Communication Amid Uncertainty

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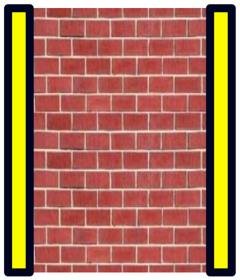
Based on
- Juba, S. (STOC 2008, ITCS 2011)
- Goldreich, Juba, S. (JACM 2011)
- Juba, Kalai, Khanna, S. (ITCS 2011)
- Haramaty, S. (ITCS 2014)
- Canonne, Guruswami, Meka, S. (ITCS 2015)
- Leshno, S. (manuscript)

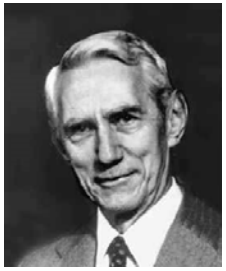
Communication vs. Computation

- Interdependent technologies: Neither can exist without other
- Technologies/Products/Commerce developed (mostly) independently.
  - Early products based on clean abstractions of the other.
  - Later versions added other capability as afterthought.
  - Today products … deeply integrated.
- Deep theories:

Well separated … and have stayed that way

Turing '36



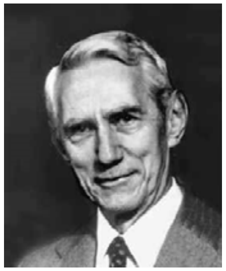


Shannon '48

Consequences of the wall

- Computing theory:
  - Fundamental principle = Universality
  - You can program your computer to do whatever you want.
  - ⇒ Heterogeneity of devices

- Communication theory:
  - Centralized design (Encoder, Decoder, Compression, IPv4, TCP/IP).
  - You can NOT program your device!
  - ⇐ Homogeneity of devices



- Contradiction! But does it matter?
  - Yes!

Sample problems:

- Universal printing:
  - You are visiting a friend. You can use their Wifi network, but not their printer. Why?
- Projecting from your laptop:
  - Machines that learn to communicate, and learn to understand each other.
- Digital libraries:
  - Data that lives forever (communication across time), while devices change.

