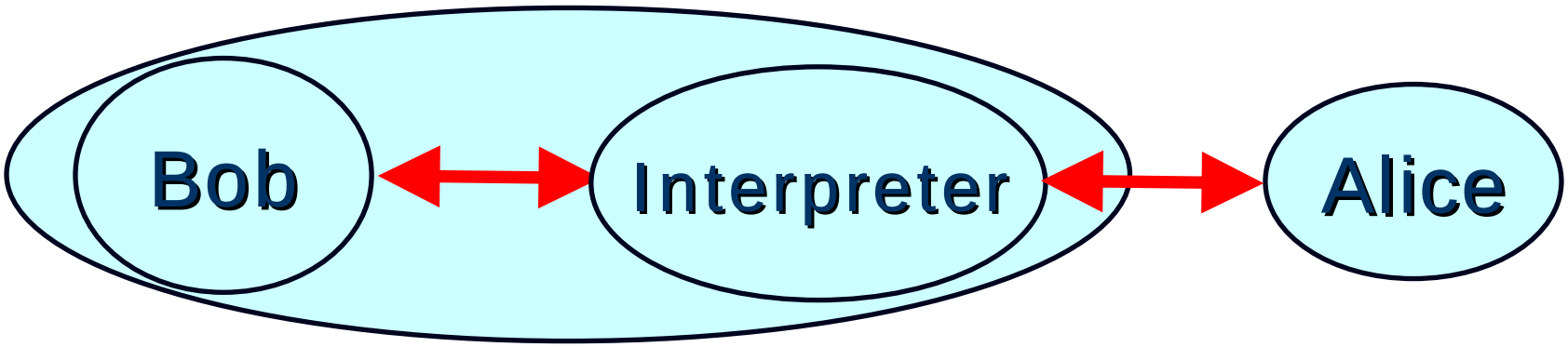


- Goal:
  - Function of transcript of interaction
  - States of environment!
  - But not Bob itself!
  - Should forgive finite prefixes.

Generic Helpfulness, Universality

- Consider a class of Alices $\mathcal{A}$, a class of Bobs $\mathcal{B}$, and some goal G
  - (G, $\mathcal{B}$)-Helpful: Helpful for some Bob in $\mathcal{B}$.
  - (G, $\mathcal{A}$)-Universal: Works with all Alices in $\mathcal{A}$.

- $\mathcal{A}$-Verifiable protocol: For every A in $\mathcal{A}$, Protocol accepts iff goal is achieved.



- Theorem: Verifiable Goals can be achieved universally.

Conclusions

- Communication of “meaning/context” is feasible; provided goals are explicit.
- Verifying “goal achievement” for non-trivial goals is the (only?) way to learn languages.
- Currently the learning is slow … is this inherent?
  - Better class of Alices?
- What are interesting goals, and how can they be verified?

Thank You!