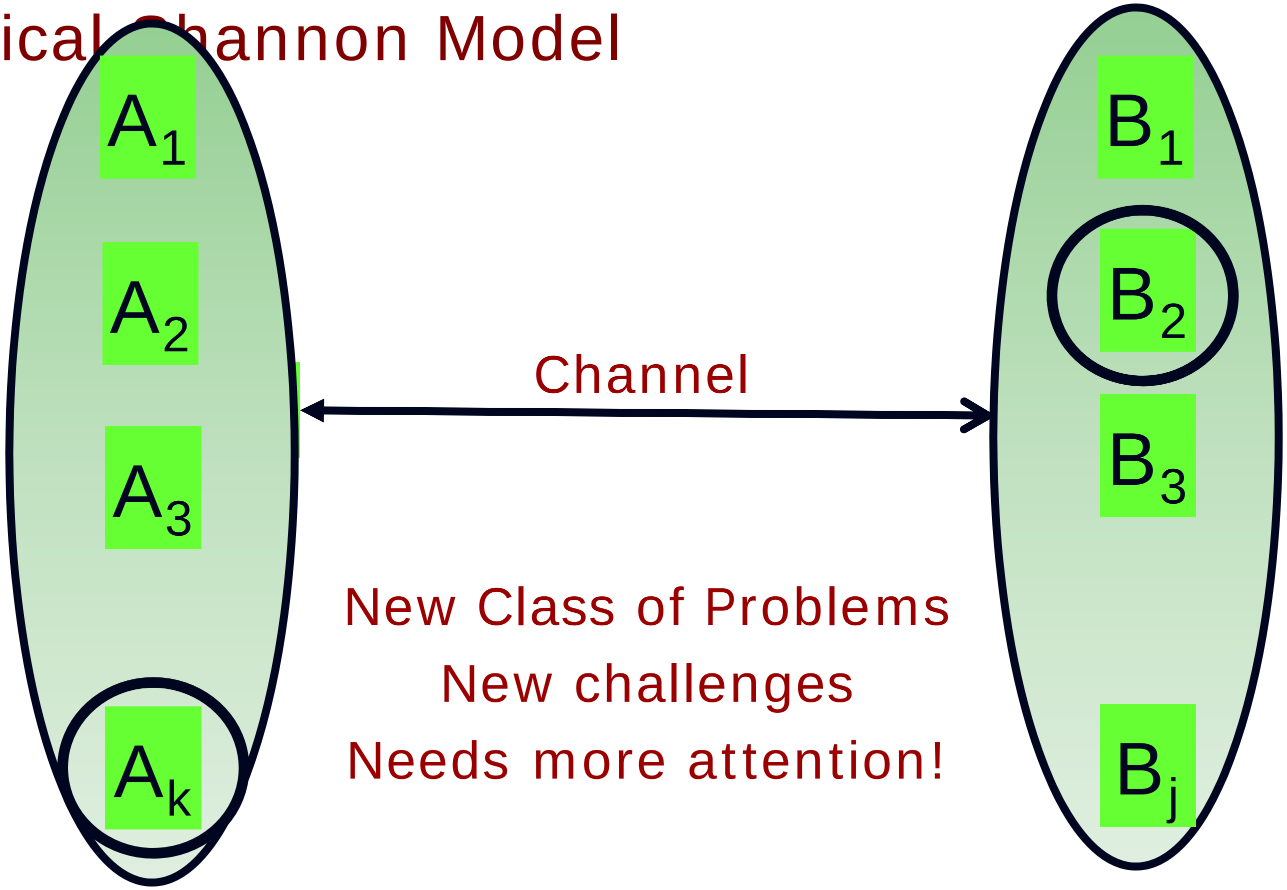
Essence of "semantics": Uncertainty

- Shannon:
  - *"The significant aspect is that the actual message is one selected from a set of possible messages"*
- Essence of unreliability today:
  - Context: Determines set of possible messages.
    - dictionary, grammar, general knowledge
    - coding scheme, prior distribution, communication protocols …
  - Context is HUGE; and not shared perfectly;

Modelling uncertainty

Uncertain Communication Model

Classical Shannon Model



Hope

- Better understanding of existing mechanisms
  - In natural communication
  - In “ad-hoc” designs

- What problems are they solving?

- Better solutions?
  - Or at least understand how to measure the quality of a solution.

II: Uncertain Compression

Human-Human Communication

$$\begin{aligned} M_1 &= w_{11}, w_{12}, \ldots \\ M_2 &= w_{21}, w_{22}, \ldots \\ M_3 &= w_{31}, w_{32}, \ldots \\ M_4 &= w_{41}, w_{42}, \ldots \\ &\ldots \end{aligned}$$

- Role of dictionary = ?
  - [Juba, Kalai, Khanna, S. 11]
- Dictionary: list of words representing message
  - words appear against multiple messages
  - multiple words per message.
- How to decide which word to use? Context!
  - Encoding: Given message, use shortest unambiguous word in current context.
  - Decoding: Given word, use most likely message in current context, (among plausible messages)
- Context = ???. Prob. distribution on messages

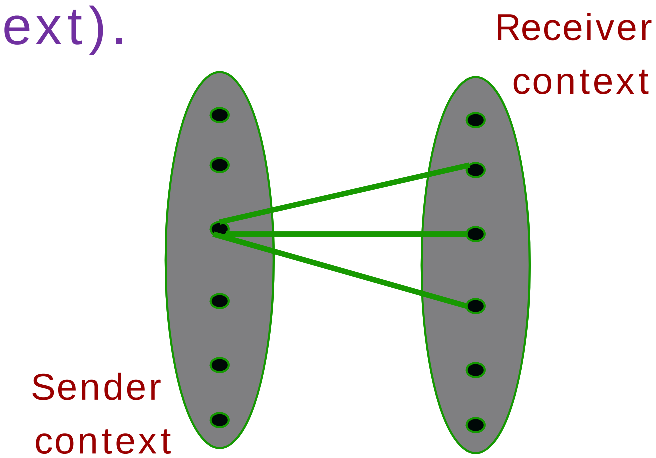
$$P_i = \text{Prob}\,[\text{message} = M_i]$$

$$
\begin{aligned}
M_1 &= w_{11}, w_{12}, \ldots \\
M_2 &= w_{21}, w_{22}, \ldots \\
M_3 &= w_{31}, w_{32}, \ldots \\
M_4 &= w_{41}, w_{42}, \ldots \\
&\ldots
\end{aligned}
$$

Human Communication - 2

- Good (Ideal?) dictionary
  - Should compress messages to entropy of context: $H(P = \langle P_1, \ldots, P_N \rangle)$.
- Even better dictionary?
  - Should not assume context of sender/receiver identical!
  - Compression should work even if sender **uncertain** about receiver (or receivers' context).

Theorem [JKKS]: If dictionary is "random" then compression achieves message length $H(P) + \Delta$, if sender and receiver distributions are "Δ-close".



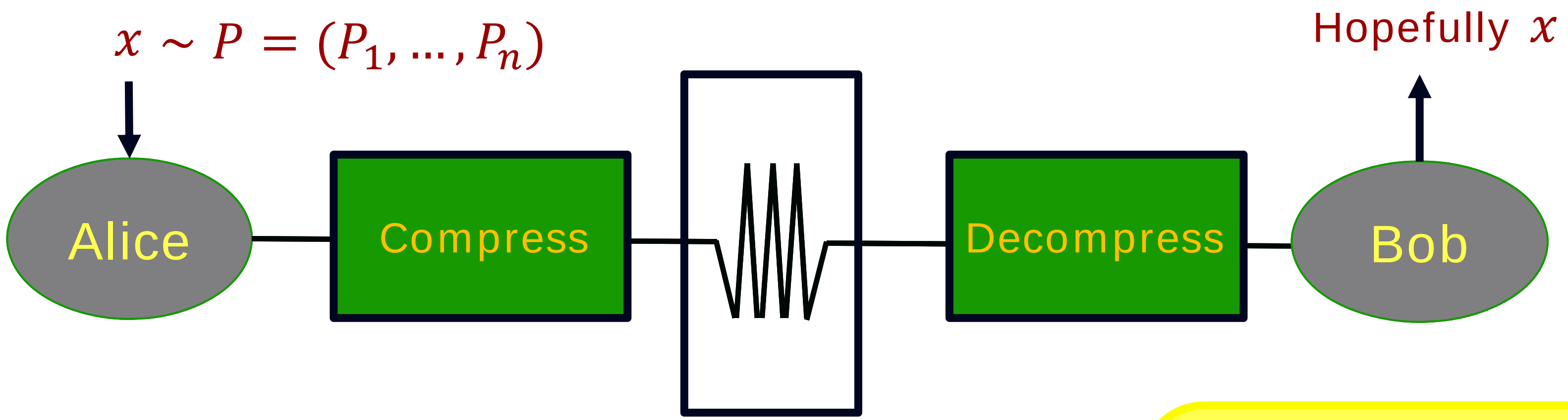
Implications

- Reflects tension between ambiguity resolution and compression.
  - Larger the gap in context (Δ), larger the encoding length.
- Coding scheme reflects human communication?
- "Shared randomness'' debatable assumption:
  - Dictionaries do have more structure.
  - Deterministic communication? [Haramaty+S,14]
  - Randomness imperfectly shared? Next ...

III: Imperfectly Shared Randomness

Communication (Complexity)

- Compression (Shannon, Noiseless Channel)

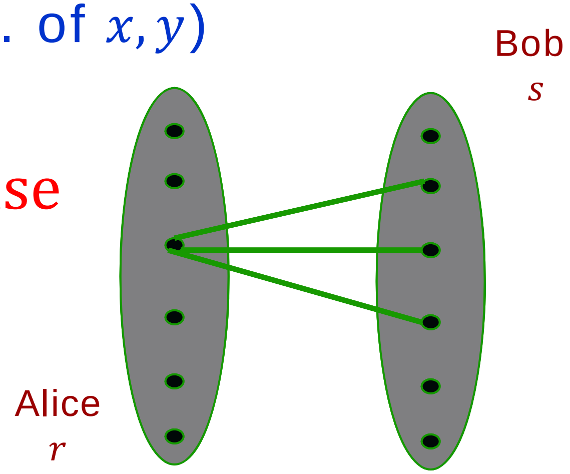


- What will Bob do with x?
  - Often knowledge of x is overkill.
  - [Yao]'s model:
    - Bob has private information y.
    - Wants to know $f(x, y) \in \{0,1\}$.
    - Can we get away with much less communication?

In general, model allows interaction. For this talk, only one way comm.

Brief history

- ∃ problems where Alice can get away with much fewer bits of communication.
  - Example: $\oplus(x, y) \triangleq \oplus_i (x_i \oplus y_i)$
  - But very few such deterministically.
- Enter Randomness:
  - Alice & Bob share random string r (ind. of x, y)
  - Many more problems; Example:
    - $\mathrm{Eq}(x, y) = 1$ if $x = y$ and 0 otherwise
      - Deterministically: $\Theta(n)$
      - Randomized: $O(1)$
- Uncertainty-motivated question:
  - Does randomness have to be perfectly shared?



Results

- [Newman '90s]:

  CC without sharing $\leq CC$ with sharing $+ \log n$
- But additive cost of $\log n$ may be too much.
  - Compression! Equality!!
- Model recently studied by [Bavarian et al.'14]
  - Equality: $O(1)$ bit protocol w. imperfect sharing
- Our Results: [Canonne, Guruwami, Meka, S.'15]
  - Compression: $O(H(P) + \Delta)$
  - Generally: k bits with shared randomness

    $\Rightarrow 2^k$ bits with imperfect sharing.
  - $k \rightarrow 2^k$ loss is necessary.

Some General Lessons

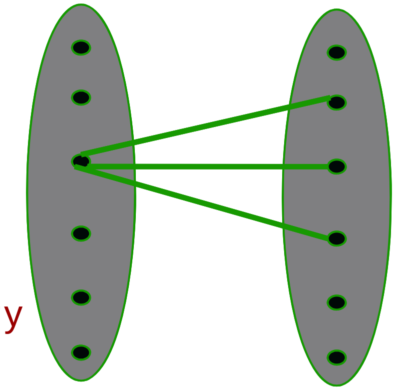
- Compression Protocol:
  - Adds “error-correction” to [JKKS] protocol.
    - Send shortest word that is far from words of other high probability messages.
    - Another natural protocol.
- General Protocol:
  - Much more “statistical”
    - Classical protocol for Equality:
      - Alice sends random coordinate of ECC(x)
    - New Protocol
      - ~ Alice send # 1’s in random subset of coordinates.

IV: Coordination

Communicate meaning?

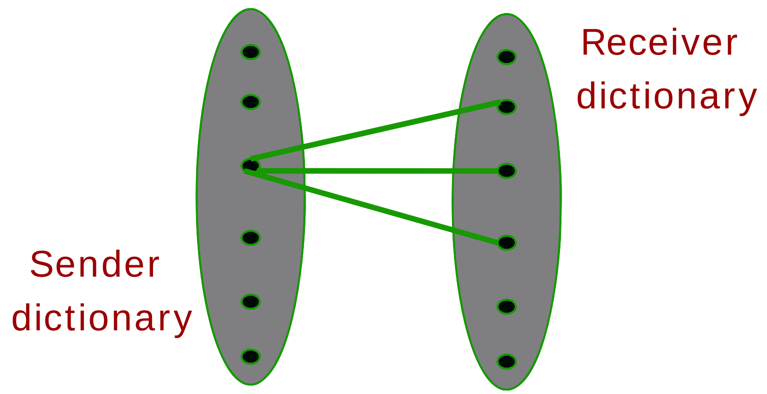
- Ultimate goal:
  - Message ⇒ Instructions.
    - What is this dictionary?
    - Can it be learned by communication?
- At first glance:
  - Ambiguity can never be resolved by communication (even a theorem [JS'08]).
- Second look:
  - Needs more careful definitions.
    - Meaning = mix of communication + actions + incentives.

(Mis) Understanding?



- Uncertainty problem:
  - Sender/receiver disagree on meaning of bits
- Definition of Understanding?
  - Sender sends instructions; Receiver follows?
    - Errors undetectable (by receiver)
    - Not the right definition anyway:
      - Does receiver want to follow instructions
      - What does receiver gain by following instructions? Must have its own "Goal"/"Incentives".
- [Goldreich,Juba,S. 2012]: Goal-oriented communication:

(Mis) Understanding?



- Uncertainty problem:
  - Sender/receiver disagree on meaning of bits
- Definition of Understanding?
  - Receiver has goals/incentives.
- [Goldreich,Juba,S. 2012]: Goal-oriented communication:
  - Define general communication problems (and goals)
  - Show that if
    - Sender can help receiver achieve goal (from any state)
    - Receiver can sense progress towards goal
  - then
    - Receiver can achieve goal.
- Functional definition of understanding.

Illustration: (Repeated) Coordination

- [Leshno,S.]
- Basic Coordination Game:
  - Alice and Bob simultaneously choose actions $\in \{0,1\}$
  - If both pick same action, both win.
  - If they pick opposite actions, both lose.
- Main challenge: Don't know what the other person will choose when making our choice.
- Repeated version:
  - Play a sequence of games, using outcome of previous games to learn what the other player may do next.
  - Goal: Eventual perpetual coordination.

Our setting

- Repeated coordination game with uncertainty:
  - Bob's perspective:
    - Knows his payoffs – 1 for coord.; 0 for not.
    - Does not know Alice's payoffs (uncertainty):
      - May vary with round
      - But for every action of Alice, payoff does not decrease if Bob coordinates (compatibility).
      - Knows a set S_A of strategies she may employ ("reasonable behaviors").
    - Can he learn to coordinate eventually?

Coordination with Uncertainty

- Mixes essential ingredients:
  - Communication: Actions can be use to communicate (future actions).
  - Control: Communication (may) influence future actions.
  - Incentives:
    - Bob has incentive to coordinate.
    - Alice not averse.
- What do the general results say?
  - ∃ Universal strategy U s.t.
    - ∀ Alice s.t. ∃ Bob who coordinates with Alice from any state.
    - U coordinates with Alice.

Lessons

- Coordination is possible:
  - Even in extreme settings where
    - Alice has almost no idea of Bob
    - Bob has almost no idea of Alice
    - Alice is trying to learn Bob
    - Bob is trying to learn Alice
- Learning is slow ...
  - Need to incorporate beliefs to measure efficiency. [Juba, S. 2011]
  - Does process become more efficient when languages have structure? [Open]

Conclusions

- Context in communication:
  - White elephant in the room.
    - Huge, unmentionable, weighing us down.
- Context usually imperfectly shared.
- Uncertainty + Scale $\Rightarrow$ New class of problems.
- What are new “error-correcting” mechanisms?
  - Can be build reliability on top of unreliability?

Thank You!